A Phenomenological Analysis of Heavy Hadron Lifetimes

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Abstract

A phenomenological analysis of lifetimes of bottom and charmed hadrons within the framework of the heavy quark expansion is performed. The baryon matrix element is evaluated using the bag model and the nonrelativistic quark model. We find that bottom-baryon lifetimes follow the pattern $\tau(\Omega_b) \simeq \tau(\Xi_b^-) > \tau(\Lambda_b) \simeq \tau(\Xi_b^0)$. However, neither the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ nor the absolute decay rates of the $\Lambda_b$ baryon and $B$ mesons can be explained. One way of solving both difficulties is to allow the presence of linear $1/m_Q$ corrections by scaling the inclusive nonleptonic width with the fifth power of the hadron mass $m_{HQ}$ rather than the heavy quark mass $m_Q$. The hierarchy of bottom baryon lifetimes is dramatically modified to $\tau(\Lambda_b) > \tau(\Xi_b^-) > \tau(\Xi_b^0) > \tau(\Omega_b)$: The longest-lived $\Omega_b$ among bottom baryons in the OPE prescription now becomes shortest-lived. The replacement of $m_Q$ by $m_{HQ}$ in nonleptonic widths is natural and justified in the PQCD-based factorization approach formulated in terms of hadron-level kinematics. For inclusive charmed baryon decays, we argue that since the heavy quark expansion does not converge, local duality cannot be tested in this case. We show that while the ansatz of substituting the heavy quark mass by the hadron mass provides a much better description of the charmed-baryon lifetime ratios, it appears unnatural and unpredictable for describing the absolute inclusive decay rates of charmed baryons, contrary to the bottom case.

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I. INTRODUCTION

The lifetime differences among the charmed mesons $D^+$, $D^0$ and charmed baryons have been studied extensively both experimentally and theoretically since later seventies. It was realized very early that the naive parton model gives the same lifetimes for all heavy particles containing a heavy quark $Q$ and that the underlying mechanism for the decay width differences and the lifetime hierarchy of heavy hadrons comes mainly from the spectator effects like $W$-exchange and Pauli interference due to the identical quarks produced in heavy quark decay and in the wavefunction (for a review, see [1,2]). The spectator effects were expressed in eighties in terms of local four-quark operators by relating the total widths to the imaginary part of certain forward scattering amplitudes [3–5]. With the advent of heavy quark effective theory (HQET), it was recognized in early nineties that nonperturbative corrections to the parton picture can be systematically expanded in powers of $1/m_Q$ [6,7]. Subsequently, it was demonstrated that this $1/m_Q$ expansion is applicable not only to global quantities such as lifetimes, but also to local quantities, e.g. the lepton spectrum in the semileptonic decays of heavy hadrons [8]. Therefore, the above-mentioned phenomenological work in eighties acquired a firm theoretical footing in nineties, namely the heavy quark expansion (HQE), which is a generalization of the operator product expansion (OPE) in $1/m_Q$. Within this QCD-based framework, some phenomenological assumptions can be turned into some coherent and quantitative statements and nonperturbative effects can be systematically studied. As an example, consider the baryon matrix element of the two-quark operator $\langle \Lambda_b | \bar{b}b | \Lambda_b \rangle$. The conventional quark-model evaluation of this matrix element is model-dependent:

$$\frac{\langle \Lambda_b | \bar{b}b | \Lambda_b \rangle}{2m_{\Lambda_b}} = \left\{ \begin{array}{ll}
1 & \text{NQM;} \\
\int d^3r [u_b^2(r) - v_b^2(r)] & \text{bag model,}
\end{array} \right. (1.1)$$

where $u(r)$ and $v(r)$ are the large and small components, respectively, of the quark wavefunction. However, the matrix element (1.1), which is equal to unity in the nonrelativistic quark model (NQM), becomes smaller in the bag model due to the contribution from the lower component of the quark wavefunction. In the HQE approach, it is given by [see Eq. (2.8) below]

$$\frac{\langle \Lambda_b | \bar{b}b | \Lambda_b \rangle}{2m_{\Lambda_b}} = 1 + \frac{1}{2m_b^2} \left( \frac{\langle \Lambda_b | \bar{b}(iD_\perp)^2b | \Lambda_b \rangle}{2m_{\Lambda_b}} \right) + \frac{1}{4m_b^2} \left( \frac{\langle \Lambda_b | \bar{b}\sigma \cdot Gb | \Lambda_b \rangle}{2m_{\Lambda_b}} \right) + O(1/m_b^3), \quad (1.2)$$

with $D_\perp^\mu = \partial^\mu - \nu^\mu v \cdot D$. This expression is not only model independent but also contains nonperturbative kinetic and chromomagnetic effects which are either absent or overlooked in
the earlier quark-model calculations.

Based on the OPE approach for the analysis of inclusive weak decays, predictions for the ratios of bottom hadron lifetimes have been made by several groups. The first correction to bottom hadron lifetimes is of order $1/m_b^2$ and it is model independent [9]:

\[
\begin{align*}
\frac{\tau(B^-)}{\tau(B_d)} &= 1 + \mathcal{O}(1/m_b^3), \\
\frac{\tau(B_s)}{\tau(B_d)} &= (1.00 \pm 0.01) + \mathcal{O}(1/m_b^3), \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} &= 0.98 + \mathcal{O}(1/m_b^3). 
\end{align*}
\]  

(1.3)

The $1/m_b^2$ corrections are small and essentially canceled out in the lifetime ratios. Spectator effects in inclusive decays due to the Pauli interference and $W$-exchange contributions account for $1/m_b^3$ corrections and they have two eminent features: First, the estimate of spectator effects is model dependent; the hadronic four-quark matrix elements are usually evaluated by assuming the factorization approximation for mesons and the quark model for baryons. Second, $1/m_b^3$ corrections can be quite significant due to a phase-space enhancement by a factor of $16\pi^2$. Predictions made in [10] for lifetime ratios of bottom hadrons are

\[
\begin{align*}
\frac{\tau(B^-)}{\tau(B_d)} &= 1.0 + 0.05 \left( \frac{f_B}{200 \text{ MeV}} \right)^2, \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} &\gtrsim 0.9.
\end{align*}
\]  

