NEUTRON TRANSFER REACTIONS WITH RADIOACTIVE BEAMS

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Neutron transfer experiments under inverse kinematics have been proposed as a spectroscopic tool with second-generation radioactive nuclear beam facilities. We illustrate typical experimental criteria by examples involving the spectroscopy of light neutron-rich nuclei. Calculations have been performed to estimate the energy resolution attainable. The relative importance of various contributions to the resolution is studied for a range of masses and incident energies, for both fragmentation and on-line isotope separator facilities. The detection of the heavy product of the reaction by an energy-loss spectrometer and that of the recoil light particle by a charged particle array is compared. It is shown that the coincidence detection of the heavy ion and light particle may improve the energy and angular resolution, while reducing background from projectile breakup and scattering.

1 Introduction

Transfer reactions induced by light ions, such as (p, d), (d, p), (d, 3He), and (α, t), have been a standard spectroscopic tool in nuclear physics since the mid-1960’s. Such reactions may be used to deduce spins and parities from the characteristic angular momentum (L) transfer of the reaction as well as energies of excited states. Moreover, since the cross-section magnitudes essentially reflect the probability of finding a nucleon in a given shell model orbit, the extracted spectroscopic factors allow one to learn directly about the configuration of nuclear states.

In the past, the need to use stable targets has seriously limited the application of direct transfer reactions. This limitation may be overcome by making the light particle the target and using a radioactive nuclear beam (RNB) in inverse
kinematics. A further advantage of the use of a light target is to increase the observed reaction yield through the kinematic focussing of the products. Such experiments have indeed been proposed for many new facilities [1–4]. However, the special properties of the kinematics, together with the large emittance, broad energy spread and low intensity of RNBs, combine to impose demanding experimental requirements. We discuss in this paper the consequences of these requirements, and in particular, the achievable energy resolution.

We concentrate on the inverse (p,d) and (d,p) reactions, i.e. neutron pickup and stripping, respectively. These reactions are likely to be among the first to be used with radioactive beams, because of the relatively high cross sections, the simpler kinematics and the easier spectroscopy. A review of the prospects for other light-ion transfer reactions in inverse kinematics (albeit limited to the detection of the light particle) is given by Hardy [1], while Egelhof [5] provides a further discussion of experimental methods and the physics motivations. Many of our conclusions will be generally applicable to present and future RNB facilities, but for quantitative estimates we have taken characteristics of the present [6] and forthcoming [2] RNB facilities at GANIL, Caen, with which we are most familiar.

It is, of course, impossible to find a completely general example of an inverse kinematic transfer reaction that would illustrate all the experimental desiderata. In Section 3, we have chosen to focus on two specific experiments with light neutron-rich RNBs: the first example gives a benchmark for the energy resolution, the second a benchmark for the angular resolution. The resolution criteria which we have set are 300 keV for excitation energy and 1.5° for centre-of-mass (c.m.) angle. The fact that each of these criteria would be rather easily obtained in normal kinematics with a stable beam, underlines the difficulties with inverse-kinematic RNB experiments. We shall also use as illustrations reactions with proton-rich krypton beams, which are a probable RNB available from the early startup phase of an ISOL (Isotope Separator On-Line) facility. Our discussion assumes targets of the order of several mg/cm² thick. This comparatively large thickness is mainly because of the need to compensate for the weak intensity of the RNB.

For most RNB facilities, the poor quality of the beam has to be taken into account. One approach to deal with a large energy spread of a secondary beam is to use a spectrometer with dispersion-matching to detect the forward-focussed heavy ion. Another approach is to detect the light particle (proton or deuteron), since the energy of the light particle is rather insensitive to the beam energy, as will be shown in Sec. 2. Large area charged-particle arrays are a natural choice for this strategy. However, a potentially large contribution from the energy loss and small angle scattering in the target is then of concern. The various contributions to the energy resolution and the important effect of the laboratory to c.m. scaling on them are considered in Sec. 4. The angular
resolution may also be an issue for both the beam and detected particle. This is discussed in Sec. 5.

Simulations described in Sec. 6.2 discuss possible improvements to the c.m. energy resolution through the detection of the light recoil in kinematic coincidence with the heavy ion. Such a coincidence requirement would have the additional benefit of rejecting projectile breakup and inelastic scattering contamination of pickup spectra, which is discussed in Sec. 6.1.

2 Inverse Kinematics

Fig. 1 shows the particle energy vs. laboratory angle for a typical \( p(A, d)A^{-1} \) pickup reaction, where the outgoing deuteron is detected. The transitions to the ground state and to a fictitious state at 5 MeV excitation in the residual nucleus are illustrated. One sees that the kinematics solution is double-valued in energy, with the deuteron focussed well forward of 90° in the laboratory frame. The higher kinetic energy solution corresponds to the “backward-going” c.m. situation, i.e. \( \theta_{cm} > 90° \), and hence corresponds to small reaction cross-sections. The lower kinetic energy solution is thus the one of interest. As the detection angle increases, the energy difference between deuterons corresponding to given energy levels in the final nucleus increases, which means that the intrinsic energy resolution of the detector becomes less and less critical. However, the required angular resolution becomes increasingly demanding, as the rate of change of outgoing deuteron energy with angle increases, and becomes unattainably small as the detection angle approaches the “turnover” point. A paradoxical feature of the kinematics in this region is that the deuteron energy increases with excitation energy in the final system.

As an alternative to the detection of the light particle, one can detect the heavy product of the reaction. The kinematics of the heavy-ion angle and energy, for the same beam and target as above, are shown in Fig. 2. Here it is the higher energy branch of the curve that corresponds to the “forward-going” c.m. situation. Note that the heavy ion is restricted to a very small angular range in the laboratory frame. This is an advantage from the point of view of a small angular coverage being required to measure a complete distribution, but also implies that a fine angular resolution is necessary to achieve even a moderate energy resolution.

Stripping reactions such as \( d(A, p)A^{+1} \) give quite different kinematics for the light recoil particle (Fig. 3). The important forward c.m. angles correspond to backward laboratory angles in inverse kinematics. A comparison of the c.m. angular spacing in Figs. 1 and 3 shows that the laboratory angular resolution is less demanding for the stripping reaction than for pickup. On the other hand,
Fig. 1. Kinematical diagram for $p(^{12}\text{Be}, d)^{11}\text{Be}$ at 30 MeV per nucleon. The right-hand panel is a detailed view of the forward-going c.m. portion of the left-hand panel, with a selection of c.m. angles indicated.

Fig. 2. Kinematical diagram for $p(^{12}\text{Be}, ^{11}\text{Be})d$ at $E_t/A = 30$ MeV.

the separation of the 5 MeV “excited state” from the ground state in terms of the proton kinetic energy (Fig. 3) is only half the corresponding separation in the deuteron kinetic energy (Fig. 1). Thus the effect on the energy resolution caused by the energy loss in the target, for example, is greater for stripping reactions than for pickup reactions. The form of the kinematics for $d(A, A^{+1})p$ (not illustrated) is similar to that for $p(A, A^{-1})d$ shown in Fig. 2. Again, the separation of excited states in terms of the laboratory energy of the detected particle is about one half of that for the corresponding pickup reaction.

As is apparent from the above discussion, in these inverse kinematics transfer reactions a broadening in energy of the detected particle $E_f$ is far from the
Fig. 3. Kinematical diagram for $d^{(11}\text{Be},p)^{12}\text{Be}$ at 30 MeV per nucleon.

Table 1

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Be or Kr detected</th>
<th>$p$ or $d$ detected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$dE_x/dE_f$</td>
<td>$dE_f/dE_i$</td>
</tr>
<tr>
<td>$^{12}\text{Be} + p \rightarrow ^{11}\text{Be} + d$</td>
<td>-0.70</td>
<td>0.98</td>
</tr>
<tr>
<td>$^{11}\text{Be} + d \rightarrow ^{12}\text{Be} + p$</td>
<td>-1.40</td>
<td>1.00</td>
</tr>
<tr>
<td>$^{77}\text{Kr} + p \rightarrow ^{76}\text{Kr} + d$</td>
<td>-0.38</td>
<td>1.00</td>
</tr>
<tr>
<td>$^{76}\text{Kr} + d \rightarrow ^{77}\text{Kr} + p$</td>
<td>-1.61</td>
<td>1.00</td>
</tr>
</tbody>
</table>

same as the energy resolution of the excited state spectrum $E_x$ (i.e., measured in the c.m.). A given change in $E_f$ produces a smaller change in $E_x$ for inverse $(p,d)$ reactions, but a greater one for inverse $(d,p)$ reactions. In both cases, the recoil energy of the light ion is only weakly dependent on variations in the incident beam energy $E_i$ – an important advantage when the beam-energy spread may be large. These points are illustrated numerically in Table 1 for beryllium beams of $E_i/A = 30$ MeV and for for krypton beams of $E_i/A = 10$ MeV. We denote the change in $E_x$ per $E_f$ as $dE_x/dE_f$, and that of $E_f$ by $E_i$ as $dE_f/dE_i$. In fact, for strongly inverse kinematics, these ratios are not strictly constant, and need to be evaluated at the specific $E_x$ and $E_i$ under consideration for accuracy. The values for $dE_x/dE_f$ listed are evaluated at an excitation energy $E_x = 200$ keV. This ratio decreases slowly with $E_f$. The values listed for $dE_f/dE_i$ are evaluated for a nominal 1 MeV added to the base beam energy $(E_i)_0$. This ratio increases slowly with $E_i$. 

