Electron Electric Dipole Moment from 
CP Violation in the Charged Higgs Sector

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Abstract

The leading contributions to the electron (or muon) electric dipole moment due to CP violation in the charged Higgs sector are at the two–loop level. A careful analysis of the model-independent contribution is provided. We also consider specific scenarios to demonstrate how charged Higgs sector CP violation can naturally give rise to large electric dipole moments. Numerical results show that the electron electric dipole moment in such models can lie at the experimentally accessible level.

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Introduction

Experiment has established that neither parity (P) nor charge conjugation (C) are unbroken symmetries of nature. Furthermore, kaon physics show that the product CP also fails to be an exact symmetry. The CPT theorem then implies that time-reversal (T) is necessarily broken as well, leading to the expectation of a T-odd electric dipole moment (EDM) for one or more of the elementary particles.

The Standard Model (SM) of electroweak interactions explains the CP violation in the $K - \bar{K}$ system as the result of a single complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It also predicts an electron EDM $d_e$ (which is non-zero only starting at the four-loop level) of about $8 \times 10^{-41} \text{e} \cdot \text{cm}$ [1]. The muon EDM in the SM is similarly about $2 \times 10^{-38} \text{e} \cdot \text{cm}$, while the neutron EDM (calculated from the up and down quark EDMs) is estimated [2] to be less than $10^{-31} \text{e} \cdot \text{cm}$.

The experimental limits (given at 95% C.L.) are several orders of magnitude above these predictions, with the limit on the electron EDM $|d_e| < 6.2 \times 10^{-27} \text{e} \cdot \text{cm}$ [3]. The limit on $d_\mu$ is even further removed from the SM prediction, with $|d_\mu| < 1.1 \times 10^{-18} \text{e} \cdot \text{cm}$ [4], although a proposed experiment at Brookhaven National Laboratory hopes to measure the muon EDM at the level of about $10^{-22} \text{e} \cdot \text{cm}$ [5]. The neutron EDM limit is $|d_n| < 11 \times 10^{-26} \text{e} \cdot \text{cm}$ [6]. Clearly, measurement of a non-zero electron, muon, or neutron EDM close to current or proposed limits would point to physics beyond the Standard Model.

New sources of CP violation can come from complex couplings or vacuum expectation values (VEV) associated with the Higgs boson sector. A significant electric dipole moment for elementary fermions can be generated if CP violation is mediated by neutral Higgs-boson exchange [7, 8, 9, 10, 11]. Dominant contributions come from one-loop or two-loop diagrams. The one-loop terms are proportional to $(m/v)^3$ (with one factor of $m/v$ due to an internal mass insertion), while the two-loop terms are proportional [9] to $m/v$, with $m$ being the fermion mass and $v = 246 \text{ GeV}$ the vacuum expectation value of the SM Higgs field. The one-loop contributions are thus strongly suppressed relative to the two-loop terms, by a factor of $(m/v)^2$.

Exhaustive studies [10] have been carried out on the electron EDM generated by neutral Higgs-boson sector CP violation. The corresponding charged Higgs contribution to $d_e$, on
the two Higgs-doublet Model (2HDM) II[12] usually cannot contain charged Higgs-related CP violation, due to the discrete symmetry imposed to enforce natural flavor conservation (NFC) [13]. Since there is only a single charged Higgs boson in this model, there can be no CP violation from Higgs boson mixing. The discrete symmetry then rules out the remaining possibility of intrinsically complex Higgs-fermion couplings. There are, however, several other simple models, like the 2HDM III[14, 15] without natural flavor conservation or the three Higgs doublet model (3HDM)[16] with NFC, which can easily contain sufficient CP violation in the charged Higgs sector to produce an electron EDM at an observable level.

