String-Scale Baryogenesis

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Abstract

Baryogenesis scenarios at the string scale are considered. The observed baryon to entropy ratio, $n_B/s \sim 10^{-10}$, can be explained in these scenarios.

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1 Introduction

Baryogenesis [1] is one of the important problems in particle physics and cosmology. Why is our universe made of matter, not anti-matter? How do we explain the observed value of the ratio $n_B/s \sim 10^{-10}$, where $n_B$ is the difference between the number density of baryons and that of anti-baryons, and $s$ is the entropy density? Three ingredients are necessary to dynamically generate a nonzero $n_B$ from a baryon-symmetric initial state [2]:

1. baryon number nonconservation,
2. violation of both C and CP invariance,
3. departure from thermal equilibrium.

Baryogenesis scenarios at the electroweak scale have been studied recently [3]. Baryon number conservation is violated at the electroweak scale via sphaleron processes. However, it is difficult to generate the observed baryon to entropy ratio within the Minimal Standard Model (MSM). First of all, CP violation coming only from the CKM phase is too small to explain the observed value, even if thermal plasma effects are taken into account [4]. Secondly, the electroweak phase transition should be a strong first-order phase transition in order to avoid the wash-out problem. However, this requirement gives an upper bound on the Higgs boson mass [5], which is already ruled out by LEP experiments.

Within the context of grand unified theories (GUT's) and the expanding universe all three necessary conditions are satisfied. However, GUT’s also predict super-heavy magnetic monopoles which lead to a serious cosmological problem. Another problem of GUT baryogenesis scenarios is that the baryon asymmetry produced at the GUT scale is washed out by sphaleron processes.

In this paper we will consider baryogenesis scenarios at the string scale or the Planck scale and show how the observed baryon to entropy ratio can be explained quite easily in these new scenarios. Even if the non-SUSY non-GUT MSM describes the nature well above the electroweak scale, it must be modified around the string scale or the Planck scale due to gravitational effects. Hence it is important to consider the baryogenesis scenarios at these scales.

At the string and Planck scales, the three necessary conditions for baryogenesis are satisfied. Let’s consider string inspired models or effective theories with a cut-off at the string scale. There is no reason to prohibit baryon- or lepton-number violating interactions in
theories with a cut-off. On the other hand, the MSM, which is required to be renormalizable
and gauge invariant, does not allow such interactions. Sources of CP violation at the string
scale can differ from those at the electroweak scale, and other sources than the CKM phase
are allowed at the string scale.

As for departure from thermal equilibrium, we consider two scenarios which cause nonequi-
librium distributions of matter. The first uses the so called Hagedorn temperature [6, 7].
String theory has a limiting temperature, where the higher excited states of string theory
are occupied. The decay processes of these states will cause nonequilibrium distributions.
The other is the inflation scenario [8], where inflaton decay processes cause nonequilibrium.

In this paper we will mainly consider the Hagedorn scenario. Nonzero $n_B$ is generated
during the decay of the higher excited states. It is also generated after the decay since
nonequilibrium distributions caused by the decay processes are maintained until the rates
for thermalization processes dominate the Hubble expansion rate.

The resultant baryon to entropy ratio will not have suppression factors since the theory
has only one scale, the string scale or the Planck scale. Hence, we expect the observed value
is obtained in these scenarios.

In Section 2 we present a model and show how it satisfies the three conditions for baryo-
genesis. In Section 3 we calculate the resultant lepton asymmetry by considering Boltzmann
equations and show that these scenarios can explain the observed baryon to entropy ratio.
The last section is devoted to conclusions and discussions.