5
3 Prototypical Experiments

In this section, two specific examples are given to illustrate the use of (p, d) reactions with relatively low mass radioactive beams. These examples are given as a guide to typical experimental requirements, in particular to the desired energy and angular resolution and the angular range coverage.

3.1 An $N = 8$ shell closure experiment

The nuclear physics interest in this experiment is the evolution of the intruder $1s_{1/2}$ configuration in light $N = 8$ nuclei. The method is to perform a single neutron transfer pickup from the ground state of the nucleus of interest (which has a possible intruder configuration), and measure the yield to states with known shell model structure in the residual nucleus. The ultimate goal of such a study would be a determination of the amount of $1s$ and $0p$ contributions to the ground state wave function of the classic halo nucleus $^{11}\text{Li}$. Such a measurement is, however, at present beyond the available secondary beam intensity, besides being complicated by the particle instability of $^{10}\text{Li}$.

A possible initial experiment on an $N = 8$ nucleus is the $p(^{12}\text{Be}, ^{11}\text{Be})d$ reaction. The ground state configuration of $^{12}\text{Be}$ has been described in shell model terms to be a mixed $0h\omega$-$2h\omega$ configuration [7]. The single neutron transfer yield to the $1h\omega$ $1s_{1/2}$ and $0h\omega$ $0p_{1/2}$ levels would measure this admixture. These levels are represented by the $1/2^+$ "intruder" ground state and $1/2^-$ $320$ keV first excited state of $^{11}\text{Be}$. A pure $0h\omega$ configuration in the $^{12}\text{Be}$ ground state would lead only to the $1/2^-$ final state, whereas the mixed configuration will lead also to the $1/2^+$ state [8]. Moreover, three-body models which have been used successfully to describe levels in $^{11}\text{Li}$ [9], are now available for the $^{12}\text{Be}$ ground state as $^{10}\text{Be}$ plus two valence neutrons [10]. These calculations include the possibility of core excitation and show that the relative intensities of the $(p_{1/2})^2$ and $(s_{1/2})^2$ neutron component relative to the core in $^{12}\text{Be}$ are substantially different in the inert-core and excited-core calculations.

The basic experimental requirement is, at least for the forward angles (say, $\theta_{cm} \leq 10^\circ$), the ability to measure relative yields of two states separated by $320$ keV in the excitation energy spectrum.
3.2 An experiment to measure the ground state spin of neutron-rich carbon isotopes

The nucleus $^{19}$C is of considerable interest because of the weak binding of its valence neutron, which appears to lead to a one-neutron halo according to the evidence of narrow breakup momentum distributions [11,12]. The ground state may be an "intruder" $1s$ state, in a similar fashion to $^{11}$Be [13]. Shell model calculations with a $0p$-$1s0d$ cross-shell interaction indeed predict that the ground state is $1/2^+$, but a neighbouring $5/2^+$ level lies only 100 keV away [11], which is well within the expected accuracy of the calculations. The shape of the forward angular distribution of the $p(^{19}$C,$^{18}$C)d reaction might be used to distinguish between $L = 0$ and $L = 2$ transfer, if not the actual $J^*$, and would thus give important information about the ground state structure of $^{19}$C. Presently, the beam intensity of $^{19}$C (less than 100 pps at GANIL) put this experiment out of reach. However, like $^{19}$C, the ground state spin and parity for $^{17}$C are not known experimentally. Again, the energies of the $5/2^+$, $3/2^+$, and $1/2^+$ levels given by the shell model are too close to constitute a prediction. Knowledge of the $^{17}$C ground-state spin would help determine the shell model interaction in this $(A, Z)$ region, which would lead to better confidence in the predictions for $^{19}$C. Thus, $p(^{17}$C,$^{16}$C)d is another suitable prototypical reaction to consider.

What are the basic experimental requirements in this case? The shapes of $(p,d)$ angular distributions for the different possible $^{17}$C ground state spin assignments are illustrated in Fig. 4 by calculations with the zero-range DWBA code DWUCK4 [14]. Test calculations have shown that, while the exact location and sharpness of extrema in the cross sections depend on the choice of optical model potential, there is always a clear distinction between the different $L$-transfers. In particular, the $L = 0$ cross section always has a maximum at $0^\circ$ and falls off rapidly with angle. Nevertheless, it would be important to have, or to obtain in the same experiment, proton and deuteron elastic scattering data with appropriate beams or targets. Experiments to obtain similar elastic scattering data in inverse kinematics have already taken place – e.g., $^{11}$Li [15], $^6$He [16,17], and $^{10,11}$Be [16] scattering on proton targets. From the transfer reaction calculations in Fig. 4, it is seen that at $E_t/A = 30$ MeV one would want to measure cross sections out to at least $15^\circ_{c.m.}$ to distinguish clearly between $L = 0$ and $L = 2$ transfer. As noted above, measurements near $0^\circ$ are particularly valuable. The required energy resolution is not at all demanding: the first excited state in $^{16}$C is separated by 1.77 MeV from the ground state.

The basic experimental requirement is to obtain sufficient angular resolution to distinguish the shapes of the predicted angular distributions, bearing in mind some degree of model dependency. From an inspection of Fig. 4, the
Fig. 4. Calculated angular distributions for $p^{(17}C, ^{16}C)d$ at 30 MeV per nucleon for $1s_{1/2}$, $0d_{5/2}$ and $0d_{3/2}$ neutron transfers. The cross sections have arbitrary normalisation.

required resolution appears to be roughly $1.5_\text{cm}^\circ$.

4 Excitation Energy Measurements

4.1 Inherent energy and angular resolution of the detectors

It follows from the angular constraints of the inverse kinematics that a good choice would be to detect the forward-focussed heavy ion in a spectrometer and the light particle in an array of position-sensitive silicon detectors. Hence, we concentrate the discussion on such configurations. If the transfer reaction leads to an unbound state in the residual nucleus, the spectrometer option may not be feasible. Moreover, for reactions where the kinematic focussing is insufficient, the spectrometer could be replaced by a compact detector array. These options are addressed later in Sec. 6.3.

The spectrometer performs several functions. Through the separation of the beam from the particles of interest, $0^\circ$ operation is possible. Further, if the momentum of the heavy ion is measured in an energy-loss spectrometer, the energy spread of the secondary beam is automatically compensated through dispersion matching. In the SPEG spectrometer [18] at GANIL, an analysing dipole with a dispersion of 9.86 cm/% $\Delta p/p$ is used to make a dispersive focus at the target. The theoretical momentum resolution of the spectrometer is $1 \times 10^{-4}$ $\Delta p/p$ for a standard beam emittance of $5\pi$ mm mrad. However, during experiments in which secondary beams from the SISSI device [6] were
elastically scattered off proton and carbon nuclei, the momentum resolution of the SPEG spectrometer was about $5 \times 10^{-4}$ [16,17], which is significantly poorer than the resolution routinely achieved with a primary beam. It appears that this loss in resolution owes mainly to the angular divergence of the beam, and could be ameliorated by the use of beam tracking detectors to measure directly the incident angle of each beam particle. In addition, for inverse kinematic scattering, such as on a proton target, there is a strongly angle-dependent kinematical factor, $k$, which can be corrected only to an accuracy dependent on the precision of the angle measurements in the focal plane.

The above considerations help to specify reasonable limits for the performance of a model spectrometer in these reactions. There is an “uncorrectable kinematical broadening” owing to the finite angular resolution of the detectors used to track the incident beam particles and also the ejectile trajectories near the focal plane of the spectrometer. For the resolutions of these detectors, we take 0.11° and 0.05° [18], respectively. By folding the two angular resolutions together, we take $\Delta \theta$ as 0.12° in our calculations. The focussing and other optical effects are assumed to contribute to an “intrinsic resolution” $\Delta p/p$ for the spectrometer of $3 \times 10^{-4}$ in addition to (and separate from) any uncorrectable kinematical broadening.

