EFFECTIVE GRAVITY AND $OSp(N,4)$ INVARIANT MATTER

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ABSTRACT

We re-examine the $OSp(N,4)$ invariant interacting model of massless chiral and gauge superfields, whose superconformal invariance was instrumental, both in proving the all-order no-renormalization of the mass and chiral self-interaction lagrangians, and in determining the linear superfield renormalization needed. We show that the renormalization of the gravitational action modifies only the cosmological term, without affecting higher-order tensors. This could explain why the effect of the cosmological constant is shadowed by the effects of newtonian gravity.

March 1997

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1 Introduction

The introduction of locally supersymmetric theories [1] was motivated by the wish for a unified description, encompassing the theories of elementary particles and gravity. The resulting supergravity is not renormalizable, but its large symmetry provides powerful constraints, such as the vanishing of the lepton anomalous magnetic moment, as required by the supersymmetry of the theory [2]. The finiteness of the fermionic (and therefore also the bosonic) contribution to it, which can be traced back to an effective chiral symmetry in the gravitino sector [3], has been checked in [4,3] and, more recently in [5], making use of supersymmetry preserving regularization schemes.

An introduction to the effective action in quantum gravity can be found in [6]. The quantum corrections to the low-energy limit of a theory coupling gravity to scalar fields have been computed [7]. The equivalence principle has been invoked, in order to reduce the terms with an arbitrary number of derivatives in the effective theory [8]. This principle also constrains the spin-1 and spin-0 partners of the graviton in the $N = 2, 8$ supergravity multiplets [9,10].

$OSp(N, 4)$ invariant models in the fixed four-dimensional background of anti-de Sitter space ($AdS_4$) occur as both ground state solutions of gauged extended supergravity theories (see [11] and references therein) and vacuum configurations for superstrings [12]. The $OSp(N, 4)$ invariant generalization of the Wess-Zumino model [13] is the simplest one. We recall that the Wess-Zumino model with softly broken supersymmetry in de Sitter space plays a role in the Affleck-Dine mechanism [14] for baryogenesis, in contrast to the maximal symmetry of $AdS_4$ which grants the existence of global supersymmetry. This mechanism is effective for supersymmetric grand unified theories, as the quantum corrections do not affect the flat directions in the superpotential, owing to the no-renormalization theorem [15].

Very recently, the one-loop effective potential along a flat direction in this model has been calculated [16]. In [17] the one-loop effective action was computed for scalar-QED, taking into account the large-scale configurations that change the topology. Also, the question of the dynamics of a superstring propagating in $AdS_4$, with the $OSp(1, 4)$ supersymmetry group, deserves further study, within a geometrical framework, especially in connection with the underlying algebraic structure of the $W$-algebra extension of two-dimensional conformal symmetry (see, e.g. [18-21]).

The renormalization procedure for $OSp(N, 4)$ invariant theories breaks the naturality implied by the no-renormalization theorem and so allows all classically invariant counterterms to appear in the divergent structure of the quantum effective action [22-34]. This breakdown of the no-renormalization theorem in $AdS_4$ forces us to introduce a linear superfield in the effective action, for the purpose of renormalization. However, the corresponding modification of the classical potential does not induce the breaking of supersymmetry invariance [25,11]. The superconformal invariance of the model with interacting chiral and real gauge superfields in $AdS_4$, following the line suggested in Ref. [35], allowed us to prove to all orders in the perturbative series the non renormalization of the mass nor the cubic interaction action [34].

We organize the present work as follows. We begin in sec. 2 with recalling the superfield formulation of the $AdS_4$ interacting model of chiral and gauge superfields. The superconformal invariance of the massless model allows us to implement an expansion in the curvature effects, in terms of the interaction vertices of the quantum model. In sec. 3 we present the
renormalization of the gravitational action, based on the use of $OSp(N, 4)$ superfield techniques, and propose some interpretation for the shadowing of the effect of the cosmological constant by the effects of newtonian gravity. We draw our conclusions in sec. 4.

2 Interacting chiral and real gauge superfields

For the purpose of fixing notations, we briefly recall in this section a superspace approach \[35,33,34\] to the $OSp(N, 4)$ invariant theory of a supergravity multiplet coupled to interacting chiral and real gauge supermultiplets.

