Mean Charge of the Light Sea Quarks in the Proton

Susumu Koretune
Department of Physics, Fukui Medical University,
Matsuoka, Fukui 910-1193, Japan

The modified Gottfried sum rule multiplied by $3/2$ can be interpreted as the one to measure the mean $I_3$ of the $[(\text{quark}) - (\text{antiquark})]$ in the proton. Based on this interpretation we find the sum rule which can be understood as the one to measure the mean charge of the light sea quarks (u,d,s) in the proton, and show that it takes the value 0.23 for the proton and 0.34 for the neutron.
The Gottfried sum rule [1] is usually interpreted as the difference between the charge square of the quarks in the proton and in the neutron. Many years ago, however, this sum rule was related to the bilocal quantity, [2] and several years ago this quantity was related to the kaon-nucleon scatterings. [3, 4] I called this sum rule ‘the modified Gottfried sum rule’. It takes the form;

\[ \int_0^1 \frac{dx}{x} \left\{ F_{2p}^{ep}(x, Q^2) - F_{2n}^{en}(x, Q^2) \right\} = \frac{1}{3} \left( 1 - \frac{4f_K^2}{\pi} \int_{m_K m_N}^{\infty} \frac{d\nu}{\nu^2} \sqrt{\nu^2 - (m_K m_N)^2} \left\{ \sigma_{K^+N}(\nu) - \sigma_{K^+P}(\nu) \right\} \right) \]

where \( \sigma_{K^+N}(\nu) \) is the total cross section of the \( K^+ N \) scatterings and \( f_K \) is the kaon decay constant. Through the experimental values of these quantities, the right-hand side of this sum rule was estimated as \( 0.26 \pm 0.03 \). Equation (1) is derived essentially from the fact that both sides of Eq. (1) are related to the quantity

\[ \frac{1}{3\pi} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} A_3(\alpha, 0), \]

where \( A_3(\alpha, 0) \) is a bilocal quantity which we will explain soon. The left-hand side of Eq. (1) multiplied by \( \frac{3}{2} \) can be expressed in the parton model as

\[ \int_0^1 dx \left\{ \frac{1}{2} u - \frac{1}{2} d \right\} + \int_0^1 dx \left\{ \frac{1}{2} \lambda_u - \frac{1}{2} \lambda_d \right\} - \int_0^1 dx \left\{ - \frac{1}{2} \lambda_u + \frac{1}{2} \lambda_d \right\} = \frac{1}{2} + \frac{1}{2} \int_0^1 dx \left\{ \lambda_u - \lambda_d + \lambda_u - \lambda_d \right\} \]

where \( \lambda_i \) is the sea quark of the \( i \) quark. Thus we can understand it as the mean \( I_3 \) of the \( [(\text{quark}) - (\text{antiquark})] \) in the proton. This agrees with Eq. (2) in the sense that it has the meanings of the mean \( I_3 \) of something. Though the bilocal quantity in our formalism is not necessarily that defined by the quark field, we can give clear correspondence between them.[5] As far as the \( n = 1 \) moment of the structure function \( F_2 \) is concerned, \( A_3(\alpha, 0) \) can be considered as the quantity defined by

\[ \left\langle p \left| \frac{1}{2i} \left[ \bar{q}(x) \gamma^\mu \frac{1}{2} \lambda_\alpha q(0) - \bar{q}(0) \gamma^\mu \frac{1}{2} \lambda_\alpha q(x) \right] \right| p \right\rangle_c = p^\mu A_\alpha(px, x^2) + x^\mu \tilde{A}_\alpha(px, x^2), \]

on the null-plane \( x^+ = 0 \). It should be noted that as a particular property of the method to reach the fixed-mass sum rule in the null-plane formalism
the state $|p\rangle$ can be taken in any frame. [6] Thus we can even take the rest frame which may be useful in low energy models such as chiral quark models and soliton models. Now we can see why the contribution from the antiquark is multiplied by $-1$ in Eq. (2). Decomposing the quark field into the particle mode and the anti-particle one, we find that because of the integral of the type $P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha}$, the contribution from the anti-particle mode gets an additional factor $-1$. Hence the sea quark and its antiquark contribute additively to the sum rule. Compared with this, the Adler sum rule [7] corresponds to the mean $I_3$ of the [(quark)+(antiquark)] in the proton. Hence the contribution from the sea quark and its antiquark to the sum rule cancels out exactly, and it measures the mean $I_3$ of the valence quark being equal to the $I_3$ of the proton. This is the fundamental difference between the Gottfried sum rule and the Adler sum rule. The experimental study starting from the Ellis-Jaffe sum rule [8, 9] and the Gottfried sum rule [10] in recent ten years has shown that the hadronic vacuum is very important in understanding the deep structure of the hadron. HERA [11] also has shown that the theoretical understanding of the pomeron which should be related to this hadronic vacuum is very important. In view of these situations it is important to have a model-independent constraint on the hadronic vacuum which has a clear physical meaning. Here we give the sum rule which can be understood as the mean charge of the light sea quarks in the proton, where the 'light sea quarks' are those of $u$, $d$, and $s$ types.