At the beam energies we are considering here – up to 30 MeV per nucleon – the maximum recoil energies of protons and deuterons is about 15 MeV. Particles of these energies can be stopped in high resolution silicon detectors. Assuming the use of position-sensitive detectors, perhaps backed by detectors of several millimeters of (position-insensitive) Si(Li), it is reasonable to assume 40 keV resolution in the deposited energy.

For these light particles detected in a silicon array, there is “kinematic broadening” without compensation. This is determined by the angular resolution, which is essentially governed by the distance of the detector from the target, and is thus usually a compromise between the number of detectors available and the angular coverage desired. We have chosen 0.54° for the angular resolution, which represents a 1-mm detector acceptance (or silicon strip width) folded with a beam position measurement of 1-mm accuracy at a distance of 150 mm. These figures are consistent with those planned for the MUST array of silicon strip detectors [19]. A detector with resistive charge-division readout might slightly improve this performance.

Table 2 concerns the detection of the heavy ion in an energy-loss spectrometer at an angle corresponding to approximately $10^\circ_{\text{cm}}$. The columns $\Delta \theta$ and $\Delta p$, give the limiting resolution in $E_x$ arising from the angular and momentum resolution of the spectrometer as described above.

In Table 3, which deals with the case of the detection of the light recoil parti-
Table 2
Major contributions in keV to the resolution of the excitation energy spectra of single neutron stripping and pickup reactions in inverse kinematics, where the heavy ion is detected in a spectrometer. The detection angle corresponds to 10°_cm. The last column is an approximate estimate as a sum in quadrature of the net effect of five non-Gaussian contributions. Other symbols are explained in the text.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_i/A$ (MeV)</th>
<th>$\theta_{lab}$</th>
<th>Origin of contribution</th>
<th>$\Sigma_{quad}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^{(12}\text{Be}, ^{11}\text{Be})d$</td>
<td>30</td>
<td>1.07°</td>
<td>$\Delta \theta$ $\Delta p$ $E_{stragg}$ $\Theta_{1/2}$ $dE/dx$</td>
<td>23 259</td>
</tr>
<tr>
<td>$p^{(12}\text{Be}, ^{11}\text{Be})d$</td>
<td>15</td>
<td>1.06°</td>
<td>84 71 99 74 37 169</td>
<td></td>
</tr>
<tr>
<td>$p^{(77}\text{Kr}, ^{76}\text{Kr})d$</td>
<td>30</td>
<td>0.16°</td>
<td>1404 811 808 723 56 1952</td>
<td></td>
</tr>
<tr>
<td>$p^{(77}\text{Kr}, ^{76}\text{Kr})d$</td>
<td>10</td>
<td>0.10°</td>
<td>334 143 502 570 268 883</td>
<td></td>
</tr>
<tr>
<td>$d^{(76}\text{Kr}, ^{77}\text{Kr})p$</td>
<td>10</td>
<td>0.21°</td>
<td>1140 614 2177 1859 1321 3408</td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Major contributions in keV to the resolution of the excitation energy spectra of single neutron pickup and stripping reactions in inverse kinematics, where the light particle is detected in a silicon detector. Symbols as described in text and Table 2.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_i/A$ (MeV)</th>
<th>$\theta_{lab}$</th>
<th>Origin of contribution</th>
<th>$\Sigma_{quad}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^{(12}\text{Be},d)^{11}\text{Be}$</td>
<td>30</td>
<td>19.0°</td>
<td>$\Delta \theta$ $\Delta E_f$ $\Delta E_i$ $\Theta_{1/2}$ $dE/dx$</td>
<td>649 685</td>
</tr>
<tr>
<td>$p^{(12}\text{Be},d)^{11}\text{Be}$</td>
<td>15</td>
<td>17.8°</td>
<td>66 72 55 89 984 995</td>
<td></td>
</tr>
<tr>
<td>$p^{(77}\text{Kr},d)^{76}\text{Kr}$</td>
<td>30</td>
<td>15.0°</td>
<td>124 55 64 63 186 249</td>
<td></td>
</tr>
<tr>
<td>$p^{(77}\text{Kr},d)^{76}\text{Kr}$</td>
<td>10</td>
<td>6.0°</td>
<td>26 24 23 19 775 777</td>
<td></td>
</tr>
<tr>
<td>$d^{(76}\text{Kr},p)^{77}\text{Kr}$</td>
<td>10</td>
<td>155.3°</td>
<td>52 93 37 60 1309 1316</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Target thickness effects

Primarily because of the need to compensate for the weak radioactive beam intensity, we have chosen a comparatively large thickness for the target material of 3 mg/cm² polypropylene, (CH₂CHCH₃)_n. Another reason for a thick
target arises if the beam spot on the target is large and charged particle detectors are being used. Then we envisage the necessity for a tracking detector immediately in front of the target, and the target thickness needs to be substantially thicker than that of the tracking detector to minimize background from reactions in the latter.

In Tables 2 and 3, two target dependent effects are listed: small angle multiple scattering $\Theta_{1/2}$, and the effect of the energy loss difference between the beam and the detected particle $dE/dx$ in the total target thickness. In addition, the contribution of the energy straggling $E_{stragg}$ on entrance and exit is given in Table 2. This is omitted for Table 3, since it is a small contribution when the light particle is detected (less than 50 keV in the examples studied).

We have not included a contribution from possible target non-uniformity in either table. Polypropylene foils are typically given tolerances of $\pm 10\%$ for the point to point non-uniformity. Without knowing the frequency distribution of thicknesses in a target, it is not possible to predict the effect of non-uniformity on the energy resolution. One would not expect as many points on the target with thickness at the extreme tolerances as there are points close to the mean thickness. In other words, it would seem to be pessimistic to assume that the effect on the energy resolution is given by the energy loss times the tolerance. Although it almost certainly overestimates the variation across the beam spot, the broadening from a single step variation $\Delta t$ in the thickness may be used as a qualitative guide to where one should be especially careful in selecting target material. Values from this simple estimate for the reactions discussed in this section are given in Table 4 and details are given in Appendix A.6. From this table, one can see that target thickness variations are indeed an important consideration for low energy reactions with heavy beams when the heavy ion is detected in a spectrometer. On the other hand, the effect is much reduced when the light particle is detected.

### 4.3 Q-value effects

As seen from Table 3 the energy-loss difference $dE/dx$, caused by the target thickness, dominates the broadening in experiments where the light particle is detected. One might at first think that the resolution would then worsen in proportion to the square of the atomic number of the beam $z^2$, in accordance with the Bethe-Bloch equation. Certainly the energy loss increases with $z^2$, however, since the mass of the beam also increases, the $z^2$ factor tends to be offset by the decreasing kinematic factor $dE_f/dE_i$ discussed in Sec. 2. In fact, in many cases, the energy loss of the light detected particle causes a much greater spread in the excitation energy spectrum than does the energy loss of the beam. The light particle energy, and hence the stopping power,
Table 4
Qualitative comparisons of the change in the deduced excitation energy caused by a step variation $\Delta t$ of 10% in the thickness of a nominally 3 mg/cm$^2$ polypropylene foil:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_i/A$ (MeV)</th>
<th>$\delta E_{\Delta t}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p($^{12}$Be, $^{11}$Be)d</td>
<td>30</td>
<td>65</td>
</tr>
<tr>
<td>p($^{12}$Be, $^{11}$Be)d</td>
<td>15</td>
<td>112</td>
</tr>
<tr>
<td>p($^{77}$Kr, $^{76}$Kr)d</td>
<td>10</td>
<td>3004</td>
</tr>
<tr>
<td>p($^{12}$Be, d)$^{11}$Be</td>
<td>30</td>
<td>36</td>
</tr>
<tr>
<td>p($^{77}$Kr, d)$^{76}$Kr</td>
<td>10</td>
<td>44</td>
</tr>
<tr>
<td>d($^{76}$Kr, p)$^{77}$Kr</td>
<td>10</td>
<td>64</td>
</tr>
</tbody>
</table>

is quite strongly dependent on the reaction Q-value, thus a wide variety of incident beams show a fairly smooth, monotonic decrease of the $E_x$ resolution ($\Sigma_{quad}$ in Table 3) with falling $Q$. This is illustrated in Fig. 5, for which beam energies per nucleon characteristic of a fragmentation facility (30 MeV) were used throughout.