2 A Model

In this section we will present a model of string scale baryogenesis. There has been
progress in the study of string models without SUSY or GUT recently [9], which we think
are interesting. Hence, as an effective theory of string theory, we consider a model whose
matter content is the same as that of the MSM. For simplicity, we consider lower-dimensional
operators. Let’s consider the following model.

$$
\mathcal{L} = \mathcal{L}_{\text{MSM}} + \frac{1}{4} \frac{g_{st}^2}{m_{st}} h_{ij} \epsilon^{\alpha^\beta} (\epsilon_{ab} \epsilon_{cd} + \epsilon_{ad} \epsilon_{cb}) l^i_{\alpha^a} \phi^b \phi^c + h.c.
$$
\[
\frac{1}{4} \frac{g_{st}^2}{m_{st}^2} \epsilon^{\alpha \beta} \epsilon^{\gamma \delta} [C_{ijkl} (\delta_{ac} \delta_{bd} + \delta_{ad} \delta_{bc}) + C'_{ijkl} (\delta_{ac} \delta_{bd} - \delta_{ad} \delta_{bc}) + C''_{ijkl} \epsilon_{ab} \epsilon_{cd}] \\
\times l^a_\alpha l^b_\beta l^c_\gamma l^d_\delta, \tag{2.1}
\]

where \( l \)'s are the lepton doublets and \( \phi \) is the Higgs doublet. The string coupling constant and the string scale are \( g_{st} \) and \( m_{st} \), respectively. Also, \( i, j, k \) and \( l \) are generational indices, \( \alpha, \beta, \gamma \) and \( \delta \) are spinorial indices, and \( a, b, c \) and \( d \) are \( SU(2)_L \) gauge-group indices.

The first term violates lepton number conservation since it is a Majorana-type coupling. The second term is included to incorporate CP violation. Through a unitary transformation the coefficient of the first term, \( h_{ij} \), can be rotated to real diagonal form. However, if the second term is present we cannot guarantee that both terms can be rotated to real form simultaneously. With two or more generations, this model violates CP invariance. Henceforth, we will consider exactly two generations for simplicity.

In the remainder of this section we will speculate on the departure from thermal equilibrium. First of all, let’s consider how the universe would have been in the context of string theory if thermal equilibrium had been maintained \([6, 7]\). Since the density of states is exponentially rising in string theory, it has a limiting temperature, the Hagedorn temperature,

\[
T_H = \begin{cases} 
\frac{m_{st}}{2 \sqrt{2} \pi} \sim 5.93 \times 10^{16} \text{GeV} & \text{(type II)}, \\
\frac{m_{st}}{(2 + \sqrt{2}) \pi} \sim 4.92 \times 10^{16} \text{GeV} & \text{(heterotic string)}. 
\end{cases} \tag{2.2}
\]

While the universe is contracting, the energy density increases but the temperature remains just below the Hagedorn temperature. When the energy density is low compared to the string scale, matter is composed of particles, or the excitations of short strings whose lengths are of the order of \( m_{st}^{-1} \). However, the high-energy limit of the single-string density of states is found to be \([7]\)

\[
\omega(\epsilon) = \frac{\exp(\epsilon/T_H)}{\epsilon}. \tag{2.3}
\]

When we consider the microcanonical ensemble it turns out that long strings traverse the entire volume of the universe in sufficiently high energy density.

Next we consider whether the thermal equilibrium is actually realized by comparing the rates for thermalization processes with the Hubble expansion rate. When matter is composed of particles, the rates for thermalization processes are \( \Gamma_{th} \sim g_\star \alpha^2 T \), and the Hubble expansion rate is \( H = 1.66 g_\star^{1/2} T^2 / m_{pl} \), where \( m_{pl} \) and \( g_\star \) are the Planck mass and the number of matter species, respectively, and \( \alpha = g_{st}^2 / 4\pi \). Hence, above the temperature \( T_{eq} \sim g_\star^{1/2} \alpha^2 m_{pl} / 1.66 \sim 3.8 \times 10^{16} \text{GeV} \), or above the energy density \( \rho_{eq} \sim (1.6 \times 10^{17} \text{GeV})^4 \),
the thermalization processes are too slow to maintain equilibrium distributions. However, in
the long-string phase, the rates for thermalization processes are proportional to the densities
of the string-bits, $E/V$, where $E$ and $V$ are the total energy and the volume of the universe,
respectively. Since the Hubble expansion rate is proportional to the square root of the
energy density, $\sqrt{E/V}$, the rates for thermalization processes will dominate the Hubble rate
for sufficiently high energy density. Therefore, there will be a critical energy density, $\rho_*$, above
which equilibrium distributions are realized. Below $\rho_*$ interactions freeze out and
matter distributions are merely affected by the expansion of the universe and depart from
thermal equilibrium distributions.