(1.4)

Experimentally [11], while the $B^-$ and $B_d$ lifetimes are very close, it appears that the $\Lambda_b$ lifetime is shorter than the $B$ meson one:

\[
\begin{align*}
\frac{\tau(B^-)}{\tau(B_d)} &= 1.06 \pm 0.04, \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} &= 0.79 \pm 0.06 \quad \text{(world average)},
\end{align*}
\]  

(1.5)

where the world average value for $\tau(\Lambda_b)/\tau(B_d)$ is dominated by LEP experiments [11]. It is evident that the conflict between experiment (1.5) and theoretical expectations from (1.3) or (1.4) is striking and intriguing. This has motivated several subsequent studies trying to understand the enhancement of the $\Lambda_b$ decay rate [9,12–15]. For example, a model-independent analysis in [9] gives

\[
\begin{align*}
\frac{\tau(B^-)}{\tau(B_d)} &\simeq 1 + 0.03B_1 + 0.004B_2 - 0.70\varepsilon_1 + 0.20\varepsilon_2, \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} &\simeq 0.98 - 0.17\varepsilon_1 + 0.20\varepsilon_2 - (0.012 + 0.021\tilde{B})r,
\end{align*}
\]  

(1.6)

where $\varepsilon_i$, $B_i$, $\tilde{B}$, $r$ are the hadronic parameters to be introduced below in Sec. II. Note that while the ratio $\tau(B^-)/\tau(B_d)$ is predicted to be greater than unity in [10] [see (1.4)], it
is argued in [9] that the unknown nonfactorizable contributions in (1.6) characterized by \( \varepsilon_i \) make it impossible to have reliable predictions on the magnitude of the lifetime ratio and even the sign of corrections. Since the measured ratio of \( \tau(B^-)/\tau(B_d) \) is very close to unity, it follows from (1.6) that \( \varepsilon_1 \approx 0.3 \varepsilon_2 [9] \). Then it is clear that the data for the ratio \( \tau(\Lambda_b)/\tau(B_d) \) cannot be accommodated by the theoretical prediction (1.6) without invoking a too large value of \( r \) or \( \tilde{B} \), which is expected to be order unity. It is reasonable to conclude that the \( 1/m_b^3 \) corrections in the heavy quark expansion do not suffice to describe the observed lifetime differences between \( \Lambda_b \) and \( B_d \).

In order to employ the OPE approach to compute inclusive weak decays of heavy hadrons, some sort of quark-hadron duality has to be assumed (for an extensive discussion of quark-hadron duality and its violation, see [16,17]). Consider the inclusive semileptonic decay. The OPE cannot be carried out on the physical cut in the complex \( v \cdot q \) plane since \( T_{\mu\nu} \), the time-ordered product of two currents, along the physical cut is dominated by physical intermediate hadron states which are nonperturbative in nature. To compute \( T_{\mu\nu} \) or the Wilson coefficients by perturbative QCD, the OPE has to be performed in the unphysical region far away from the physical cut. The question is then how to relate the operator product expansion for \( T_{\mu\nu} \) in the unphysical region to the physical quantities in the physical Minkowski space. Since the physically observable quantity is related to the imaginary part of \( T_{\mu\nu} \), it can be reliably computed by deforming the contour of integration into the unphysical region \([18,16]\), provided that the physical quantity involves certain integrals of \( T_{\mu\nu} \) in the physical region. This procedure is called “global duality” \([16]\). Global quark-hadron duality also means that the hadronic cross section is dual or matching to the OPE-based quark cross section. However, unlike the total cross section in \( e^+e^- \) annihilation, there is a small portion of the contour near the physical cut where global duality can no longer be applied. As stressed in \([16]\), one must resort to local duality to justify the use of the OPE in this small region. Fortunately, the contribution is of order \( \Lambda_{QCD}/m_Q \) and can be neglected for quantities smeared over an energy scale of order \( \Lambda_{QCD} \).

Global quark-hadron duality for inclusive semileptonic decays, namely the matching between the hadronic and OPE-based expressions for decay widths or smeared spectra in semileptonic \( B \) and \( \Lambda_b \) decays has been explicitly proved to the first two terms in \( 1/m_b \) expansion and the first order in \( \alpha_s \) in the Shifman-Voloshin (SV) limit \([19]\). The hadronic decay rate is calculated by summing over all allowed exclusive decay channels. In the SV limit for \( B \) meson
decays via $b \to c$ transitions, the dominant hadronic final states are the $D$ and $D^*$. (At zero recoil, the quark-mixing-favored semileptonic decays of a $B$ meson in the heavy quark limit can only produce a $D$ or $D^*$ meson \cite{19,20}.) The exclusive decay rates or distributions for $B \to (D+D^*)\ell\bar{\nu}$ depend on hadron masses, whereas the inclusive decay rates evaluated by the OPE depend on quark masses. Global duality is then proved by showing explicitly the equality of inclusive and exclusive decay rates. Note that this proof of global duality in QCD is valid only in the SV limit. Beyond this limit, it becomes difficult to sum over all allowed exclusive semileptonic decay channels and evaluate all of them. It was shown recently in \cite{21} that a proof of quark-hadron global duality in the general kinematic region to order $(\Lambda_{QCD}/m_B)^2$ can be achieved in the PQCD-based factorization approach, which is formulated in terms of meson-level kinematics rather than the quark-level one. It was demonstrated explicitly in \cite{21} that the integrated quark-level spectrum equals to the hadron-level spectrum and that linear $1/m_b$ corrections to the total decay rate are nontrivially canceled out, in agreement with the OPE expectation \cite{6,7}.  