Fig. 5. Calculated energy resolution as a function of reaction Q-value for p(A, d)A$^{-1}$ (solid circles) and d(A, p)A$^{+1}$ (open squares), assuming beam energies per nucleon of 30 MeV. The points are labelled by the beam nucleus. The target thickness is 3 mg/cm$^2$ and $\theta_{cm} = 10^\circ$.

An emphatic demonstration of the effect of $dE_f/dE_i$ is given by one of the examples included in Fig. 5, that of p($^{137}$Xe, d)$^{136}$Xe. If the nuclear reaction takes place at the back of the target, the only energy-loss consideration is that of the heavy-ion beam (135 MeV). Through the $dE_f/dE_i$ factor, this results
Fig. 6. Calculated energy resolution as a function of reaction Q-value for beam energies characteristic of an ISOL-type facility. See caption to Fig. 5.

in a decrease in the deuteron energy of only 198 keV. On the other hand, if the reaction takes place at the front of the target, the energy loss of the deuteron is 361 keV directly. The contribution of the difference of these two changes in deuteron laboratory energy to the excitation energy spectrum is 275 keV. The overall energy resolution from all sources is about 350 keV, which is better than might naively be expected for a transfer reaction with a $z = 54$ beam on a target of 3 mg/cm$^2$.

The difference between the energy loss in the target of the beam and the detected particle, and consequently the broadening of the energy resolution, naturally becomes greater as $E_i/A$ is reduced. Figure 6 is a similar plot to Fig. 5 except that the beam energies per nucleon range from 22 MeV for $A = 11$ to 5 MeV for $A = 137$. These are more realistic beam energies available from an ISOL-type RNB facility than the constant $E_i/A = 30$ MeV used in Fig. 5. The energy resolution in all cases is poorer than that at $E_i/A = 30$ MeV, with the positive Q-value reactions especially affected.

A consequence of the trends in Figs. 5 and 6 is that inverse (d, p) reactions, which tend to have positive Q-values, tend to have poorer energy resolution than the corresponding (p, d) reaction [compare also the p($^{77}$Kr, d) and d($^{76}$Kr, p) examples in Table 3]. We note that although in an inverse (d, p) kinematic experiment with a stable beam [20] a remarkable 80 keV resolution has been obtained in the excitation energy spectrum of d($^{136}$Xe, p)$^{137}$Xe (Q-value = 1.801 MeV), this was only achieved with a very thin target (deuterium absorbed in a 100-µg/cm$^2$ titanium foil).

The Q-value of the reaction has further important bearings on low-energy transfer reactions. If negative, the Q-value will limit the available phase space
for the final state and reduce the reaction cross section. Obviously, if the Q-value is more negative than the available energy per nucleon $E_i/A$, then the reaction is not possible at all. This introduces a dilemma for transfer reactions with ISOL-type low energy beams on the proton-rich side. One would like to use a neutron pickup reaction for such beams, to produce species that are even more proton-rich. Unfortunately, the corresponding reaction Q-values tend to be strongly negative, leading to small reaction cross sections. The more advantageous positive Q-value, obtained by performing the neutron stripping reaction, has the drawback of requiring a more proton-rich beam, with a corresponding lowering of production intensity.

4.4 Summary of implications for reactions with light neutron-rich beams

The first row of Table 2 shows the expected energy resolution for the prototypical $N = 8$ shell closure experiment described in Sec. 3.1. The beam energy is such as might be obtained from a fragmentation production mechanism, as employed with SISSI at GANIL. The calculations show that the required resolution of 320 keV is achievable at $\theta_{cm} = 10^\circ$. The dominant contributions at $E_i/A = 30$ MeV are from the uncertainty in the beam angle measurement and from the momentum resolution of the spectrometer and associated focal plane detector. The former is less severe at more forward angles, while the latter only slowly varies with angle. From further calculations, we estimate that the primary objective of the experiment is feasible for $\theta_{cm} < 15^\circ$.

In contrast, the first row of Table 3 shows that the required resolution is much more difficult to achieve by measuring the recoil deuteron. The dominant contribution in this case is the difference in energy loss of the beam and the deuteron in the target. In order to reduce this contribution to the 320 keV level, one would need a 1-mg/cm$^2$ target.

We now turn to the energy resolution achievable with beams from a two-stage ISOL-type facility such as SPIRAL [2]. The principal advantage of SPIRAL is the high RNB intensities at low energies [21], allowing the spectroscopy of exotic nuclei near the Coulomb barrier. The energy per nucleon of these beams will range from a few MeV up to a maximum of 25 MeV.

At a low beam energy the kinematic factor, i.e. the rate of change of the energy of the detected particle with angle, is also small. This is an important benefit for reactions of the type $p(A, A^{-1})d$, since it will considerably reduce the required angular resolution for the heavy ion (compare the $\Delta \theta$ contribution for the $E_i/A = 30$ and 15 MeV results in Table 2).

Although a lower incident beam energy naturally leads to greater energy-loss and straggling effects from the target, these do not necessarily add significantly
to the energy resolution in the case of heavy-ion detection. Because of the scaling by which the energy of the detected particle is connected to the c.m. energy, a comparison of the $E_i/A = 30$ and 15 MeV results for $(^{12}\text{Be},^{11}\text{Be})$ in Table 2 shows that the effects of energy straggling and multiple scattering on the excitation energy resolution are quite similar at the two incident energies. The energy-loss difference between the $^{11}\text{Be}$ and the deuteron is worse at the lower energy, although not prohibitively so. As already noted, an increase in stopping power also makes target uniformity more important (see Table 4). On the other hand, the beam spot from an ISOL-type facility is likely to be better defined than that from a fragmentation one. From the calculations in Table 2, it appears that one could accept a 10 mg/cm² polypropylene target and still resolve the 320-keV doublet in $^{11}\text{Be}$.

Thus the favoured option for light neutron-rich beams is to detect the beam-like particle. The energy resolution is improved at lower bombarding energies.

### 4.5 Summary of implications for $A > 30$ beams

For beams heavier than about mass 30, the precision of the scattering angle measurement constrains the excitation energy resolution when the heavy ion is measured in a spectrometer. For a beam in the region of mass of 80 [see the p($^{77}\text{Kr},^{76}\text{Kr})d example in Table 2] even this may not be the limiting factor (assuming that one could maintain 0.12° angular resolution). There, the broadening from a thick target, such as the one considered (3 mg/cm²), is dominant. Both these effects continue to worsen as the mass $A$ and atomic number $z$ of the beam increase.

The detection of the light recoil particle by a silicon array may thus be a more attractive proposition for $A > 30$ beams. A comparison of the contributions to the energy resolution for p($^{77}\text{Kr},d$)$^{76}\text{Kr}$ in Table 3 with the corresponding entry for p($^{77}\text{Kr},^{76}\text{Kr})d$ in Table 2 shows that the light particle option is marginally better. Moreover, Table 4 shows that there is considerably less sensitivity to target non-uniformity when the deuteron is detected rather than the $^{76}\text{Kr}$. An immediate conclusion from Fig. 6 is that one will struggle to resolve nuclear levels unless they are separated by at least 1 MeV.

A survey of level spacings in even-even and even-odd nuclei with $30 < A < 100$ reveals that a reasonable amount of spectroscopy should be possible if one attains a resolution of better than 500 keV. This criterion is met for most of the negative Q-value p(A, d)A⁻¹ reactions at $E_i/A = 30$ MeV in Fig. 5. However, it is apparent that the potential applications for the d(A, p)A⁺¹ reactions are much more limited unless a thin target can be used. The spectroscopy of odd-odd final nuclei, with typical level spacings of the order of tens of keV, will be
even more constrained.

In summary, the favoured option for heavy beams is to detect the light ejectile. The resolution is less degraded for higher bombarding energies. Target thickness effects are critical, especially for inverse (d, p) reactions.

5 Angular Distribution Measurements

5.1 Required angular coverage

For experiments where the only measurement is that of the heavy ion in a spectrometer, we have already emphasised the need for beam tracking detectors in order to achieve acceptable energy resolution. Without such detectors, there will be a direct contribution to the angular resolution from the beam emittance. Even with beam tracking, the measurement of an angular distribution becomes extremely challenging when most of the differential cross section is covered within a few tenths of a degree in the laboratory.

It may be more practical to measure the angle of the recoil light particle, which is usually spread over a relatively large range. The important factor governing the required extent of the angular coverage is the laboratory to c.m. conversion. For a given beam energy, this is strongly dependent on the Q-value of the reaction and not, as one might at first expect, on the mass of the heavy ion. For positive and slightly negative Q-values, such as for stripping reactions and pickup reactions with neutron-rich radioactive beams, the laboratory angles of the light particle are well spread out. However, for very negative Q-values, which includes proton-rich beams in pickup reactions, the laboratory angles are restricted to what may be an inconveniently-small angular range.