With charged Higgs sector CP violation, the one-loop contribution is suppressed as in the neutral Higgs case but suffers an additional factor of \( m_\nu/m_e \), where \( m_\nu \) is the mass of the electron neutrino. If no right-handed neutrino exists, or the neutrino is massless, the two-loop diagrams are unequivocally the leading contribution. It is also important to note that the recent measurement of the decay rate of \( b \to s\gamma \) by the CLEO collaboration [17] stringently constrains[18] the mass of the \( H^\pm \) only in the 2HDM II. The constraint is easily evaded in 2HDM III[15, 19] and other extensions.

In this letter, we make a study of the model-independent two–loop contribution to the electric dipole moments of the electron and muon from charged Higgs sector CP violation. Useful formulas are given. We discuss specific models to see how CP nonconservation can arise in the charged Higgs sector. Numerical results show the electron electric dipole moment can naturally lie within reach of experiment. We also present results for the corresponding contributions to \( d_\mu \) and \( d_n \).

**General Formalism**

Before presenting the details of our work, we comment on the model-independence of our analysis. First, one may have several charged Higgs bosons in a given model, with one or more of them contributing to the electron EDM. Unless there is a significant level of degeneracy, possible cancellations should be mild. In the following, for simplicity we thus only consider contributions from the lightest charged Higgs. Depending on the particular
model, the neutral Higgs bosons may also contribute: either directly (from neutral Higgs sector CP violation as mentioned above), or indirectly, in a model-dependent fashion involving vertices with neutral and charged Higgs bosons. We shall ignore the latter in our discussion, which is of course also equivalent to assuming decoupling of the neutral Higgs boson(s) by a factor of $m_{H^+}^2/m_{H^0}^2 \ll 1$.

We study the CP violation due to the following interactions,

$$
\mathcal{L} = \frac{g}{\sqrt{2}} \left( \frac{m_t}{M_W} c_t \bar{t}_R b_L H^+ + \frac{m_e}{M_W} c_e \bar{\nu}_L e_R H^+ + \bar{\nu}_L \gamma^\alpha b_L W^\alpha_+ + \bar{\nu}_L \gamma^\alpha e_L W^\alpha_+ \right)
- m_e \bar{e}_L e_R - m_t \bar{t}_L t_R + \text{H.c.} \quad (1)
$$

To highlight the physics involved, we only illustrate the most important contribution from the top-bottom generation; our study can easily be generalized to the three generations case. The bottom quark mass $m_b$ is also set to zero. We have written Eq.(1) so that the complex mixing parameters $c_t$ and $c_e$ signal deviations from the 2HDM II. If $c_t c_e^*$ has a non-zero imaginary part, the phase is intrinsic to the lagrangian and cannot be rotated away by redefinition of any or all of the fields in Eq.(1); redefining the fields to remove the phases from the first two terms would only shift a complex phase to one or more of the remaining terms.

The two-loop charged Higgs contribution involves Feynman diagrams such as the one shown in Fig. 1. We first present a simple expression for the one-loop sub-diagram with fermion in the loop, that is, the truncated three-point Green’s function $\Gamma^{\mu\nu} = \langle 0 | [H^-(p) A^\mu(k) W^{-\nu}(-q)]_+ | 0 \rangle$. We note that $\Gamma^{\mu\nu}$ is the off-shell extension of the rate for $H^+ \to W^+ \gamma$ given in Ref.[20], and that both $\Gamma^{\mu\nu}$ and its charge conjugate contribute to $d_e$. The relevant Feynman diagrams can also be found in Ref.[20]. As discussed above, we ignore the model-dependent contributions to $\Gamma^{\mu\nu}$ that involve neutral Higgs, or alternatively assume $m_{H^+}^2 \ll m_{H^0}^2$.