Unlike the semileptonic inclusive decays in which the use of the OPE is validated by deforming the contour away from the physical cut, it is pointed out in \cite{16} that there is no external momentum $q$ in inclusive nonleptonic decays which allows analytic continuation into the complex plane. Therefore, the OPE is \textit{a priori} not justified in this case and local duality has to be invoked in order to apply the OPE directly in the physical region. It is obvious that local quark-hadron duality is less firm and secure than global duality, although its validity has been proved to the first two terms in $1/m_Q$ expansion and the first order in $\alpha_s$ in the SV limit under the factorization hypothesis \cite{22}. It should be stressed that quark-hadron duality is \textit{exact} in the heavy quark limit, but its systematical $1/m_Q$ expansion is still lacking. It is very likely that $1/m_Q$ corrections to quark-hadron duality behave differently for inclusive semileptonic and nonleptonic decays. Motivated by the conflict between theory and experiment for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$, it was suggested in \cite{14} that the assumption of local duality is not correct for nonleptonic inclusive width and that the presence of linear $1/m_b$ corrections is strongly indicated by the data. Moreover, the $1/m_b$ corrections are well

\footnote{The absence of linear $1/m_b$ corrections to decay widths is trivial in the SV limit since the inclusive decay rates depend on $\Delta M = m_B - m_D$ rather than $m_B$, and $\Delta M = \delta m + \mathcal{O}(1/m_b^2)$ with $\delta m = m_b - m_c$.}
described by the simple ansatz that the heavy quark mass \( m_Q \) is replaced by the decaying hadron mass in the \( m_Q^5 \) factor in front of all nonleptonic widths. It is easily seen that the factor \( (m_B/m_{\Lambda_b})^5 = 0.73 \) is very close to the observed value of \( \tau(\Lambda_b)/\tau(B_d) \).

Under this ansatz, a much better description of lifetimes of bottom and charmed hadrons was shown in [14]. Irrespective of the lifetime ratio problem, there is another important reason why this ansatz is welcome. The absolute decay rate of the \( B \) meson predicted in the OPE approach is at least 20% smaller than the experimental value (see Sec. III below). We shall show in Sec. III that the discrepancy between theory and experiment is greatly improved when the nonleptonic width scales with \( m_B^5 \).

In the aforementioned factorization approach of [21], the nonleptonic width \( \Gamma_{\text{had}}^{\text{NL}} \) of bottom hadrons scales with \( m_{H_b}^5 \). Local duality means that a replacement of meson-level kinematics by quark kinematics, for example, \( m_{H_b} = m_b(1 + \tilde{\Lambda}_{H_b}/m_b + \cdots, \cdots \) etc., will turn \( \Gamma_{\text{had}}^{\text{NL}} \) into \( \Gamma_{\text{OPE}}^{\text{NL}} \), the OPE-based decay rate. Consequently, the relation between violation of local duality and the above-mentioned ansatz will become natural in the factorization approach.

In the present paper we will study spectator effects in inclusive nonleptonic and semileptonic decays and analyze the lifetime pattern of heavy hadrons. In particular, we focus on the lifetimes of heavy baryons and study the implications of broken local duality. We will demonstrate that the lifetime hierarchy of bottom baryons is dramatically modified when the quark mass is replaced by the hadron mass in nonleptonic widths. The layout of this paper is organized as follows. In Sec. II we give general heavy quark expansion expressions for inclusive nonleptonic and semileptonic widths and pay attention to the evaluation of baryon four-quark matrix elements and the nonperturbative parameter \( \lambda_2 \) for baryons. We then study bottom-hadron lifetimes in Sec. III and apply the ansatz mentioned above. In Sec. IV we examine the applicability of the same prescription to charmed baryon decays. Discussions and conclusions are given in Sec. V.

II. FRAMEWORK

In this section we write down the general expressions for the inclusive decay widths of heavy hadrons and evaluate the relevant hadronic matrix elements. It is known that the inclusive decay rate is governed by the imaginary part of an effective nonlocal forward transition operator \( T \). When the energy released in the decay is large enough, the nonlocal effective action
can be recast as an infinite series of local operators with coefficients containing inverse powers of the heavy quark mass $m_Q$. Under this heavy quark expansion, the inclusive nonleptonic decay rate of a heavy hadron $H_Q$ containing a heavy quark $Q$ is given by [6,7]

$$\Gamma_{\text{NL}}(H_Q) = \frac{G_F m_Q^5}{192\pi^3 N_c} \xi \frac{1}{2m_{H_Q}} \left\{ \left( c_1^2 + c_2^2 + \frac{2c_1 c_2}{N_c} \right) \times \right.$$

$$\left[ (I_0(x,0,0) + I_0(x,x,0))\langle H_Q|\bar{Q}Q|H_Q \rangle \right.$$ 

$$- \frac{1}{m_Q^2}(I_1(x,0,0) + I_1(x,x,0))\langle H_Q|\bar{Q}\sigma \cdot GQ|H_Q \rangle \right\}$$

$$- \frac{4}{m_Q^2} \frac{2c_1 c_2}{N_c} \langle I_2(x,0,0) + I_2(x,x,0)\rangle\langle H_Q|\bar{Q}\sigma \cdot GQ|H_Q \rangle \right\}$$

$$\left. + \frac{1}{2m_{H_Q}}\langle H_Q|\mathcal{L}_{\text{spec}}|H_Q \rangle + O(1/m_Q^4), \right) \tag{2.1}$$

where $\sigma \cdot G = \sigma_{\mu\nu}G^{\mu\nu}$, $c_1$, $c_2$ are Wilson coefficient functions, $N_c = 3$ is the number of color, the factor $\xi$ takes care of the relevant CKM matrix elements, for example, $\xi = |V_{cb}V_{ud}|^2$ for quark-mixing-favored bottom decay, $I_0$, $I_1$ and $I_2$ are phase-space factors:

$$I_0(x,0,0) = (1 - x^2)(1 - 8x + x^2) - 12x^2 \ln x,$$

$$I_1(x,0,0) = \frac{1}{2}(2 - x) \frac{d}{dx}I_0(x,0,0) = (1 - x)^4,$$

$$I_2(x,0,0) = (1 - x)^3 \tag{2.2}$$

for $b \to c\bar{u}d$ ($x = m_c^2/m_b^2$) or $c \to s\bar{u}d$ ($x = m_s^2/m_c^2$) transition and

$$I_0(x,0) = v(1 - 14x - 2x^2 - 12x^3) + 24x^2(1 - x^2) \ln \frac{1 + v}{1 - v},$$

$$I_1(x,x,0) = \frac{1}{2}(2 - x) \frac{d}{dx}I_0(x,x,0),$$

$$I_2(x,x,0) = v(1 + \frac{x}{2} + 3x^2) - 3x(1 - 2x^2) \ln \frac{1 + v}{1 - v} \tag{2.3}$$

for $b \to cc\bar{s}$ transition with $v \equiv \sqrt{1 - 4x}$.