As the beam energy is lowered the change in the laboratory to c.m. angle scaling with beam energy is modest, but the significant change in the linear momentum transfer means that the angular distribution is pushed out to larger c.m. (and hence, laboratory) angles. From the sharp cut-off model [22], the c.m. angle corresponding to, e.g., the first minimum in an $L = 0$ angular distribution, follows:

$$k \sin(\theta_{cm}^{1/2}) \approx \text{constant}, \quad (1)$$

where $k$ is the scattering wave number. As a specific example, we use the $p^{(17}C,^{16}C)d$ reaction introduced in Sec. 3.2. We have empirically determined the constant for this reaction to be 0.27 from a series of DWBA calculations at different incident proton energies. Equation (1) provides a rough guide to the
Table 5
Predicted angle of the first minimum in the $s_{1/2}$ transfer for $^{17}C + p \rightarrow ^{16}C + d$ as a function of $^{17}C$ beam energy per nucleon, following Eq. (1).

<table>
<thead>
<tr>
<th>$E_i/A$ (MeV)</th>
<th>$k$ (fm$^{-1}$)</th>
<th>$\theta^{\text{geo}}_{\text{cm}}$ (°)</th>
<th>$\theta_{16}^C$ (°)</th>
<th>$\theta_d$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.462</td>
<td>35.8°</td>
<td>3.07°</td>
<td>51.8°</td>
</tr>
<tr>
<td>10</td>
<td>0.653</td>
<td>24.4°</td>
<td>2.02°</td>
<td>42.9°</td>
</tr>
<tr>
<td>15</td>
<td>0.800</td>
<td>19.7°</td>
<td>1.61°</td>
<td>37.2°</td>
</tr>
<tr>
<td>20</td>
<td>0.924</td>
<td>17.0°</td>
<td>1.38°</td>
<td>33.3°</td>
</tr>
<tr>
<td>25</td>
<td>1.033</td>
<td>15.2°</td>
<td>1.22°</td>
<td>30.4°</td>
</tr>
<tr>
<td>30</td>
<td>1.131</td>
<td>13.8°</td>
<td>1.11°</td>
<td>28.1°</td>
</tr>
</tbody>
</table>

maximum c.m. angle needed to be measured in an experiment to distinguish between $L = 0$ and other $L$-transfers. This distinction has particular relevance in studies of halo nuclei and their neighbours. The maximum angle $\theta^{\text{geo}}_{\text{cm}}$ is given in Table 5 for several energies, together with the corresponding angles in the laboratory frame for the detection of $^{16}C$ and the deuteron in inverse kinematics.

5.2 Angular resolution requirements

For the $p(^{17}C, ^{16}C)d$ example discussed in Sec. 3.2, the angular resolution required at $E_i/A = 30$ MeV was estimated to be about $1.5^\circ_{\text{cm}}$. This corresponds to an angular resolution in the laboratory of $0.12^\circ$ if the $^{16}C$ were measured and $2.7^\circ$ if the deuteron were detected. The former requirement is challenging, but is consistent with our previous assumptions about beam tracking and detector resolutions. The deuteron angle measurement should present no great difficulty, at least for relatively thin targets (see comments in Sec. 5.3 below). Similarly, the angular range coverage out to $15^\circ_{\text{cm}}$ corresponds to a maximum angle of about $1.2^\circ_{\text{lab}}$ for the spectrometer and to about $30^\circ_{\text{lab}}$ for the charged particle array (Table 5). Each of these approaches is achievable, with the detection of the deuteron being favoured. On the other hand, the discussion of the energy resolution in Sec. 4 favours the detection of the heavy ion. This dichotomy suggests that there are benefits from coincidence measurements, which will be discussed in Sec. 6.2. At a lower incident energy of $E_i/A = 15$ MeV, life is easier by about 50% for the experimenter. Scaling by $\theta^{\text{geo}}_{\text{cm}}$, the equivalent c.m. resolution is about $2.1^\circ$, which corresponds to roughly $0.18^\circ_{\text{lab}}$ for the spectrometer and to $3.9^\circ_{\text{lab}}$ for the charged particle detector.
5.3 Target thickness effects

For beams where the energy loss in the target is significant in comparison with the total energy, there is another consequence of the inverse kinematics to be considered. The relationship between the c.m. angle and the laboratory detection angle (light or heavy particle) may vary greatly with the interaction depth in the target. This effect clearly impedes the measurement of an angular distribution. As an example, we consider again the $^{77}\text{Kr}, d)^{76}\text{Kr}$ reaction at $E_i/A = 10$ MeV. If the nuclear reaction takes place at the front of the target, at the full beam energy of 770 MeV, the c.m. angle for a deuteron detected at $6.03^\circ_{\text{lab}}$ is $10^\circ$. If, on the other hand, the reaction occurs after the beam has lost 123-MeV energy in a 3-mg/cm$^2$ polypropylene target, the corresponding c.m. angle for a deuteron detected at the same $6.03^\circ_{\text{lab}}$ is about $16^\circ$. The two extreme cases entail a change in c.m. angle of roughly $6^\circ$. Of course the deuteron energy is also changed, (indeed, this is part of the energy loss broadening). The angular information may be recovered in a kinematically complete experiment where the energies and angles of both particles in the exit channel are measured. In that case, the interaction depth in the target can be reconstructed, as demonstrated by Bennett et al. [23], and hence the true c.m. angle. Note that when the energy loss of the beam in the target is small, as in $^{12}\text{Be}, d)^{11}\text{Be}$ at 360 MeV ($dE_i = 0.96$ MeV in 3 mg/cm$^2$ polypropylene), the change in c.m. angle is correspondingly negligible: about $2 \times 10^{-3}$ degrees at $10^\circ_{\text{lab}}$.

6 Kinematic Coincidence Measurements

6.1 Background from breakup, scattered beam, and target contaminants

In the $^{12}\text{Be}$ prototypical experiment, the $^{11}\text{Be}$ from the direct reaction (at $E \approx 353$ MeV) must be distinguished from the one-neutron breakup process in the target. The single-neutron separation energy $S_{1n}$ of $^{12}\text{Be}$ being 3.2 MeV, the threshold for the breakup process corresponds to a $^{11}\text{Be}$ energy of about 356.8 MeV for $E_i/A = 30$ MeV. The breakup contamination of the 2-body spectrum hence commences at $^{11}\text{Be}$ energies a few MeV above that for the ground state, and it may be expected to have a peak at an equivalent two-body excitation energy of about 30 MeV. This background can be removed by requiring a coincidence with the recoil light particle from the 2-body reaction at the appropriate kinematic angle. The limited particle identification requirements for the deuteron can be achieved by kinematic reconstruction of the energy signal against the identified heavy ion in the spectrometer. An array of ten $5 \times 5$ cm$^2$ detectors could easily provide 30% coincidence efficiency.
Another consideration, especially for neutron pickup experiments, is the scattering and breakup of the beam inside the spectrometer, which is necessarily operated at 0°. For neutron transfer reactions, the rigidity $Bp$ of the beam will be close to that of the particles of interest. Thus, for inverse (p, d) reactions, some inelastically scattered beam particles may have the same $Bp$ as for the 2-body reaction in the target and be difficult to separate by the focal plane detector. This scattering background can be rejected through the coincident detection of the deuteron in a similar manner to the rejection of the target breakup background.

A further advantage of the coincidence technique is the separation of events arising from the transfer reaction on hydrogen from those arising from the same reaction on other constituents of the target – notably, carbon in the case of a polymer target. Usually the recoiling carbon (or other heavy ion) will have an energy of a few MeV and may not even penetrate the dead layer of a charged particle array; otherwise, it could be stopped by a thin foil. While a gas target may be quite feasible for non-spectrometer experiments (see, e.g., the proposed experimental arrangements considered by Hardy [1]), the extended targets used in dispersion-matched spectrometers are commonly solid polymer ones, such as polythene. One might, of course, measure a spectrum on a pure carbon target and subtract this from the (CH)$_n$ spectrum, but the low-count rate from the intensity of the radioactive beam may make this a less attractive option.

We note that a target backing may also give rise to a background from, e.g., evaporated light particles [20], and is another potentially serious source of unwanted counts. However, a backing does not give rise to an energy-loss difference contribution to the energy resolution. Neither does a target backing contribute to the straggling in inverse-kinematic (d, p) reactions when the proton is detected at angles greater than 90°$_{\text{lab}}$.