Decay processes of higher excited states begin when the energy density decreases to
$\rho_{\text{decay}} \sim (3.7 \times 10^{17}\text{GeV})^4$, where the decay rates $\sim \alpha m_{\text{st}}$ dominate the Hubble expansion
rate. These processes are not adiabatic since the matter distributions have departed from
equilibrium ones.

During the decay processes of higher excited states, the baryon asymmetry as well as
entropy is generated. We can make a rough estimate,
\[ n_B/s \sim \alpha^2 \sim 10^{-3}, \] (2.4)
since the first nontrivial contribution to CP violation comes from the interference of the
lowest-order diagrams and the one-loop corrections.

However, if many processes occur at $\rho = \rho_{\text{decay}}$, some cancellations among the decay
processes can decrease the above result. These cancellations might be possible if many
excited states are taken into account since ten-dimensional superstring theory has no CP
violation originally. They might be explained also since CPT invariance and unitarity assures
the following relation: $\sum_X \Gamma(X \to b) = \sum_X \Gamma(X \to \bar{b})$.

We cannot make a precise estimate since we don’t know the dynamics of the decay
processes in detail. Hence let’s consider the following case: Nonzero $n_B$ is generated after the
decay while it is not generated during the decay due to exact cancellation. By considering this
case we can give a lower bound on $n_B/s$. We assume that the following matter distributions
are caused by the decay processes:
\[ n_{\phi} = n_{\phi^*} \neq n_l = n_{l^*} \quad (T = T_H), \] (2.5)
where $n_{\phi}$ and $n_l$ are the number densities of Higgs bosons and lepton doublets, respectively.
These nonequilibrium distributions are maintained until the temperature of the universe
decreases to $T_{\text{eq}}$. During this epoch a nonzero $n_L$ is generated. This nonzero $n_L$ will be converted into a nonzero $n_B$ of the same order of magnitude via sphaleron processes. In the next section, we will estimate the resultant lepton asymmetry by using the nonequilibrium distribution of Eqn.(2.5) as an initial condition.

Finally, we will make a brief comment on another scenario which causes nonequilibrium distributions, i.e., inflation. While the inflaton decays, a nonzero $n_B$ as well as entropy can be generated [8]. Even if a nonzero $n_B$ is not generated during the decay processes, some nonequilibrium distributions like those of Eqn.(2.5) are generated. Then a nonzero $n_B$ is generated after the inflaton decays.

3 Boltzmann Equations

In this section we will calculate the resultant lepton asymmetry by using the model of Eqn.(2.1) and the nonequilibrium distributions of Eqn.(2.5). We consider the processes, $ll \leftrightarrow \phi^*\phi^*$ and processes related by CP conjugation. The first nonzero contributions to the generation of nonzero $n_L$ come from the interference terms of the tree-level amplitudes and the one-loop corrections shown in Fig.1. They are proportional to $\mathcal{I}(\sum_{k,l} h^*_{ij}C_{ijkl}h_{kl})\mathcal{I}(I)$, where $I$ is the factor coming from the loop integrations. However, they vanish if we naively sum over the indices for generations $i,j$, since $C_{ijkl}^* = C_{klij}$ as is evident form the Lagrangian of Eqn.(2.1). Hence, we consider the processes shown in Fig.2 in order to produce a different number density for generation 1 and generation 2.
For simplicity, we will make the following assumption: the distributions for matter remain near equilibrium, $\rho \sim \exp\left(\frac{E - \mu}{T}\right)$ and $\mu \ll T$. Thus the Boltzmann equations for the above processes are as follows:

\[ \dot{Y}_l + \Gamma_{\text{th}}(Y_l - 1) = 0, \]
\[ \dot{Y}_\phi + \Gamma_{\text{th}}(Y_\phi - 1) = 0, \]
\[ Y_{1-2} + \Gamma_{\text{th}}Y_{1-2} = \frac{3}{2}\left(\frac{g_{st}^2}{m_{st}}\right)^2(h_{11}^2 - h_{22}^2)T^3(Y_\phi^2 - Y_l^2), \]

where $Y_i = n_i/n^{(\text{eq})}$. $Y_{1-2} = Y_{l1} - Y_{l2}$, $Y_L = Y_{l1} + Y_{l2} - (Y_{l1}^* + Y_{l2}^*)$. $\Gamma_{\text{th}} \approx g_\ast \alpha^2 T$ is the rate for thermalization. We took the convention where the coefficient of the Majorana-type interactions, $h_{ij}$, is real diagonal form.

Eqns.(3.1) represent the thermalization processes which reduce the nonequilibrium distributions of Eqn.(2.5) imposed as an initial condition to the equilibrium ones. Eqn.(3.2) represents the processes which produce the difference in the number densities between the generations. Finally, the third equation, Eqn.(3.3), represents the production of a nonzero $n_L$. The right-hand sides of Eqns.(3.2) and (3.3) are given by calculating the amplitudes shown in Fig.2 and Fig.1, respectively, and performing the phase space integrations.

The Boltzmann equations are integrated to give

\[ Y_L = -\frac{27}{2\pi} \frac{g_{st}^{10}}{m_{st}^6} \left( \frac{m_{pl}}{1.66g_\ast^{1/2}} \right)^2(h_{11}^2 - h_{22}^2)h_{11}h_{22}\Im(C_{1122}) \]
\[ \times \int_{T_H}^{T} dT' T'^2 Y_l(T') \exp(-T_{\text{eq}}/T') \]
\begin{equation}
\times \int_{T_H}^{T'} dT'' \exp(T_{eq}/T'')(Y_\phi(T'')^2 - Y_i(T'')^2), \tag{3.4}
\end{equation}

where

\begin{equation}
Y_i(T) = 1 + (Y_i(T_H) - 1) \exp(-T_{eq}/T + T_{eq}/T_H), \tag{3.5}
\end{equation}

for \( i = l, \phi \), and \( T_{eq} = g_*^{1/2} \alpha^2 m_{pl}/1.66 \sim 3.75 \times 10^{16} \text{GeV} \) is the temperature below which thermalization processes begin. Note that the Hubble expansion rate appears when we change the variable from time to temperature. The final result is as follows:

\begin{equation}
Y_L = -\frac{27}{2\pi} \frac{1}{12} \frac{g_{st}^{10}}{(1.66 g_*^{1/2})^2} \left( \frac{m_{pl}}{m_{st}} \right)^2 \left( \frac{T_H}{m_{st}} \right)^4 J \tag{3.6}
\end{equation}

\begin{equation}
\approx 8.4 \times 10^{-8} J, \tag{3.7}
\end{equation}

where

\begin{equation}
J = (h_{11}^2 - h_{22}^2) h_{11} h_{22} \Im(C_{1122}) K, \tag{3.8}
\end{equation}

\begin{equation}
K = 12(T_{eq}/T_H)^4 \int_{T_{eq}/T_H}^{\infty} dz z^{-4} (\exp(-z) + A_i \exp(-2z)) \times \int_{T_{eq}/T_H}^z dz' z'^{-2} (2(A_\phi - A_l) + (A_\phi^2 - A_l^2) \exp(-z')) \tag{3.9}
\end{equation}

\begin{equation}
\approx 0.538(A_\phi - A_l) + 0.173 A_i (A_\phi - A_l) + 0.108(A_\phi^2 - A_l^2) + 0.0355 A_i (A_\phi^2 - A_l^2), \tag{3.10}
\end{equation}

\begin{equation}
A_i = (Y_i(T_H) - 1) \exp(T_{eq}/T_H). \tag{3.11}
\end{equation}