The dimension-six four-quark operators $\mathcal{L}_{\text{spec}}$ in (2.1) describe spectator effects in inclusive decays of heavy hadrons and are given by [3–5]

$$\mathcal{L}_{\text{spec}} = \frac{G_F m_Q^2}{2\pi} \xi (1 - x)^2 \left\{ (c_1^2 + c_2^2)\langle \bar{Q}Q \rangle\langle \bar{q}_1 q_1 \rangle + 2c_1 c_2\langle \bar{Q}q_1 \rangle\langle \bar{q}_1 Q \rangle \right\}$$

$$- \frac{G_F m_Q^2}{6\pi} \xi \left\{ \frac{1}{2}(1 - x)^2 \langle (1 + \frac{x}{2})(\bar{Q}Q)\langle \bar{q}_2 q_2 \rangle - (1 + 2x)\bar{Q}\sigma(1 - \gamma_5)q_2^\alpha q_2(1 + \gamma_5)Q^\alpha \right\}$$

$$+ (2c_1 c_2 + N_c c_2^2)(1 - x)^2\left\{ (1 + \frac{x}{2})(\bar{Q}q_2)\langle \bar{q}_2 Q \rangle - (1 + 2x)\bar{Q}(1 - \gamma_5)q_2\bar{q}_2(1 + \gamma_5)Q \right\} \right\}$$
\[- \frac{G_F^2 m_Q^2}{6 \pi} \xi \left\{ c_1^2 \sqrt{1 - 4x} \left[ (1 - x)(\bar{q}b q_3) - (1 + 2x)\bar{Q}^\alpha (1 - \gamma_5) q_3 \bar{q}_3 q_3 (1 + \gamma_5) Q^\alpha \right] \\
+ (2c_1c_2 + N_c c_2^2) \sqrt{1 - 4x} \left[ (1 - x)(\bar{q}b q_3)(\bar{q}_3 q) - (1 + 2x)\bar{Q}(1 - \gamma_5) q_3 q_3 (1 + \gamma_5) Q \right] \right\} \\
- \frac{G_F^2 m_Q^2}{6 \pi} \xi \left\{ c_1^2 (1 - x)^2 \left[ (1 + \frac{x}{2})(\bar{q}b q_3)(\bar{q}_3 q) - (1 + 2x)\bar{Q}^\alpha (1 - \gamma_5) q_3 \bar{q}_3 q_3 (1 + \gamma_5) Q^\alpha \right] \\
+ (2c_1c_2 + N_c c_2^2)(1 - x)^2 \left[ (1 + \frac{x}{2})(\bar{q}b q_3)(\bar{q}_3 q) - (1 + 2x)\bar{Q}(1 - \gamma_5) q_3 \bar{q}_3 q_3 (1 + \gamma_5) Q \right] \right\}, \quad (2.4)\]

where \((\bar{q}q) \equiv \bar{q}' \gamma_\mu (1 - \gamma_5) q, \) and \(\alpha, \beta\) are color indices. Note that for charm decay, \(Q = c, \) \(q_1 = u, \) \(q_2 = d, \) and \(q_3 = s\) and for bottom decay, \(Q = b, \) \(q_1 = u, \) \(q_2 = d, \) \(q_3 = s.\)

The last term in (2.4) is due to the constructive interference of the \(s\) quark and hence it occurs only in charmed baryon decays. The third term in (2.4) exists only in bottom decays with \(c\bar{c}\) intermediate states. For inclusive nonleptonic decays of heavy mesons, the first term in (2.4) corresponds to a Pauli interference and the second and third terms to \(W\)-exchange contributions. For heavy baryon decays, the first term is a \(W\)-exchange contribution and the rest are interference terms. The phase-space suppression factors e.g. \((1 - x)^2, \sqrt{1 - 4x}, \cdots\) etc. in (2.4) are derived in [23,9].

Several remarks are in order. (i) There is no linear \(1/m_Q\) corrections to the inclusive decay rate due to the lack of gauge-invariant dimension-four operators [18,6], a consequence known as Luke’s theorem [24]. Nonperturbative corrections start at order \(1/m_Q^2.\) (ii) It is clear from Eqs. (2.1) and (2.4) that there is a two-body phase-space enhancement factor of \(16\pi^2\) for spectator effects relative to the three-body phase space for heavy quark decay. This implies that spectator effects, being of order \(1/m_Q^2,\) are comparable to and even exceed the \(1/m_Q^3\) terms. (iii) For charmed meson decay, the \(1/N_c\) correction to \(\Gamma_{NL}\) characterized by the term \((2c_1c_2/N_c)\langle H_c | c\bar{c}| H_c \rangle\) is found to be compensated by the nonperturbative gluonic effect [i.e. the term proportional to \(I_2(x, 0, 0)\)]. This cancellation is small for \(B\) meson decay due to the smallness of \(1/m_b^2.\) This indicates that the rule of discarding \(1/N_c\) terms [25] is operative in charm decays but not so for the \(B\) meson case. (iv) Thus far the Wilson coefficients and four-quark operators in Eq. (2.4) are renormalized at the heavy quark mass scale. Sometimes the so-called hybrid renormalization [5,26] is performed to evolve the four-quark operators (not the Wilson coefficients !) from \(m_Q\) down to a low energy scale, say, a typical hadronic scale \(\mu_{had}.\) The underlying reason is that the factorizable approximation for meson matrix elements and the quark model for baryon matrix elements are believed to be more reliable at
the scale $\mu_{\text{had}}$. The evolution from $m_Q$ down to $\mu_{\text{had}}$ will in general introduce new structures such as penguin operators. However, in the present paper we will follow [9] to employ (2.1) and (2.4) as our starting point for describing inclusive weak decays since it is equivalent to first evaluating the four-quark matrix elements renormalized at the $m_Q$ scale and then relating them to the hadronic matrix elements renormalized at $\mu_{\text{had}}$ through the renormalization group equation, provided that the effect of penguin operators is neglected.