6.2 Improved resolution of final states

As suggested by a comparison of the $\Delta\text{theta}$ contributions in Tables 2 and 3, there may be advantages in a measurement of the scattering angle of the recoil light particle in a charged-particle array in place of a measurement of the heavy-ion angle in a spectrometer. The latter measurement is rather demanding and is an important factor in the limiting resolution. On the other hand, the energy resolution from a measurement of the light ion energy is often poor, being dominated by energy loss effects in the target. These considerations suggest that improved resolution should be obtained by a combination of the light particle-angle and heavy-ion energy.
Computer simulations of kinematic coincidence experiments have been performed under the following scheme. There is a loop over the nominal detected heavy-ion angle, and at each value a large number of events is simulated. For each event, random spreads in the incident beam energy and angle are generated, weighted by Gaussian probability distributions. Next, an interaction depth in the target is randomly chosen. From this interaction depth, the energy loss and the FWHM of the energy straggling and of the small angle scattering is determined for each of the beam particle, the outgoing heavy ion and the outgoing light particle. The parameters concerning the beam particle are used to correct and spread the incident energy and angle. The two-body kinematics are then performed, based upon the adjusted beam energy and the scattering angle of the heavy ion. The relationship between the scattering angles $\theta_{HI}^{\text{true}}$ and $\theta_{d}^{\text{true}}$, and the detection angles $\theta_{HI}$ and $\theta_{d}$ is illustrated in Fig. 7. Since we explicitly consider the use of a dispersion-matched energy-loss spectrometer for the detection of the heavy ion, the outgoing energy of the heavy ion $E'_{HI}$ is corrected for that part of the spread in beam energy not arising from the target thickness. This correction is performed by adding back the difference $\Delta E_{i}$ of the actual beam energy (before the target) from the central value, multiplied by an angle-independent scaling factor, \textit{viz}:

$$E'_{HI} = E_{HI} + \Delta E_{i} \times \frac{dE_{f}}{dE_{i}}$$

(2)

where the scaling factor $dE_{f}/dE_{i}$ is evaluated at $0^\circ$ only. This simple correction has been found to be adequate for the reaction studied, and approximates well the actual operation of the spectrometer.
The resulting energy parameters \( E'_{HI} \) and \( E_d \) are then adjusted for the energy loss of the outgoing particles in the target and "smeared" by Gaussian probability distributions to simulate the energy loss and straggling of these particles in the target. Similarly, \( \theta_{HI} \) and \( \theta_d \) are smeared to simulate the multiple small angle scattering in the target. There is a further Gaussian spreading to account for the inherent resolution of the detectors (spectrometer and charged particle array).

The input parameters for an example simulation were chosen to match those of the calculations discussed in Sec. 4.4, except that here the beam angle resolution is explicitly separated from that of the detector. The reaction is \( p^{(12}\text{Be},^{11}\text{Be})d \) at a central beam energy of 360 MeV. We take FWHM spreads in the beam energy and angle of of 1.0% and 0.11°, respectively. The spectrometer and focal plane detector resolutions for \( E_{Be} \) and \( \theta_{Be} \) are 211 keV \( (3 \times 10^{-4} \text{ in } \Delta p/p) \) and 0.05° FWHM, respectively. The target is 3 mg/cm\(^2\) polypropylene as usual.

Figure 8 is a plot of the simulated energy signal of \(^{11}\text{Be}\) detected in the dispersion-matched spectrometer, against the angle of the same heavy ion. The counts are the sum of those for the reaction leading equally to either the ground state or to the 320-keV excited state of \(^{11}\text{Be}\). On the right hand side are shown projections of the measured \(^{11}\text{Be}\) energy for three regions of the angle domain (indicated by dashed vertical lines on the scatterplot). The width of the projections in angle are the same as the FWHM of the angular resolution in \( \theta_{Be} \). It is seen that the 320-keV level is resolved only at the most forward angles.

Figure 9 is a similar plot to the above, except that the detected angle of the recoil deuteron is substituted for that of the \(^{11}\text{Be}\). The detection resolution assumed for \( \theta_d \) is 0.54° FWHM. The vertical axis is the measured energy of the \(^{11}\text{Be}\), as for Fig. 8. The improvement in the resolution over Fig. 8, both in the scatterplot and in the projected \( E_{Be} \) histograms, is clear. The greater angular range of the deuteron allows a less demanding angular resolution compared to that for the heavy ion, and yet a better separation of the ground and excited states is obtained. Even with a simulated detector resolution of 1°\(_{lab}\), the resolution of the 320-keV doublet is lost only at the largest angles.

This kinematic coincidence technique appears to be most suited to light mass beams, or, more generally, to when thick targets need to be used. For beams of, say, A > 30, and targets no thicker than several mg/cm\(^2\), a measurement of the energy and angle of the light particle (\( p \) or \( d \)) in singles mode is more appropriate, as shown by the direct calculations in Sec. 4.5. Further computer simulations of different combinations of the energy and angle signals for light and heavy mass beams have confirmed the results of the direct calculations.
Fig. 8. Energy of $^{11}$Be detected in a dispersion-matched spectrometer against the laboratory angle of the $^{11}$Be, with projections on the $E_{Be}$ axis of three vertical slices in $\theta_{Be}$. As a reference, $2_{\text{lab}}^\circ$ corresponds to $18.8^\circ_{\text{cm}}$.

Fig. 9. As for Fig. 8, except that the detected angle of the recoil deuteron is substituted for that of the $^{11}$Be. The vertical axis remains $E_{Be}$.

6.3 Correction for the energy spread in light RNBs

The correction for the beam energy is the most critical feature in the calculations of the previous section, which essentially reflects that it is the energy
of the heavy ion which is being measured. An uncorrected 1% spread in the beam energy would destroy the approximately 260-keV resolution achieved in Fig. 9. Regarding the beam angle measurement, simulations have shown that the energy resolution obtained is not nearly so sensitive to its precision, especially if $\theta_d$ is used in conjunction with $E_{HI}$. Even with a beam angle spread of 1°, the 320-keV states in the $^{11}$Be final state are still well resolved.

Thus the energy-loss mode for the spectrometer is a crucial feature for the study of transfer reactions with light RNBs produced by the fragmentation method where an energy spread of the order of 1% could be expected. However, if timing detectors can be placed in the beam, it may be possible to correct the beam energy event-by-event to the order of 0.1% by a time-of-flight measurement. Furthermore, ISOL beams may have an intrinsic energy spread of this order (for the SPIRAL project at GANIL, this would imply the need for single turn extraction from the post-accelerating cyclotron [2]). In these cases, one might be able to detect the heavy ion in a position-sensitive silicon detector close to 0°, and still maintain sub-MeV resolution.

An alternative to the energy-loss spectrometer in general might be to make an energy-dispersive focus at the target and correct the beam energy by measuring the beam position event-by-event. The required accuracy in the beam position measurement is not great: If, for example, one takes the 9.86 cm/% $\Delta p/p$ dispersion of the analysing magnet in the first half of the SPEG spectrometer at GANIL [18], an accuracy of 0.5 cm is sufficient to obtain $10^{-3}$ in $\Delta E/E$. The heavy ions would be detected in an array of silicon position-sensitive detectors, which would ideally have sufficient dynamic range to detect also the light ions with appropriate resolution. The advantage would be a large increase in the solid angle of acceptance for the heavy ions, especially at low bombarding energies where the kinematic focussing is reduced.

To avoid the need for an extended target (as above), the beam could be re-focussed to a spot after its position was tagged at a prior dispersive plane. A mass achromat such as LISE [24] at GANIL, could be employed for this purpose. For the first stage of LISE (1.71 cm/% $\Delta p/p$), the position resolution required at the intermediate dispersive plane for a 0.1% precision in the beam energy is 0.85 mm. The tagging detector would have to be thin, because energy-loss and angular straggling would be detrimental to the performance of the spectrometer. Although these requirements are demanding, thin-foil position-sensitive detectors with such performance have been made (see, for example, ref. [25]).
7 Summary and conclusions

The (p,d) and (d,p) reactions are potentially useful spectroscopic tools in inverse kinematics with radioactive beams. Their advantages include simplicity of interpretation (reliability of model calculations) and large transfer cross sections for single particle (or hole) states.