In order to choose $AdS_4$ as a background space for the matter and gauge model, we set the supergravity prepotential $H$ to zero and introduce the background only through the compensator $\phi$. Its equation of motion

$$D^2 \phi = \alpha \phi^2$$

(2.1)

can be obtained from the action for supergravity with the cosmological term

$$S = -\frac{3}{\kappa^2} \int d^4 x d^4 \theta E^{-1} + (\alpha \frac{1}{\kappa^2} \int d^4 x d^2 \theta \phi^3 + h.c.)$$

(2.2)

yielding the solution with a regular behaviour at infinity

$$\phi = \frac{1}{1 - \alpha \alpha x^2 / 4} - \frac{\alpha \theta^2}{(1 - \alpha \alpha x^2 / 4)^2}$$

(2.3)

with the inverse determinant $E^{-1} = \overline{\phi} \phi$. Then, by applying this solution to the construction of invariant actions of the general and chiral type, we can formulate different supersymmetric matter models in the given background. We recall the expression of the covariant derivatives in terms of the $\phi$ field

$$\nabla_{\alpha} = \phi^{-1} \overline{\phi}^{1/2} D_{\alpha} , \quad \nabla_a = \overline{\phi}^{-1} \phi^{1/2} D_a$$

(2.4)

The theory of interacting chiral ($\eta$) and real gauge superfields is described (in the gauge-chiral representation) by the action \[35\]

$$S(\eta, \overline{\eta}, V) = \int d^4 x d^4 \theta E^{-1} \overline{\eta}_j [\exp(V)]^j_i \eta^i + \int d^4 x d^2 \theta \phi^3 W^2 + \int d^4 x d^2 \theta \phi^3 (m \frac{1}{2} \eta^2 + \lambda \frac{1}{6} \eta^3) + h.c.]$$

(2.5)

with $V^i = V^A(T_A)^i_j$, and where $(T_A)^i_j$ is a matrix representation of the generators of the gauge group that leaves this action invariant. This model possesses a partial superconformal invariance, which has been exploited \[33,34\], in order to treat perturbatively the effects of the background curvature, when carrying out the renormalization procedure that yields its quantum effective action. All $\phi$-dependence in the free-field functional integral can be removed by carrying out a superconformal transformation, in accordance with the canonical weights of the matter and gauge fields and their sources

$$\hat{\eta} = \phi \eta , \quad \hat{W}_a = \phi^{3/2} W_a , \quad \hat{J} = \phi^2 J , \quad \hat{J}_V = \phi \overline{\phi} J_V$$

(2.6)
Hence the quantum model can be described in the most natural form in terms of the transformed fields, i.e. defining the effective action in terms of the hatted fields.

The definition of \( \hat{W} \) in terms of the familiar derivatives in flat background reads

\[
\hat{W}_\alpha = iD^2 D_\alpha V
\]  

(2.7)

The covariantization of this expression in the Yang-Mills chiral representation

\[
W_\alpha = i(\nabla^2 + \alpha)e^{\exp(-V)}\nabla_\alpha e^{\exp(V)}
\]  

(2.8)

gives \( W_\alpha \) in terms of the background covariant derivatives (2.4). After the superconformal transformation, the gauge-fixing procedure can be carried out, along the line of the flat background theory. The resulting gauge propagator reads (in the Fermi-Feynman gauge, with \( \xi = 1 \))

\[
<VV>_0 = -\frac{1}{p^2}\delta^4(\theta - \theta')
\]  

(2.9)

It is worthwhile to notice that, as a consequence of superconformal invariance, the ghost propagators and vertices of the flat space-time theory, along with the usual flat space-time \( D \)-algebra, remain intact in \( AdS_4 \).

Enforcing the boundary conditions needed, in order to preserve supersymmetry for the scalar and spinor propagators in \( AdS_4 \) [32], and evaluating the free-field functional integral, yields the vacuum expectation values [33]

\[
<T\hat{\eta}(x')\hat{\eta}(0)> = \frac{1}{4\pi^2}D^2D^2\delta^4(\theta - \theta')\frac{1}{(x')^2}, \quad <T\hat{\eta}(x')\hat{\eta}(0)> = \frac{1}{16\pi^2}(|\alpha|^2 + 2|\alpha|^3\theta\theta')
\]  

(2.10)