Let us first derive the hypercharge sum rule in the $SU(3)$ flavor group. Explicit forms of the various sum rules in our formalism including their derivation are reviewed in Ref. [4]. According to this, it is straightforward to obtain the sum rule

$$
\frac{1}{2\pi} \frac{2\sqrt{3}}{3} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} A_s(\alpha, 0) = \int_0^1 \frac{dx}{x} \left\{ F_2^{\pi}(x, Q^2) + F_2^{\nu}(x, Q^2) - 3F_2^{\rho}(x, Q^2) - 3F_2^{\epsilon}(x, Q^2) \right\},
$$

(5)

$$
\frac{1}{2\pi} \frac{2\sqrt{3}}{3} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} A_s(\alpha, 0) = \frac{1}{3} [2I_\pi - I_\rho^p - I_\rho^K],
$$

(6)

where $I_\pi, I_\rho^p$ and $I_\rho^K$ are defined in Ref. [4] by assuming the smooth extrapolation to the on-shell quantity as

$$
I_\pi = g_A^2(0) + \frac{2f_\pi^2}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2} \left(\nu^2 - m_\pi^2 m_N^2\right)^{1/2} \left\{ \sigma^{\pi+N}(\nu) + \sigma^{\pi-N}(\nu) \right\} - \nu s^b \beta_{\pi N},
$$

(6)
\[ I_p = (g_A^{p\pi}(0))^2 + (g_A^{pN}(0))^2 + \frac{2f_K^2}{\pi} \int_{\nu_0^K}^\infty \frac{d\nu}{\nu^2} (\nu^2 - m_K^2m_N^2)^{1/2} \]

\[ \{\sigma^{K^+}(\nu) + \sigma^{K^-}(\nu)\} - \nu s^b \beta_{KN} + \frac{2f_K^2\beta_{KN}}{\pi} \ln\left(\frac{1}{2\nu_0^K}\right) + U_p. \] \hspace{1cm} (8)

\[ I_n = (g_A^{n\pi}(0))^2 + \frac{2f_K^2}{\pi} \int_{\nu_0^K}^\infty \frac{d\nu}{\nu^2} (\nu^2 - m_K^2m_N^2)^{1/2} \{\sigma^{K+}(\nu) + \sigma^{K^-}(\nu)\} \]

\[ -\nu s^b \beta_{KN} + \frac{2f_K^2\beta_{KN}}{\pi} \ln\left(\frac{1}{2\nu_0^K}\right) + U_n. \] \hspace{1cm} (9)

Here the intercept of the pomeron is assumed as \( \alpha_P(0) = 1 + b \) with \( b = 0.0808 \) \hspace{1cm} (12) and \( s \) is defined as \( s = m_\pi^2 + m_N^2 + 2\nu \) for \( I_p \) and \( s = m_K^2 + m_N^2 + 2\nu \) for \( I_K^p \) and \( I_K^n \). The quantities \( \beta_{\pi N} \) and \( \beta_{KN} \) correspond to the residues of the pomeron which subtract the infinity in the above each integral. The quantities \( \nu_0^\pi \) and \( \nu_0^K \) are defined as \( \nu_0^\pi = m_\pi m_N \) and \( \nu_0^K = m_K m_N \). The terms \( U_p \) and \( U_n \) are the contributions below the \( KN \) threshold. Using Adler-Weisberger sum rules for the kaon, \hspace{1cm} (13) we can express these terms by the integral over the \( KN \) total cross sections. Then, through the experimental values of \( \pi N \) and \( KN \) total cross sections, \( I_\pi, I_K^p \) and \( I_K^n \) were estimated as \( I_\pi \approx 5.17 \), \( I_K^p \approx 2.39 \) and \( I_K^n \approx 1.61 \). Thus the right-hand side of the sum rule (6) is \( \frac{1}{3}[2I_\pi - I_K^p - I_K^n] \approx 2.12 \). Subtracting the contribution from the valence quarks from this, we find that the mean hypercharge of the light sea quarks is \( (2.12 - 1)/2 \approx 0.56 \). The reason \( (2.12 - 1) \) is divided by 2 is that the sea quarks and their antiquarks contribute additively. Similarly, the mean \( I_3 \) of the light sea quarks given by the modified Gottfried sum rule is \( 3(0.26 - 1)/3 \approx -0.055 \). Thus we obtain the sum rule of the mean charge of the light sea quarks in the proton and its value as