Specific physics goals will determine the best experimental configuration. However, considerations of the kinematics and consequent energy broadening factors in the excited state spectrum will almost always be important factors. That the laboratory to c.m. scaling cannot be neglected has been shown by the calculations described in Sec. 4. The angular resolution of the detected particle is often critical, as discussed in Secs. 4.5 and 5. These various considerations, illustrated by prototypical experiments, indicate that for relatively low mass beams detection of the heavy ion in an energy-loss spectrometer is a convenient approach to obtain an energy resolution of a few hundred keV. Here, the sensitivity to the beam energy spread is addressed through a dispersed beam spot, which also suggests that if the beam position is measured event-by-event, a silicon detector array might be used to detect the heavy-ion in place of the spectrometer. A charged particle array for the coincidence detection of the light particle is most likely to be necessary to remove background from the spectrum, especially in the (p,d) transfer direction. Simulations have shown that the energy resolution can be substantially improved by combining the measurements of the heavy-ion energy (or momentum) in an energy-loss spectrometer and the the angle of the recoil light particle in a charged-particle array.

For heavy beams, on the other hand, a spectrometer is less useful, and one has to rely on detecting the outgoing light particle in singles mode. The energy resolution in such experiments will be limited by target thickness effects. Nevertheless, for certain inverse kinematics (p,d) reactions at, say, $E_i/A = 30$ MeV, light-particle detection can give c.m. resolution at the 300 keV level even for targets of several mg/cm$^2$ thickness. Better energy resolution will be possible with thinner targets, but this may preclude the use of tracking detectors.

A promising aspect for the future is that the instrumental energy and angular resolution requirements will be more relaxed at the beam energies provided by future ISOL-type facilities such as SPIRAL than at the energies provided by the projectile fragmentation method. Radioactive beams of high intensity will not only allow transfer reactions to be performed with exotic isotopes, but will also permit collimation of beams close to the stability line, thus giving better emittance quality.
Acknowledgement

We are indebted to the theory group at the University of Surrey for ideas concerning the $^{12}$Be proposal, to Wolfgang Mittig for illuminating discussions, and to Françoise Auger and John Kelley for helpful communications. JSW would like to thank the members of the Physics Department at Surrey for the hospitality shown during his short, but productive, visit. Financial support from the Centre National de la Recherche Scientifique and the Engineering and Physical Sciences Research Council is gratefully acknowledged.

A Program to estimate energy resolution in inverse kinematics reactions

A utility program, FERGIE, has been developed to facilitate computation of the energy resolution governing inverse kinematics experiments. It is not restricted to neutron transfer reactions (for example, calculations for elastic and inelastic scattering reactions are possible), but does assume two-body initial and final states. FERGIE is not a Monte-Carlo program, but rather it directly calculates the sensitivity of the energy resolution to various experimental parameters.

The program makes use of several standard routines for energy loss and multiple scattering calculations, adapted for inverse kinematics. For portability reasons, nuclear masses may be entered "by hand", although the program can access a nuclear mass excess database [26]. The nuclear masses should be entered as precisely as possible, certainly to the keV level and should include any nuclear excitation energy. The calculated reaction Q-value is printed out by the program for verification.

The target material, for the purposes of the energy loss and multiple scattering calculations, is entered separately from the specification of the target nucleus in the reaction. It can either be an element (entered as an atomic symbol) or one of a list of possible compounds, such as polypropylene. The stopping power of a compound is calculated from the stopping power of its elements scaled according to the fraction by weight. Ionisation potentials for compounds are taken from the compilation of ref. [27] where available. The increased effective thickness of target material traversed by an outgoing particle at a non-zero angle is taken into account. The target itself can be at an angle to the perpendicular to the beam, as would be necessary if one wanted to detect particles close to 90° to the beam direction.

For inverse kinematic (p, d) reactions, the kinetic energy is typically double-valued for a given laboratory angle. The relativistic two-body kinematics rou-
tine, uses either the “first” or the “second” kinematic solution, as selected by the user. Generally, one wants the solution which corresponds to the forward-going c.m. kinematics, since this will obviously have the larger cross section. For the case of the detection of the heavy product one wants the first kinematic solution. When the light particle is detected in stripping reactions, such as $d(A, p)A^+$, one again wants the first kinematic solution, but at backward laboratory angles, whereas in pickup reactions, e.g. $p(A, d)A^-$, it is the second kinematic solution which corresponds to the forward-going c.m. situation.

The various contributions to the energy broadening are described in Secs. A.1 through A.6 below. The equations give the detected particle energy in the laboratory frame. These are converted within the program to the excitation energy of the residual nucleus (in the c.m. frame) by multiplication by the absolute value of $dE_x/dE_f$, calculated at the appropriate value of $E_f$.

There are few experimental data on reactions with RNBs with which to compare the predictions of the program. The elastic scattering data of Cortina-Gil et al. [16,17] measured with a spectrometer were used in the present work to set realistic input parameters in the program for the case of heavy-ion detection. To evaluate the light-ion detection mode, we first compare with an experiment in which a stable $^{136}$Xe beam bombarded a deuterated titanium target [20]. The energy resolution of the PIN diode array used to detect the outgoing protons is given in Ref. [28] as 35 keV. Assuming that the beam emittance and energy spread were negligible contributions in this case, we used FERGIE to estimate the experimental resolution. We find that at the detection angle considered (120.8°) the dominant contributions to the excitation energy broadening are those of the detector resolution (59 keV), the energy loss difference (43 keV), and the multiple scattering in the target (39 keV). These add in quadrature to give the $\approx 80$ keV resolution seen experimentally. As a second comparison with experiment, we consider the study by Suomi-järvi et al. [29] of inelastic proton scattering in inverse kinematics using both a degraded $^{40}$Ar beam and a radioactive secondary beam of $^{38}$S. With the given experimental parameters, in both cases the principal contribution to the resolution in the excitation energy spectrum is predicted to be the angular resolution. However, if the angular acceptance of the detectors is assumed to be the only angular factor, we obtain a total resolution of about 600 keV, whereas roughly 700 keV was observed for the degraded $^{40}$Ar beam and 800 keV for the $^{38}$S beam. In fact, the range in angles subtended by the large beam spot size on the tilted target was somewhat greater than detector angular resolution [30], accounting for the apparent discrepancy. A dominant contribution from the beam spot size is also a plausible explanation for the poorer resolution with the $^{38}$S secondary RNB compared to the degraded primary beam.
A.1 Angular broadening

This is the kinematic effect wherein the energy of the detected particle varies rapidly as a function of angle. An estimate of the angular resolution $\Delta \theta$ (in degrees) is required in the input to the program. This might be either the angular resolution of the detectors (light particle detection) and/or the angular spread in the beam (heavy ion detection). However, the increased angular spread from multiple scattering in the target is estimated internally by the program and is dealt with separately (Sec. A.4). Note that both $dE_I/d\theta$ and $d^2E_I/d\theta^2$ are calculated and used by the program. The second order term can be significant for strongly inverse kinematic transfer reactions near $0^\circ$ where the first order term vanishes. For example, in $p(^{77}\text{Kr}, ^{78}\text{Kr})d$ at $0.1^\circ$, $d^2E/d\theta^2$ is ten times greater than $dE/d\theta$. Higher order terms are negligible because of the multiplicative $(\Delta \theta)^n/n!$ factor.

The calculated contribution is

$$\delta E_{\Delta \theta} = \left| \frac{dE_I}{d\theta} \Delta \theta + \frac{1}{2!} \frac{d^2E_I}{d\theta^2}(\Delta \theta)^2 \right| \text{keV.} \quad (A.1)$$

A.2 Detector resolution, Doppler broadening and beam energy spread

In the program the ‘detector resolution’ covers the energy broadening directly from the resolution limit of the detector or spectrometer. There is a choice of entering either an inverse momentum resolution of a spectrometer $R$, or a fixed energy resolution $\Delta E_{\text{det}}$ in keV. We will discuss the use of a spectrometer first. At each scattering angle $\delta E_{\Delta \theta}$ is calculated from $2E_I/R$. There is in addition an option to calculate the Doppler broadening from the gamma emission of an excited state in flight around a spectrometer. The “worst case” uniform distribution of emission angles is assumed, leading to $\delta E_\gamma = 2\beta E_\gamma$. Beene and DeVries [31] give a detailed treatment of the effect on the broadening when the angular distribution of the $\gamma$ emission is considered properly. In the case of the $320 \text{ keV } \gamma$ emission from the $p(^{12}\text{Be}, ^{11}\text{Be}\ast)d$ reaction at $E_i/A = 15 \text{ MeV}$, the simple formula gives an additional contribution to the broadening in the c.m. of 117 keV. The $\gamma$ de-excitation broadening is not output separately, but folded into the contribution from the energy resolution of the detector. Thus,

$$\delta E_{\Delta \theta} = \left\{ \left( \frac{2E_I}{R} \right)^2 + \delta E_\gamma^2 \right\}^{1/2} \text{keV,} \quad (A.2)$$

where $E_I$ and $E_\gamma$ are in keV (the prime indicates that $E_\gamma$ is already in the c.m. system – this is properly taken into account in the program).
For the intrinsic energy resolution of a charged particle array, $\Delta E_{\text{det}}$ (in keV), the contribution is simply:

$$\delta E_{\Delta E_j} = \Delta E_{\text{det}} \text{ keV.} \quad (A.3)$$

Further, the energy spread of the beam in keV can be input (if not considered to be cancelled by the use of an energy-loss spectrometer), and this gives a contribution:

$$\delta E_{\Delta E_i} = \Delta E_i \left| \frac{dE_f}{dE_i} \right| \text{ keV.} \quad (A.4)$$