From the generating functional one can read the vertex contributions involving the field \( \hat{\eta} \). It turns out [33,34] that there are no \( \phi \)'s at every such vertex (in \( D = 4 \)), with the only exception of a vertex quadratic in \( \hat{\eta} \), which appears with a factor \( \phi \). The conclusion is that, for our purposes, we can handle the quantum system of \( N = 1 \) super Yang-Mills coupled to matter scalar superfields in \( AdS_4 \) in a way similar to the corresponding theory in a flat background, with the only difference of including in the Feynman rules the additional quadratic vertex

\[
\frac{1}{2}m\phi + h.c.
\]  

(2.11)

The residual explicit \( \phi \) dependence of the latter reflects the deviation of the theory from a superconformal one, owing to the introduction of a mass term. A remarkable feature of the above rescaling, which effectively removes \( \phi \) from the superconformal invariant part of the superfield action (with the caveat of possible anomalous contributions [33]), is that it takes automatically into consideration the need to resort to some perturbative approach in the effects of the curvature of the background space, leading us naturally to introduce the above implicit expansion in the compensator \( \phi \).

3 The renormalization of the gravitational action

Herewith we describe our main result, in an attempt to explain the shadowing of the cosmological constant. We build upon our previous work [33,34] and carry out the renormalization
of the gravitational action induced by $OSp(N, 4)$ invariant matter multiplets in curved space. Here great care is needed, as every sign at each step is crucial.

We start by discussing the renormalization of the gravitational action induced by a matter chiral superfield, through the presence of the Feynman diagram in figure 1. This yields the following divergent contribution:

$$am^2 \frac{1}{\epsilon} \int d^4 x d^4 \theta \phi \bar{\phi}, \quad (3.1)$$

with $a > 0$ in any case. One can doubt about the fact of interpreting this diagram as a renormalization of the pure supergravity action

$$-\frac{3}{\kappa^2} \int d^4 x d^4 \theta \phi \bar{\phi}, \quad (3.2)$$

or the cosmological superspace action

$$\frac{\alpha}{\kappa^2} \int d^4 x d^2 \theta \phi^3 + h.c. \quad (3.3)$$

There are clear reasons to support this point of view, which goes in the direction of considering the above contribution as a renormalization of the second term, i.e. the cosmological action. In order to pursue this idea, we can translate the result in components, to read as follows:

$$am^2 \alpha^2 \frac{1}{\epsilon} \int d^4 x \sqrt{-g}. \quad (3.4)$$

Here, and throughout this section, we denote with subscript indices $R, B$ the renormalized and the bare parameters of the action, respectively.
Figure 2: $O(g^2)$ two-loop corrections to the cosmological renormalization factor $Z_\alpha$ in (3.7); the wavy lines denote the gauge superfields.

On the other hand, we know that the final expression for the gravitational actions has to be
\[ S = -\frac{1}{\kappa^2} \int d^4x \sqrt{-g} R(r) + \frac{6}{\kappa^2} \int d^4x \sqrt{-g} \alpha_R^2 , \] (3.5)
where, let us say, $R$ is restricted to metrics of the anti-de Sitter type, with arbitrary radius $r$. Then it is clear to us that the contribution (3.4) can be consistently interpreted as a renormalization of only the cosmological term, i.e. the second term on the r.h.s. of Eq. (3.5). This interpretation is also compatible with the fact that the renormalization of the higher order gravitational tensors (i.e. the contributions of order $\alpha^4 R$) does not take place.

In our opinion, this is not a coincidence, rather it means that any vacuum diagram can be seen as a renormalization of only one parameter in the gravitational action, namely $\alpha_R$.

So, let us write then
\[ am^2 \frac{1}{\epsilon} \int d^4x d^4 \theta \phi \bar{\phi} = am^2 \frac{1}{\epsilon} \alpha_R \int d^4x d^4 \theta \phi^3 , \] (3.6)
where we make use of the equation of motion (2.1) for the chiral compensator superfield $\phi$.

Considering this contribution alone, we would have
\[ \alpha_B = \alpha_R - am^2 \alpha_R \frac{1}{\epsilon} \kappa^2 \equiv Z_\alpha \alpha_R , \] (3.7)
but then it is obvious that this renormalization factor $Z_\alpha$ cannot lead to a running $\alpha_R$, since the above $m$ and $\kappa$ parameters do not depend on the renormalization scale $\mu$ (the $m$ parameter that appears here cannot be anything, other than the bare mass $m_B$).