\begin{equation}
(3.12)
\end{equation}

In the estimations of Eqns.(3.7) and (3.10) we used the following values: \( g_{st}^2/4\pi = 1/45 \), \( g_* = 106.75 \), \( m_{pl} = 1.22 \times 10^{19} \text{GeV} \), \( m_{st} = 5.27 \times 10^{17} \text{GeV} \) and \( T_H = 4.92 \times 10^{16} \text{GeV} \). When \( T_{eq}/T_H \ll 1 \), thermalization processes can be neglected, and the integration in Eqn.(3.9) approaches

\begin{equation}
K \to (1 + A_l)[2(A_\phi - A_l) + A_\phi^2 - A_l^2] \quad (T_{eq}/T_H \ll 1). \tag{3.13}
\end{equation}

The result in Eqn.(3.10) was calculated numerically with the value \( T_{eq}/T_H \approx 0.763 \). Note that the result has no suppression as far as \( T_{eq}/T_H \) is of the order one.

The lepton to entropy ratio is

\begin{equation}
n_L/s \approx Y_L/g_* \approx 7.9 \times 10^{-10} J, \tag{3.14}
\end{equation}

where \( J \) is given by Eqn.(3.8) and it is of the order one. This nonzero \( n_L \) will be converted into nonzero \( n_B \) of the same order of magnitude via sphaleron processes. Therefore, the
observed baryon asymmetry can be generated after the decay processes of higher excited states of string theory.

4 Conclusions and Discussions

In this paper we consider baryogenesis scenarios at the string scale or the Planck scale. At these scales the three necessary conditions for baryogenesis are satisfied. We have shown that the observed baryon asymmetry can be generated after the decay processes of higher excited states of string theory. If we take into account the baryogenesis during the decay processes, we will obtain a larger value for the baryon to entropy ratio. Too large a value could be diluted afterwards by considering entropy generation in the confinement-deconfinement phase transition, for example. Therefore, the observed baryon to entropy ratio can be explained in these scenarios.

Finally we give some comments on the model of Eqn.(2.1). We have considered Majorana-type interactions plus four-fermion interactions other than the MSM in order to introduce CP violation. It seems that only Majorana-type interactions would be enough since there are Yukawa couplings in the MSM already. The Yukawa plus Majorana-type interactions,

\[ y_{ij} e^{\alpha l_i} l_{\alpha}^j + \frac{1}{2} g_{st} h_{ij} e^{\alpha l_i} e_{\alpha l} l_{\alpha}^j + h.c., \quad (4.1) \]

would also work, since, after the Yukawa coupling is brought to the form of a real diagonal matrix via a unitary transformation, no degrees of freedom remain to insure a real Majorana-type coupling. However, because the Yukawa coupling is small, the resultant lepton asymmetry is too small to explain the observed value of \( n_B/s \). Indeed, a nonzero \( n_L \) is generated, for example, through the processes shown in Fig.3. The result is of the order,

\[ n_L/n_\gamma \sim \left( h \frac{g_{st}^2}{m_{st}} \right)^4 y^4 \frac{m_{pl}}{1.66 g_{1/2} T_H} \]

\[ \sim 1.8 h^4 \times 10^{-14}. \quad (4.2) \]

Here \( h \) and \( y \) are characteristic values for \( h_{ij} \) and \( y_{ij} \), respectively. We think \( h \) is of the order one, and we used the Yukawa coupling of the tau lepton for \( y \).

Finally, the Majorana-type terms in Eqn.(2.1) give neutrino masses of the order of

\[ m_\nu \sim h \frac{g_{st}^2}{m_{st}} v^2 \sim 3.2 h \times 10^{-5} \text{ eV}. \quad (4.3) \]
Figure 3: Diagrams which contribute to leptogenesis by means of Yukawa couplings and Majorana-type couplings. The left-handed lepton doublets and the right-handed lepton singlets are $l$ and $e$, respectively.

This value is consistent with solar neutrino experiments if we consider the vacuum oscillation or take into consideration the magnetic field in the sun [10].

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References