### A.3 Energy straggling

The energy loss straggling in the target of both the incident ion and the detected particle are calculated. The absolute values of the effect (each for the total thickness) on the excitation energy spectrum are then averaged. Normally one might follow a procedure of calculating the two stragglings (beam and detected particle) for half the target thickness and then add the two in quadrature. We consider an average to be a better approximation for inverse scattering, especially when the light particle is detected, because a change in the beam energy affects $E_x$ in a quite different way to a change in the kinetic energy of the detected particle. The "target" material specified in the input is independent of the scattering target nucleus for the kinematics.

Two formulae for the width $\sigma(E_f)$ of the energy loss straggling have been compared. The first is from [32] based on [33]:

$$\sigma^2(E) = 1.55 \times 10^{-4} z_{\text{eff}}^2 z_i t \frac{A_t}{A_i} H_\beta \text{ keV}^2 \quad (A.5)$$

where $z_{\text{eff}}$ is the effective charge state of the moving ion (calculated from the formula in [34]), $t$ is the target thickness in mg/cm$^2$, and

$$H_\beta = 1 + \frac{I}{3m_e e^2 \beta} \ln \left( \frac{2m_e e^2 \beta}{I} \right),$$

where $I$ is the ionisation potential of the target atoms in eV.

The second formulation, which has been used in the present work, is from Tschelår [35], implemented in the same manner as in ref. [36]. Here the square of the width is given by:

$$\sigma^2(E) = \sigma_0^2 t Q \frac{1 - \beta^2/2}{1 - \beta^2} \times 10^{-6} \text{ keV}^2 \quad (A.6)$$
where the non-relativistic Bohr value is

$$\sigma_B^2 = 4\pi N\sigma_{el}^2 z_{eff}^2 (z_t/A_t),$$

and $Q$ approximates an integral involving the stopping power. For $^{12}\text{Be}$ incident on 3 mg/cm$^2$ polypropylene foils, Eqs. (A.6) and (A.5) agree to within 5%. For a $^{77}\text{Kr}$ beam, Eq. (A.6) gave an energy-loss straggling 21% greater than Eq. (A.5). However, the effect of the latter, reduced by the c.m. transformation, is small compared with the effect of the energy-loss straggling of the deuteron, for which the two formulae agree closely.

The final result as a FWHM is

$$\delta E_{stragg} = \frac{2.35}{2} \left( \sigma(E_i) \left| \frac{dE_i}{dE} \right| + \sigma(E_f) \right) \text{keV.} \quad (A.7)$$

where $\sigma(E_i)$ is evaluated for the $z_{eff}$ of the beam and $\sigma(E_f)$ is evaluated for the $z_{eff}$ of the ejectile.

### A.4 Multiple scattering in the target

We use the formula from the "Review of Particle Properties" [37] for the angular width, converted here to a FWHM:

$$\Theta_{MS} = \frac{32.0}{\sqrt{\beta cp}} z \sqrt{t/X'_0 [1 + 0.038 \ln(t/X'_0)]} \text{MeV,} \quad (A.8)$$

where $p$, $\beta c$ and $z$ are the momentum, velocity and charge number of the ion, and $t/X'_0$ is the thickness of the scattering medium in terms of radiation lengths. The radiation length is calculated from the simple parameterisation of Dahl as suggested in [37], except here we convert to the same units as $t$:

$$X'_0 = \frac{7.164 \times 10^5 A_t}{z_t(z_t + 1) \ln(287/\sqrt{z_t})} \text{mg/cm}^2.$$  

Optionally, an older set of formulae from Marion and Young [38] may be selected in the program. This gives (for FWHM of angular spread):

$$\Theta_{MS} = 37.8 \chi_{\omega} z \sqrt{z_t(z_t + 1) tB/(A_t \beta cp)} \quad (A.9)$$

where $B$ is found by solving

$$B - \ln B = b$$

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Table A.1
Coefficients for the polynomial fit to $\chi_w$ as a function of $B$ in Eq. (A.10).

<table>
<thead>
<tr>
<th>$a_j$</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>5.4062 $10^{-1}$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>1.0969 $10^{-1}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-1.2005 $10^{-2}$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>6.1864 $10^{-4}$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-1.2129 $10^{-8}$</td>
</tr>
</tbody>
</table>

with

$$b = \ln[2730(z + 1)z^{1/3}z_i^2] - 0.1544$$

and $\chi_w$ is found from the graph in ref. [38] to which we have made a fourth-order polynomial fit:

$$\chi_w = \sum_{j=0}^{4} a_j B^j. \quad (A.10)$$

The coefficients $a_j$ are listed in Table A.1. Equation (A.9) tends to give multiple scattering angles roughly 30% larger than Eq. (A.8). In the present work, we have used results from the more modern formula, Eq. (A.8).

In the same manner as Eq. (A.1), the contribution is

$$\delta E_{MS} = \left| \frac{dE_i}{d\theta} \Theta_{MS} + \frac{1}{2!} \frac{d^2E_i}{d\theta^2} \Theta_{MS}^2 \right| \quad (A.11)$$

The effect of the multiple scattering of the beam and that of the detected particle on the excitation energy spectrum are both calculated using Eq. (A.11) and then averaged. The averaging procedure is preferred over an addition in quadrature for similar reasons as those given in Sec. A.3.

A.5 Energy loss difference

This concerns the energy loss differences of the beam and the detected particle in the target. The formulae used for the stopping powers are from Ziegler [34] except for $E_i/A < 1.5$ MeV. Below 1.5 MeV per nucleon, we use the approximation

$$-dE/dx = K \sqrt{E}$$

where $K$ is the stopping power determined at 1.5 MeV per nucleon. This follows the prescription of Northcliffe and Schilling [39].
The broadening effect is a consequence of the variation in the depth at which the reaction takes place in the target. For the case of the detection of the heavy ion, the stopping powers of the incident and detected particles \( (dE_i/dx \text{ and } dE_f/dx, \text{ respectively}) \) are usually quite similar, so the effect is small. (For neutron transfer, the \( z \) is, of course, unchanged). The same is not usually true when the light particle is detected, although the transformation of the laboratory energies of beam and light recoil can provide some unexpected results. As an example, in \( p^{(12}\text{Be},d) \), the maximum energy loss of the beam (reaction taking place near the back face of the target) has an effect on the excitation energy spectrum that is only one twentieth of the effect arising from the maximum energy loss of the deuteron (reaction near the front of the target). In the inverse \( (d,p) \) reaction, since the proton is detected in the backward hemisphere, the extreme situations are either maximum energy losses by both the heavy ion and the proton or no energy loss at all.

In both cases [(p, d) and (d, p)] the contribution to the broadening is taken to be the \textit{full} energy loss difference rather than half the difference:

\[
\delta E_{dE/dx} = \left| \frac{dE_i}{dx} \frac{dE_f}{dE_i} \pm \frac{dE_f}{dx} \right| t \text{ keV},
\]

(A.12)

where the stopping powers are in keV per mg/cm\(^2\) and the positive sign used when the particle is detected in the backward hemisphere (same side of the target as the incident beam); the negative sign otherwise. Note that \( dE_f/dE_i \) can be positive or negative, depending on whether a decrease of the beam energy results in a decrease or increase in the laboratory energy of the detected particle.

\[A.6\] Target Non-uniformity

A simple estimate of the effect of target non-uniformity is provided in the program. This is meant to be taken as a comparative guide rather than a quantitative evaluation. If one considers the reaction to take place in the middle of the target of thickness \( t \), for a step variation \( \Delta t \) mg/cm\(^2\) in the thickness of the target, and taking an average of the energy loss of the beam and detected particle, the change in \( E_x \) is:

\[
\delta E_{\Delta t} = \left( \frac{dE_i}{dx} \frac{dE_f}{dE_i} \right) \frac{\Delta t}{2} \text{ keV}. \tag{A.13}
\]
References