For inclusive semileptonic decays, apart from the heavy quark decay contribution there is an additional spectator effect in charmed-baryon semileptonic decay originating from the Pauli interference of the $s$ quark [27]. It is now ready to deduce the inclusive semileptonic widths from (2.1) and the last term in (2.4) by putting $c_1 = 1$, $c_2 = 0$ and $N_c = 1$:

$$\Gamma_{\text{SL}}(H_Q) = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{\text{CKM}}|^2 \eta(x, x, 0) \left[ I_0(x, 0, 0) \langle H_Q|\bar{Q}Q|H_Q \rangle - \frac{1}{m_Q^2} I_1(x, 0, 0) \langle H_Q|\bar{Q}\sigma \cdot GQ|H_Q \rangle \right]$$

$$- \frac{G_F^2 m_c^2}{6\pi} |V_{cs}|^2 \left[ \frac{1}{2m_{H_c}} (1 - x)^2 \left( (1 + \frac{x}{2}) (\bar{c}s) (\bar{s}c) - (1 + 2x) \bar{c}(1 - \gamma_5) \bar{s}(1 + \gamma_5) c \right) \right] , \quad (2.5)$$

where $\eta(x, x, 0)$ with $x = (m_\ell/m_Q)^2$ is the QCD radiative correction to the semileptonic decay rate. Its general analytic expression is given in [28]. The special case $\eta(x, 0, 0)$ is given in [29] and it can be approximated numerically by [30]:

$$\eta(x, 0, 0) \approx 1 - \frac{2\alpha_s}{3\pi} \left[ (\pi^2 - \frac{31}{4}) (1 - \sqrt{x})^2 + \frac{3}{2} \right] . \quad (2.6)$$

With $x = 0$ and the replacement $\alpha_s \rightarrow \frac{3}{4} \alpha$, (2.6) is reduced to the well-known QED correction to the muon decay. The second term in Eq. (2.5) occurs only in the semileptonic decay of $\Xi_c$ and $\Omega_c$ baryons.

We next turn to the 2-body matrix elements $\langle H_Q|\bar{Q}Q|H_Q \rangle$. The use of the equation of motion

$$\bar{Q}Q = \bar{Q}\gamma_0 Q + \frac{1}{2m_Q^2} \bar{Q}(iD_\perp)^2 Q + \frac{1}{4m_Q^2} \bar{Q}\sigma \cdot GQ + O(1/m_Q^3), \quad (2.7)$$

with $D_\perp^\mu = \partial^\mu - v^\mu v \cdot D$, leads to

$$\frac{\langle H_Q|\bar{Q}Q|H_Q \rangle}{2m_{H_Q}} = 1 - \frac{K_H}{2m_Q^2} + \frac{G_H}{2m_Q^2} , \quad (2.8)$$

with

$$K_H \equiv - \frac{1}{2m_{H_Q}} \langle H_Q|\bar{Q}(iD_\perp)^2 Q|H_Q \rangle = - \lambda_1 ,$$

$$G_H \equiv \frac{1}{2m_{H_Q}} \langle H_Q|\bar{Q} \frac{1}{2} \sigma \cdot GQ|H_Q \rangle = d_H \lambda_2 . \quad (2.9)$$
The mass of the heavy hadron $H_Q$ is then of the form

$$m_{H_Q} = m_Q + \tilde{\Lambda}_{H_Q} - \frac{\lambda_1}{2m_Q} - \frac{d_H\lambda_2}{2m_Q},$$  \hspace{1cm} (2.10)

where the three nonperturbative HQET parameters $\tilde{\Lambda}_{H_Q}$, $\lambda_1$, $\lambda_2$ are independent of the heavy quark mass and in general $\tilde{\Lambda}_{H_Q}$ is different for different heavy hadrons. Since $\sigma \cdot G \sim \vec{S}_Q \cdot \vec{B}$ and since the chromomagnetic field is produced by the light cloud inside the heavy hadron, it is clear that $\sigma \cdot G$ is proportional to $\vec{S}_Q \cdot \vec{S}_\ell$, where $\vec{S}_Q$ ($\vec{S}_\ell$) is the spin operator of the heavy quark (light cloud). More precisely,

$$d_H = -\langle H_Q | 4\vec{S}_Q \cdot \vec{S}_\ell | H_Q \rangle = -2[S_{tot}(S_{tot} + 1) - S_Q(S_Q + 1) - S_\ell(S_\ell + 1)].$$  \hspace{1cm} (2.11)

Therefore, $d_H = 3$ for $B$, $D$ mesons, $d_H = -1$ for $B^*$, $D^*$ mesons, $d_H = 0$ for the antitriplet baryon $T_Q$, $d_H = 4$ for the spin-$\frac{1}{2}$ sextet baryon $S_Q$ and $d_H = -2$ for the spin-$\frac{3}{2}$ sextet baryon $S_Q^*$. It follows from (2.10) that

$$\lambda_2^\text{meson} = \frac{1}{4}(m_{P^*}^2 - m_P^2) = \begin{cases} 0.12 \text{GeV}^2 & \text{for B meson;} \\ 0.14 \text{GeV}^2 & \text{for D meson,} \end{cases}$$

$$\lambda_2^\text{baryon} = \frac{1}{6}(m_{S_Q^*}^2 - m_{S_Q}^2).$$  \hspace{1cm} (2.12)

The values of $\lambda_2^\text{baryon}$ will be fixed later. As for the kinetic energy parameter $\lambda_1$ we use [31]

$$\lambda_1^\text{meson} \sim \lambda_1^\text{baryon} = -(0.4 \pm 0.2) \text{ GeV}^2.$$  \hspace{1cm} (2.13)

This leads to

$$m_b - m_c = (\langle m_B \rangle - \langle m_D \rangle) \left(1 - \frac{\lambda_1}{2\langle m_B \rangle \langle m_D \rangle} \right) = (3.40 \pm 0.03) \text{ GeV},$$  \hspace{1cm} (2.14)

where $\langle m_P \rangle = \frac{1}{2}(m_P + 3m_{P^*})$ denotes the spin-averaged meson mass.