In this way, we reach the conclusion that the relevant term in $Z_\alpha$ is the two-loop contribution to the vacuum, what forces us to introduce gauge interactions in the game. In fact, when introducing gauge interactions, of all the plethora of diagrams that one can imagine
to the order $g^2$ in the gauge coupling constant, we believe there are only two that survive, after performing the $D$-algebra of the covariant derivatives, i.e. the diagrams in figure 2. We give, in the following, the computation of the first graph (at the top of figure 2), as the other one (at the bottom of figure 2) looks really frightening to compute, and anyhow it cannot change the conclusion of this story. Using integration by parts for the superspace covariant derivatives and discarding a finite remainder given in figure 3, one can identify the divergent parts of the first graph in figure 2 and the diagram in figure 4.

The diagram in figure 4 yields the amplitude $\mathcal{V}$

$$
\mathcal{V} = m^2 g^2 \int \frac{d^D k}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \frac{1}{k^2 q^2} \frac{1}{(k - p)^2} \frac{1}{(k + q)^2} , \quad (3.8)
$$

where we work in dimension $D = 4 - \epsilon$. The first integral can be carried out, yielding the amplitude

$$
\mathcal{V} = m^2 g^2 \frac{1}{(2\pi)^{4-\epsilon}} \frac{1}{\pi^{\epsilon/2}} \frac{1}{\Gamma(\frac{\epsilon}{2})} B\left(1 - \frac{\epsilon}{2}, 1 - \frac{\epsilon}{2}\right) I , \quad (3.9)
$$

where

$$
B(1 - z, 1 - w) \equiv \int_0^1 dy \frac{1}{y^z (1 - y)^w} \quad (3.10)
$$

and we define the momentum integral

$$
I \equiv \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \frac{1}{(k - p)^2} \frac{1}{(k^2)^{\epsilon/2}} . \quad (3.11)
$$

Also the integral $I$ can be evaluated. Hence, using standard properties of the $B$-function,
Figure 4: \(O(g^2)\) two-loop Feynman diagram corresponding to the amplitude \(\mathcal{V}\) in (3.8); the two independent momenta (loop variables) of the internal lines are explicitly indicated, together with the momenta of the external compensators.

we get the result

\[
\mathcal{V} = m^2 g_R^2 \frac{1}{(2\pi)^{8-2\epsilon}} \pi^{4-\epsilon} \frac{2}{\epsilon} \Gamma(\epsilon) \frac{1}{1 - \epsilon} \frac{1}{[\Gamma(1 - \frac{\epsilon}{2})]^3} \frac{1}{\Gamma(2 - 3\epsilon/2)} \left(\frac{\mu^2}{p^2}\right)^{\epsilon}. \tag{3.12}
\]

Here we introduced the expression of the renormalized gauge coupling constant \(g_R\) in terms of the renormalization scale

\[
g_R = g\mu^{-\epsilon}. \tag{3.13}
\]

Two remarks are in order about this result. First of all, the leading divergence is of order \(\epsilon^{-2}\), what makes it relevant for the purpose of running \(\alpha_R\). Secondly, it contains subleading nonlocal divergencies of the form \(\log(p/\mu)\). We will have to prove that they cancel with similar terms coming from the other graph in figure 2, for this whole thing to make sense. What is important about this other graph, apart from the cancellation of the nonlocal divergencies, is that the leading divergence comes with a positive sign (as it is the case for the amplitude \(\mathcal{V}\) computed above). So let us assume that the whole contribution from figure 2 is of the form

\[
bn^2 g_R^2 \alpha_R \left(\frac{1}{\epsilon}\right)^2 \int d^4x d^2\theta \phi^3, \tag{3.14}
\]

with \(b > 0\), plus perhaps \(1/\epsilon\) subleading divergencies that are not relevant, when studying the renormalization group equation for \(\alpha_R\) to the order \(g_R^2\). Then, we have that