Our results are derived by using different choices of gauge parameters as a consistent check. In particular, the calculation becomes much simplified in the non-linear $R_\xi$ gauge[21]. Since we are interested in the soft photon limit in the study of the EDM effect at low energy, only the leading term in $k$ is kept. We also work to lowest order in $(m_e/M_W)$. Due to this
latter approximation, separation of the calculation into $\Gamma^{\mu\nu}$ and its insertion in the full two-loop graph is, despite appearances, gauge-invariant. Sub-diagrams involving a Goldstone boson are next higher order in ($m_e/M_W$). We thus obtain:

$$\Gamma^{\mu\nu} = \frac{3eg^2}{16\pi^2} \frac{c_t}{M_W} [(g^{\mu\nu} q \cdot k - q^{\mu}k^{\nu})S + iP\varepsilon^{\mu\nu\alpha\beta} p_{\alpha} q_{\beta}] ,$$  \hspace{1cm} (2)

where

$$S = \int_0^1 \frac{q_t(1-y)^2 + q_by(1-y)}{1-yq^2/m_t^2} dy , \quad P = \int_0^1 \frac{q_t(1-y) + q_by}{1-yq^2/m_t^2} dy ,$$  \hspace{1cm} (3)

and the quark charges are denoted $q_t, q_b$. The above vertex is further connected to the lepton propagator to produce EDM (see Fig. 1).

$$d_e = \left( \frac{3eg^2}{32\pi^2} \right) \left( \frac{g^2}{32\pi^2 M_W} \right) \left( \frac{m_e}{M_W} \right) \text{Im}(c_t^* c_e) (q_t F_t + q_b F_b) .$$  \hspace{1cm} (4)

Here the form factors $F_b$ and $F_t$ are given by

$$F_t = \int_0^\infty \int_0^1 \frac{m_t^2}{(M_{H^+}^2 + Q^2)(m_t^2 + yQ^2)(M_W^2 + Q^2)} \frac{(1-y)(2-y)}{2} dyQ^2 dQ^2 ,$$  \hspace{1cm} (5)

$$F_b = \int_0^\infty \int_0^1 \frac{m_t^2}{(M_{H^+}^2 + Q^2)(m_t^2 + yQ^2)(M_W^2 + Q^2)} \frac{y(2-y)}{2} dyQ^2 dQ^2 .$$  \hspace{1cm} (6)

The integrations can be carried through analytically,

$$T(z) = \frac{1 - 3z \pi^2}{z^2} - \frac{(1 - 5/2)}{z^2} \ln z - \frac{1}{z} - \left( 2 - \frac{1}{z} \right) \left( 1 - \frac{1}{z} \right) \text{Sp}(1 - z) ,$$  \hspace{1cm} (7)

$$B(z) = \frac{1}{z} + \frac{2z - 1 - \pi^2}{z^2} + \left( \frac{3}{2} - \frac{1}{z} \right) \ln z - \left( 2 - \frac{1}{z} \right) \frac{1}{z} \text{Sp}(1 - z) ,$$  \hspace{1cm} (8)

$$F_t = \frac{T(z_H) - T(z_W)}{z_H - z_W} , \quad F_b = \frac{B(z_H) - B(z_W)}{z_H - z_W} ,$$  \hspace{1cm} (9)

with $z_H = M_{H^+}^2/m_t^2$ and $z_W = M_W^2/m_t^2$. The Spence function $\text{Sp}(z)$ is defined by

$$\text{Sp}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2} = - \int_0^1 t^{-1} \ln(1-t) dt ,$$

with the normalization $\text{Sp}(1) = \pi^2/6$.  

5
Models

In order to give an idea of the natural size of this contribution to the electron EDM, we consider some specific models in this section. The goal is to illustrate how easily this mechanism can give rise to a measurable electron EDM.

2HDM III.

In Model III with two Higgs doublets, we can choose a basis so that \( \langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \) and \( \langle \phi'^0 \rangle = 0 \). Then \( \phi \) emulates most physics in the SM, and \( \phi' \) produces new physics beyond the SM. The physical charged Higgs boson \( H^+ \) is just \( \phi'^+ \).