\[ <Q>_{\text{light sea quarks}} = \frac{1}{2} \left[ \frac{1}{2\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} A_3(\alpha, 0) - \frac{1}{2} \right] + \frac{1}{2} \left[ \frac{2\sqrt{3}}{3} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} A_8(\alpha, 0) - 1 \right] = \frac{1}{6} (I_\pi + I_K^p - 2I_K^n) - \frac{1}{2} \approx 0.23. \] \hspace{1cm} (10)
Because of the large positive mean hypercharge, the mean charge becomes positive, though the mean $I_3$ is negative. It goes without saying that the mean charge of the light antiquarks in the proton is $<Q>_{\text{proton}}^{\text{light antiquarks}} \sim -0.23$. In the neutron case we find

$$<Q>_{\text{neutron}}^{\text{light sea quarks}} = \frac{1}{6}(I_\pi - 2I_K^p + I_K^n) \sim 0.34. \quad (11)$$

Since the right-hand side of Eq. (1) is $(I_K^p - I_K^n)/3$, from Eqs. (1),(2) and (5), we can easily recognize that Eqs. (10) and (11) have the correct meaning of the mean charge of the light sea quarks in the proton in the parton model. However, these results do not depend on the parton model. They give us a model independent constraint on the matrix elements of the bilocal currents in an arbitrary frame of the nucleon.

The perturbative QCD corrections to the relations (6),(10) and (11) begin from 2 loops in the anomalous dimension and they enter in the same way as that in the Gottfried sum rule. [14] Therefore they are negligible compared with the non-perturbative values given in this paper. Further, it is possible to check Eqs. (10) and (11) by the parton distributions available at present, such as the ones by the CTEQ group [15] or the MRS group. [16] Before the numerical calculation, however, we can recognize that these distributions will not satisfy Eqs. (10) and (11), since they do not satisfy the symmetry constraint $\lim_{x \to 0} x^{\alpha r(0)}\lambda_u(x, Q^2) = \lim_{x \to 0} x^{\alpha r(0)}\lambda_d(x, Q^2) = \lim_{x \to 0} x^{\alpha r(0)}\lambda_s(x, Q^2)$, which comes in our approach from the fact that the pomeron is flavor singlet, and since Eqs. (10) and (11) depend heavily on the behavior in the small $x$ region. To see this fact more concretely, we give here a typical example by the CTEQ4M initial set at $Q_0 = 1.6\text{GeV}$. By changing the lower limit of the $x$ integration to $1 \times 10^{-6}$, $1 \times 10^{-5}$, $1 \times 10^{-4}$ and $1 \times 10^{-3}$, we find that the contribution above $x = 1 \times 10^{-3}$ to Eq. (10) is 0.24, and hence it already satisfies the relation. However the contribution in the region $1 \times 10^{-4} \leq x \leq 1 \times 10^{-3}$ is 0.17, $1 \times 10^{-5} \leq x \leq 1 \times 10^{-4}$ is 0.22, and $1 \times 10^{-6} \leq x \leq 1 \times 10^{-5}$ is 0.30. Thus we see that the large contribution comes from the region below $x = 1 \times 10^{-3}$. The origin of this is clear. The strange sea quark is too small in comparison with the up and down sea quarks, while the up and down sea
quarks are very similar in this region. Since the sea quarks in the small $x$
region are poorly determined in a phenomenological analysis, we see that the
relations (10) and (11) are helpful also in this respect.

In conclusion we derived a sum rule which can be understood in the
parton model as that to measure the mean charge of the light sea quarks
in the proton and show that this charge takes the value $0.23(0.34)$ for the
proton(neutron).

References

M.Derrick et al., ibid. 316, 412(1993);ZEUS Collaboration, M.Derrick
et al., ibid. 332, 228(1994).