\[
\frac{d\alpha_B}{d\mu} = 0 = Z_\alpha \frac{d\alpha_R}{d\mu} + \frac{dZ_\alpha}{d\mu} \alpha_R \tag{3.16}
\]
\[
\frac{d\alpha_R}{d\mu} = -\frac{1}{Z_{\alpha R}} \frac{dZ_{\alpha R}}{d\mu} \alpha_R \\
\approx -\frac{1}{1 - \frac{am^2\kappa^2}{\epsilon}}\left[-2bm^2\kappa^2 g_R \frac{dg_R}{d\mu} \left(\frac{1}{\epsilon}\right)^2 \alpha_R + O(g_R^2)\right] \\
= -\frac{1}{1 - \frac{am^2\kappa^2}{\epsilon}}(bm^2\kappa^2 g_R^2 \frac{1}{\mu \epsilon}) \alpha_R + O(g_R^3) .
\]  
(3.17)

In the limit \(\epsilon \to 0\), we have then

\[
\frac{d\alpha_R}{d\mu} = \frac{b g_R^2}{a} \frac{1}{\mu} \alpha_R ,
\]  
(3.18)

so that one can easily guess the kind of theories, in which the effective cosmological constant goes to zero in the infrared, as a power of \(\mu\). We can then write that, if \(g_R^2 = \text{constant}\)

\[
\alpha_R = \alpha_0 \left(\frac{\mu}{\mu_0}\right)^{bg_R^2/a} .
\]  
(3.19)

The important thing, which we checked repeatedly, is that the exponent is positive, though it is more obscure, under which circumstances it could be bigger than one.

4 Conclusion

We cannot add much more to the above considerations, apart from the fact that, if the interpretation \(\mu^2 \approx R\) in a gravitational measurement is plausible, then this could explain why the effect of the cosmological constant is always shadowed, no matter what the value of \(\alpha_0\) might be, by the effects of newtonian gravity.

The calculation of the stress-tensor anomaly in \(AdS_4\) supersymmetry showed [36] that the choice of the vacuum, around which the model is quantized, does not affect the renormalization of the purely geometrical tensors in the effective action, nor the trace anomaly induced by matter multiplets invariant under the supersymmetry group \(OSp(N, 4)\) in curved space. These quantities are independent also from the boundary conditions for the free-field propagators, as proven in Ref. [36]. We remark, in passing, that the conformal anomaly, as an integrability condition for the supersymmetric sigma models corresponding to superstring theories, was obtained in Refs. [37-41].

Next, we wish to compare the work of Elizalde and Odintsov (E-O) [42] with our result (3.19) on the running and the consequent exponential shadowing of the cosmological constant. The two results appear to be similar, with ours as a particular case, at least at first sight. Notice however that our paper is of wider interest in what concerns keeping a global (anti-de Sitter) supersymmetry of the curved background space. Indeed our work and [42] may be considered as complementary, in many respects.

The paper by E-O contains a phenomenological analysis of theories that are finite in flat spacetime. A class of such theories is considered in [42] interacting with an external gravitational field (including a nonminimal term linear in the curvature and quadratic in the massless scalar matter field). Our calculation applies specifically to the (globally) supersymmetric background (ground state) solution for SUSY GUTS, extended SUGRA,
superconformal invariant theories (superstrings). As in our calculation the (global) supersymmetry of this ground state solution is maintained, consequently the contribution of the supersymmetric particles is determined explicitly.

In addition, the role of a nonvanishing mass for the matter fields is perfectly clear and is included in our analysis (E-O only consider a massless theory). Finally, in E-O a fine tuning is needed, in order to avoid in their Eq. (19) the unrealistic growth of Newton’s constant, and/or to obtain the screening of the cosmological constant. We obtain such screening for \( \alpha_R \) directly and without fine tuning.

We stress that we consider the superconformal invariant theories to be minimally interacting with the external gravitational background. Our main motivation is to keep the global supersymmetry of the background, which requires referring to the minimal \( n=-1/3 \) compensator \( \phi \). It was shown long ago in ref. [35] (page 336 section 5.7) that nonminimal \( n \) implies spontaneous breakdown of \( N=1 \) supersymmetry.

In our paper we have recalled several developments that simplified the procedure in practical calculations for the renormalization of our class of theories. In particular the superconformal rescaling is an improvement that can be useful in future applications. In this respect we hope that the discussion of the method given in this work will find further use.

Acknowledgement

We thank with great pleasure José González, for participating to the early stages of this research. We thank the referee of this paper for calling Ref. [42] to our attention.

References