We will follow [9] to parametrize the hadronic matrix elements in a model-independent way. For meson matrix elements of four-quark operators, we follow [9] to define the parameters $B_i$ and $\varepsilon_i$:

$$\langle \bar{B}_q | (\bar{q}b)(\bar{q}b) | \bar{B}_q \rangle = f_{B_q}^2 m_{B_q}^2 B_1,$$

$$\langle \bar{B}_q | \bar{b}(1 - \gamma_5)q(1 + \gamma_5)b | \bar{B}_q \rangle = f_{B_q}^2 m_{B_q}^2 B_2,$$

$$\langle \bar{B}_q | (\bar{b} t^a q)(\bar{q} t^a b) | \bar{B}_q \rangle = f_{B_q}^2 m_{B_q}^2 \varepsilon_1,$$

$$\langle \bar{B}_q | \bar{b} t^a(1 - \gamma_5)q t^a(1 + \gamma_5)b | \bar{B}_q \rangle = f_{B_q}^2 m_{B_q}^2 \varepsilon_2.$$  \hspace{1cm} (2.15)
where \( \bar{q'} t^a q \equiv \bar{q'} t^a \gamma_\mu (1 - \gamma_5) q \) and \( t^a = \lambda^a / 2 \). Under the factorization approximation, \( B_i \) and \( \varepsilon_i \) are given by \( B_i = 1 \) and \( \varepsilon_i = 0 \), but they will be treated as free parameters here. As a consequence of (2.15), we obtain

\[
\langle \bar{B}_q | (\bar{b} b)(\bar{q} q) | \bar{B}_q \rangle = f^2_{\bar{B}_q} m^2_{\bar{B}_q} \left( \frac{1}{3} B_1 + 2 \varepsilon_1 \right),
\]

\[
\langle \bar{B}_q | \bar{b}^\alpha (1 - \gamma_5) q^\beta q^\delta (1 + \gamma_5) b^\alpha | \bar{B}_q \rangle = f^2_{\bar{B}_q} m^2_{\bar{B}_q} \left( \frac{1}{3} B_2 + 2 \varepsilon_2 \right).
\] (2.16)

As for the baryon matrix elements of four-quark operators we have to rely on the quark model. We first consider the MIT bag model \([32]\) and define three four-quark overlap integrals:

\[
a_q = \int d^3 r [u^2_q(r)u^2_Q(r) + v^2_q(r)v^2_Q(r)],
\]

\[
b_q = \int d^3 r [u^2_q(r)v^2_Q(r) + v^2_q(r)u^2_Q(r)],
\]

\[
c_q = \int d^3 r u_q(r)v_q(r)u_Q(r)v_Q(r),
\] (2.17)

which are expressed in terms of the large and small components \( u(r) \) and \( v(r) \), respectively, of the quark wavefunction. For the antitriplet heavy baryon \( T_Q \) or the sextet heavy baryon \( \Omega_Q \) (recall that only the \( \Omega^0_c \) and \( \Omega^-_c \) of the sextet baryons decay weakly), the four baryon matrix elements

\[
\langle T_Q | (\bar{Q} q)(\bar{q} Q) | T_Q \rangle, \quad \langle T_Q | (\bar{Q} Q)(\bar{q} q) | T_Q \rangle,
\]

\[
\langle T_Q | \bar{Q}(1 - \gamma_5)q\bar{Q}(1 + \gamma_5)Q | T_Q \rangle, \quad \langle T_Q | \bar{Q}^\alpha (1 - \gamma_5)q^\beta q^\delta (1 + \gamma_5)Q^\alpha | T_Q \rangle
\]

are not all independent. First of all, we have

\[
\langle T_Q | (\bar{Q} q)(\bar{q} Q) | T_Q \rangle = -(a_q + b_q)(2m_{T_Q}),
\]

\[
\langle \Omega_Q | (\bar{Q}s)(\bar{s} Q) | \Omega_Q \rangle = -\frac{1}{3} (18a_s + 2b_s + 32c_s)(2m_{\Omega_Q}),
\] (2.18)

(see e.g., Ref. [33] for the technical detail of the bag model evaluation), where we have taken into account the fact that there are two valence \( s \) quarks in the wavefunction of the \( \Omega_Q \). Second, since the color wavefunction for a baryon is totally antisymmetric, the matrix element of \((\bar{Q} Q)(\bar{q} q)\) is the same as that of \((\bar{Q} q)(\bar{q} Q)\) except for a sign difference. Thus we follow [9] to define a parameter \( \tilde{B} \)

\[
\langle T_Q | (\bar{Q} Q)(\bar{q} q) | T_Q \rangle = -\tilde{B} \langle T_Q | (\bar{Q} q)(\bar{q} Q) | T_Q \rangle,
\]

\[
\langle \Omega_Q | (\bar{Q} Q)(\bar{s} s) | \Omega_Q \rangle = -\tilde{B} \langle \Omega_Q | (\bar{Q} s)(\bar{s} Q) | \Omega_Q \rangle,
\] (2.19)
so that $\bar{B} = 1$ in the valence-quark approximation. Third, it is straightforward to show that

$$\langle T_Q | \bar{Q}^\alpha \gamma_\mu \gamma_5 Q^\beta \bar{q}^\beta \gamma_\mu (1 - \gamma_5) q^\alpha | T_Q \rangle = 0,$$

$$\langle \Omega_Q | \bar{Q}^\alpha \gamma_\mu \gamma_5 Q^\beta \bar{q}^\beta \gamma_\mu (1 - \gamma_5) q^\alpha | \Omega_Q \rangle = 4 \left( a - \frac{b}{3} \right) (2m_{\Omega_Q}). \quad (2.20)$$