We can write down the Yukawa Lagrangian for Model III as

\[
-\mathcal{L}_Y = \eta^U_{ij} \bar{Q}_i L \phi^U_j + \eta^D_{ij} \bar{Q}_i L \phi^D_j + \xi^U_{ij} \bar{Q}_i L \phi'^U_j + \xi^D_{ij} \bar{Q}_i L \phi'^D_j + \xi^E_{ij} \bar{L}_i L \phi^E_j + \text{h.c.}
\]

Here \( i, j \) are generation indices. Coupling matrices \( \eta \) and \( \xi \) are, in general, non-diagonal. \( Q_{iL} \) and \( L_{iL} \) are the left-handed SU(2) doublets for quarks and leptons. \( U_{jR}, D_{jR}, \) and \( E_R \) are the right-handed SU(2) singlets for up-type quarks, down-type quarks and charged leptons respectively. \( \langle \phi \rangle \) generates all fermion mass matrices which are diagonalized by bi-unitary transformations, e.g.

\[
M_U = \text{diag}(m_u, m_c, m_t) = v \sqrt{2} (L_U)^\dagger \eta^U (R_U).\]

In terms of the mass eigenstates of up-type quarks \( U \), down-type quarks \( D \), charged leptons \( E \), neutrinos \( N \), the relevant interaction of the charged Higgs boson is given by

\[
\mathcal{L}_{H^+} = -H^+ \bar{U} \left[ V_K M \tilde{\xi}^D \frac{1}{2} (1 + \gamma^5) - \tilde{\xi}^U V_K M \frac{1}{2} (1 - \gamma^5) \right] D - H^+ \bar{N} \tilde{\xi}^E \frac{1}{2} (1 + \gamma^5) E + \text{h.c.},
\]

with the Kobayashi-Maskawa matrix \( V_K = (L_U)^\dagger (L_D) \), and \( \tilde{\xi}^P = (L_P)^\dagger \eta^P (R_P) \) (for \( P = U, D, E \)).

Tree-level flavor changing neutral currents (FCNC) are implied by non-zero off-diagonal elements of the matrices \( \tilde{\xi}^U,D,E \). We adopt a simple ansatz[14] for \( \tilde{\xi}^U,D,E \),

\[
\tilde{\xi}_{ij}^{U,D,E} = \lambda_{ij} \frac{g \sqrt{m_i m_j}}{\sqrt{2} M_W}.
\]

The mass hierarchy ensures that FCNC within the first two generations are naturally suppressed by small quark masses, while a larger freedom is allowed for FCNC involving the
third generations. Here $\lambda_{ij}$ can be $O(1)$ and complex. CP is already not a symmetry even if we restrict our attention to the flavor conserving diagonal entries of $\lambda_{ii}$. For simplicity, we concentrated at the contribution from the third generation of quarks and set $(V_{\text{KM}})_{tb} = 1$. The parameters $c_t$ and $c_e$ in Eq.(1) are given as,

$$c_e = -\lambda_{ee}, \quad c_t = \lambda_{tt}.$$  \hfill (13)

While $\lambda_{tt}$ cannot be significantly larger than $O(1)$ without producing strong coupling to the top quark, clearly $\lambda_{ee}$ can be much larger than $O(1)$ so that $\text{Im}(c_t^* c_e) \gg 1$ is quite allowed.

3HDM.

CP violation from the charged Higgs sector can be realized in the three Higgs doublet model[16]. The first two doublets $\phi_1$ and $\phi_2$ are responsible for the masses of the $b$–like quarks and the $t$–like quarks respectively. The charged leptons $e, \mu$ and $\tau$ only couple to $\phi_1$. The last doublet $\phi_3$ does not couple to the known fermions. In this assignment, the model preserves NFC naturally. The mass eigenstates $H_1^+$ and $H_2^+$ together with the unphysical charged Goldstone boson $H_3^+$ are linear combinations of $\phi_1^+, \phi_2^+$ and $\phi_3^+$,

$$\phi_i^+ = \sum_{j=1}^{3} U_{ij} H_j^+ \quad (i = 1, 2, 3).$$  \hfill (14)