The first relation in (2.20) is actually a model-independent consequence of heavy-quark spin symmetry [9]. Since

$$\bar{Q}^\alpha \gamma_\mu \gamma_5 Q^\beta \bar{q}^\beta \gamma_\mu (1 - \gamma_5) q^\alpha = -\bar{Q}(1 - \gamma_5)q\bar{q}(1 + \gamma_5)Q - \frac{1}{2}(\bar{Q}q)(\bar{q}Q), \quad (2.21)$$

it follows from (2.20) that

$$\langle T_Q | \bar{Q}^\alpha (1 - \gamma_5)q^\beta \bar{q}^\beta (1 + \gamma_5)Q^\alpha | T_Q \rangle = -\bar{B}\langle T_Q | \bar{Q}(1 - \gamma_5)q\bar{q}(1 + \gamma_5)Q | T_Q \rangle$$

$$= -\frac{1}{2}\bar{B}(a_q + b_q)(2m_{T_Q}),$$

$$\langle \Omega_Q | \bar{Q}^\alpha (1 - \gamma_5)s^\beta \bar{s}^\beta (1 + \gamma_5)Q^\alpha | \Omega_Q \rangle = -\bar{B}\langle \Omega_Q | \bar{Q}(1 - \gamma_5)s\bar{s}(1 + \gamma_5)Q | \Omega_Q \rangle$$

$$= \bar{B}(a_s - \frac{5}{3}b_s - \frac{16}{3}c_s)(2m_{\Omega_Q}). \quad (2.22)$$

In the nonrelativistic quark model (NQM), baryon matrix elements of four-quark operators are the same as that of (2.18) and (2.22) except for the replacement:

$$a_q \rightarrow |\psi_{Qq}(0)|^2 = \int d^3r \, u_q^2(r)u_Q^2(r), \quad b_q \rightarrow 0, \quad c_q \rightarrow 0. \quad (2.23)$$

In general, the strength of destructive Pauli interference and $W$-exchange is governed by $a_q + b_q$ in the bag model and $|\psi(0)|^2$ in the NQM. However, it is well known in hyperon decay that the bag model calculation of $a_q + b_q$ gives a much smaller value than the nonrelativistic estimate of $|\psi(0)|^2$: $a_u + b_u \sim 3 \times 10^{-3}\text{GeV}^3$, while $|\psi(0)|^2 \sim 10^{-2}\text{GeV}^3$. We shall see later that this also occurs in bottom baryon decay. As pointed out in [34], naively one may be tempted to conclude that the relativistic models are presumably more reliable. For example, the lower component of the wavefunction is needed to reduce the NQM prediction $g_A = \frac{5}{3}$ to the experimental value of 1.25. However, the difference between $a_u + b_u$ and $|\psi(0)|^2$ is not simply attributed to relativistic corrections; it arises essentially from the distinction in the spatial scale of the wavefunction especially at the origin. As a consequence, both models give a quite different quantitative description for processes sensitive to $|\psi(0)|^2$. It has been long advocated in [35] that a small value of $|\psi(0)|^2$ should be discarded since a realistic potential that fits to the orbital-excitation spectrum yields $\langle \delta(\vec{r}_1 - \vec{r}_2) \rangle \sim 10^{-2}\text{GeV}^3$. Empirically, it also appears that the NQM works better for charmed baryon decays [4,34].

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In the following we will consider the NQM estimate of baryon matrix elements. Consider $|\psi_{\Lambda_b}^\Lambda_b(0)|^2$ as an example. A straightforward calculation of hyperfine splitting between $\Sigma_b$ and $\Lambda_b$ yields [36]

$$m\Sigma_b - m\Lambda_b = \frac{16\pi}{9} \alpha_s(m_b) \frac{m_b - m_q}{m_q m_q^2} |\psi_{\Lambda_b}^\Lambda_b(0)|^2,$$

(2.24)

where the equality $|\psi_{\Sigma_b}^\Sigma_b(0)|^2 = |\psi_{\Lambda_b}^\Lambda_b(0)|^2$ has been assumed. The uncertainties in Eq. (2.24) associated with $\alpha_s(m_b)$ and the constituent quark mass $m_q$ can be reduced by introducing the $B$-meson wavefunction at the origin squared $|\psi_B^B(0)|^2 = \frac{1}{12} f_B^2 m_B$ which is related to the $B^*$ and $B$ mass difference by $m_{B^*} - m_B = \frac{32}{9} \pi \alpha_s(m_b)|\psi_B^B(0)|^2/(m_b m_q)$. Hence,

$$|\psi_{\Lambda_b}^\Lambda_b(0)|^2 = \frac{2m_q}{m_b - m_q} \frac{m\Sigma_b - m\Lambda_b}{m_{B^*} - m_B} |\psi_{\Lambda_b}^B(0)|^2.$$  

(2.25)

Another method is proposed by Rosner [13] to consider the hyperfine splittings of $\Sigma_b$ and $B$ separately so that

$$|\psi_{\Lambda_b}^\Lambda_b(0)|^2 = |\psi_{\Sigma_b}^\Sigma_b(0)|^2 = \frac{4}{3} \frac{m\Sigma_b - m\Lambda_b}{m_{B^*} - m_B} |\psi_{\Lambda_b}^B(0)|^2.$$  

(2.26)

This method is supposed to be most reliable as $|\psi_{\Lambda_b}(0)|^2$ thus determined does not depend on $\alpha_s$ and $m_q$ directly. Numerically, we find that (2.25) and (2.26) both give very similar results.