As with the CKM matrix for three quark generations in the SM, the mixing amplitude $U_{ij}$ matrix generally contains a single non-zero complex phase, which gives rise to CP nonconservation through the Yukawa couplings,

$$c_t = U_{21}(v/v_2), \quad c_e = U_{11}(v/v_1).$$  \hfill (15)

In the approximation that the lightest Higgs dominates, the index $i$ refers to the lightest charges Higgs boson. As with the 2HDM III, $\text{Im}(c_t^* c_e)$ can be much larger than one — which occurs here if $v_1 \ll v_2$ (the possibility $v_2 \ll v_1$ is constrained by maintaining perturbative coupling to the top quark).
Discussion

We have analyzed the contribution to the electron EDM due to CP violation in the charged Higgs sector. From the general structure of the typical models discussed above, we have shown that the relevant CP violating parameter $\text{Im}(c_t^* c_e)$ can be of order one or larger. In Fig. 2, we show the dependence of the electron EDM on $M_{H^+}$ for the case $\text{Im}(c_t^* c_e) = 1$. The size of $d_e$ is naturally around $10^{-26}$ e·cm, around the current limit. As noted above, we are ignoring model-dependent contributions from the neutral Higgs sector. In the event that the neutral Higgs masses are much larger than the charged Higgs mass, these other contributions are suppressed, so that Fig. 2 may be used, for example, to rule out $m_{H^+} > 200$ GeV for $\text{Im}(c_t^* c_e) \geq 1$.

The muon EDM can be easily obtained by the replacements $m_e \rightarrow m_\mu$ and $\text{Im}(c_t^* c_e) \rightarrow \text{Im}(c_t^* c_\mu)$. In this case, we would require $\text{Im}(c_t^* c_\mu) > 60$ for the calculated $d_\mu$ to rise to the proposed future limit of $d_\mu < 10^{-22}$ e·cm.

Finally, we can carry over the calculation for $d_e$ to estimate contributes to $d_n$, using SU(6) relations [22]:

$$d_n = \frac{1}{3}(4d_d - d_u),$$

where we obtain the down and up quark EDMs with replacements as made for $d_\mu$, but with an additional factor $\eta_q$ multiplying both $d_d$ and $d_u$ coming from QCD evolution of the quark mass and the quark EDM [23]:

$$\eta_q = \left( \frac{\alpha_s(m_t^2)}{\alpha_s(m_b^2)} \right)^{16/23} \left( \frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)} \right)^{16/25} \left( \frac{\alpha_s(m_c^2)}{\alpha_s(\mu^2)} \right)^{16/27}.$$

There are sizable uncertainties coming from the quark masses and the extraction of $d_n$ from $d_d$ and $d_u$, but the resulting neutron EDM should be $d_n \approx 10^{-27} (m_d/m_e) \text{Im}(c_t^* c_d) e \cdot cm$ for $m_{H^+} \approx 100$ GeV (ignoring the up quark contribution). This contribution would reach the observable limit for $\text{Im}(c_t^* c_d) > 6$. In contrast to the case of $d_e$ or $d_\mu$, there is a sizable contribution from the charged Higgs boson through the three-gluon operator [24, 25]. The relative magnitudes are highly model-dependent: in 2HDM III, the three-gluon operator may vanish even while the two-loop contributions presented here are non-zero, if $c_t$ is purely real but $c_e$ remains complex. In the 3HDM, however, the two contributions to $d_n$ are either both
zero or both non-zero. In any case, barring strong cancellations, our result places a limit of $m_{H^+} > 100$ GeV for $\text{Im}(c_i^* c_d) = 6$.

**Figures**

Fig. 1. A typical two-loop Feynman diagram for the electron EDM due to charged Higgs sector CP violation. The other diagrams for the one-loop subgraph $H^- \rightarrow W^- \gamma$ may be found in Ref.[20].

Fig. 2. Model-independent contributions to $d_e$ versus $M_{H^+}$ for $\text{Im}(c_i^* c_e) = 1/2$, 1. The horizontal line denotes the current experimental limit.

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