Defining the wavefunction ratio

$$r = \left| \frac{\psi_{\Lambda_b}^\Lambda_b(0)}{\psi_{\Lambda_b}^B(0)} \right|^2,$$

(2.27)

the baryon matrix elements in (2.18) and (2.22) can be recast to

$$\langle T_b | (\bar{b}b)(\bar{q}q) | T_b \rangle = -\bar{T} \langle T_b | (\bar{b}q)(\bar{q}b) | T_b \rangle = \frac{1}{12} f_{B_q}^2 m_{B_q} r \bar{B}(2m_{T_b}),$$

$$\langle T_b | b(1 - \gamma_5)q(1 + \gamma_5)\bar{b} | T_b \rangle = \frac{1}{24} f_{B_q}^2 m_{B_q} r (2m_{T_b}),$$

$$\langle T_b | \bar{b}^\alpha(1 - \gamma_5)q^\beta q^\beta(1 + \gamma_5)b^\alpha | T_b \rangle = -\frac{1}{24} f_{B_q}^2 m_{B_q} r \bar{B}(2m_{T_b}),$$

$$\langle \Omega_b | (\bar{b}b)(\bar{s}s) | \Omega_b \rangle = -\bar{T} \langle \Omega_b | (\bar{s}b)(\bar{b}s) | \Omega_b \rangle = \frac{1}{2} f_{B_q}^2 m_{B_q} r \bar{B}(2m_{\Omega_b}),$$

$$\langle \Omega_b | \bar{b}(1 - \gamma_5)s(1 + \gamma_5)b | \Omega_b \rangle = -\frac{1}{12} f_{B_q}^2 m_{B_q} r (2m_{\Omega_b}),$$

$$\langle \Omega_b | \bar{b}^\alpha(1 - \gamma_5)s^\beta s^\beta(1 + \gamma_5)b^\alpha | \Omega_b \rangle = \frac{1}{12} f_{B_q}^2 m_{B_q} r \bar{B}(2m_{\Omega_b}),$$

(2.28)

where $f_{B_q}$ is the decay constant of the meson $\bar{B}_q$. 

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To estimate $|\psi_{bq}(0)|^2$ and the parameter $r$ in the NQM, we find from (2.26)

$$r_{\Lambda_b} = \frac{4}{3} \frac{m_{\Sigma^*_b} - m_{\Sigma_b}}{m_{B^*} - m_B}, \quad r_{\Xi_b} = \frac{4}{3} \frac{m_{\Xi^*_b} - m_{\Xi_b}}{m_{B^*} - m_B}, \quad r_{\Omega_b} = \frac{4}{3} \frac{m_{\Omega^*_b} - m_{\Omega_b}}{m_{B^*} - m_B},$$

(2.29)

and likewise for $r_{\Lambda_c}$, $r_{\Xi_c}$ and $r_{\Omega_c}$, where $\Xi_{b,c}'$ denote spin-$\frac{1}{2}$ sextets. Heavy baryon masses have been studied in [37] in $1/m_Q$ and $1/N_c$ expansions within the HQET framework. The chromomagnetic mass splittings for charmed baryons are given by [37]

$$m_{\Sigma^*_c} - m_{\Sigma_c} = 65.7 \pm 2.3 \text{ MeV}, \quad m_{\Xi^*_c} - m_{\Xi_c} = 63.2 \pm 2.6 \text{ MeV},$$

$$m_{\Omega^*_c} - m_{\Omega_c} = 60.6 \pm 5.7 \text{ MeV},$$

(2.30)

where precise measurements of $\Sigma^*_c$ and $\Xi^*_c$ have been reported by CLEO [38]. It is evident that the heavy-quark spin-violating mass relation [37]

$$(m_{\Sigma^*_c} - m_{\Sigma_c}) + (m_{\Omega^*_c} - m_{\Omega_c}) = 2(m_{\Xi^*_c} - m_{\Xi_c})$$

(2.31)

is very accurate. It follows that

$$m_{\Sigma^*_b} - m_{\Sigma_b} = \left( \frac{m_c}{m_b} \right) (m_{\Sigma^*_c} - m_{\Sigma_c}) = 21.0 \text{ MeV}$$

(2.32)

for $m_b = 5 \text{ GeV}$, $m_c = 1.6 \text{ GeV}$ (see below). This mass splitting is substantially smaller than the preliminary result $m_{\Sigma^*_b} - m_{\Sigma_b} = (56 \pm 16) \text{ MeV}$ reported by the DELPHI Collaboration [39]. Since the measured mass difference of $\Sigma^*_c$ and $\Sigma_c$ is around 66 MeV [cf. (2.30)], a large hyperfine splitting of order 55 MeV for the $\Sigma_b$ baryon is very unlikely. Likewise,

$$m_{\Xi^*_b} - m_{\Xi_b} = 20.2 \text{ MeV}, \quad m_{\Omega^*_b} - m_{\Omega_b} = 19.4 \text{ MeV}.$$  

(2.33)

Because $\Delta m_B = m_{B^*} - m_B = 45.7 \pm 0.4 \text{ MeV}$ and $\Delta m_D = m_{D^*} - m_D \approx 143 \text{ MeV}$ [40] [note that $\Delta m_B$ and $\Delta m_D$ obey the same scaling relation as (2.32)], we find

$$r_{\Lambda_c} \cong r_{\Lambda_b} = 0.61, \quad r_{\Xi_c} \cong r_{\Xi_b} = 0.59, \quad r_{\Omega_c} \cong r_{\Omega_b} = 0.53,$$

(2.34)

and

$$|\psi_{bq}^{\Lambda_b}(0)|^2 = 0.87 \times 10^{-2} \text{GeV}^3, \quad |\psi_{bq}^{\Xi_b}(0)|^2 = 0.84 \times 10^{-2} \text{GeV}^3,$$

$$|\psi_{bq}^{\Omega_b}(0)|^2 = 0.81 \times 10^{-2} \text{GeV}^3,$$

(2.35)

Our result for $r_{\Lambda_b}$ is the same as [13] but different from [9] in which $r_{\Lambda_b}$ is given by $\frac{4}{3}(m^2_{\Sigma^*_b} - m^2_{\Sigma_b})/(m^2_{B^*} - m^2_B)$.
for $f_{B_q} = 180$ MeV [41]. An estimate in the QCD sum rule analysis yields $r \simeq 0.1 - 0.3$ [15]. Therefore, the NQM estimate of $|\psi_{bq}(0)|^2$ is indeed larger than the analogous bag model quantity: $a_q + b_q \sim 3 \times 10^{-3}\text{GeV}^3$. However, for the charmed baryon we obtain $|\psi_{cq}^A(0)|^2 = 3.8 \times 10^{-3}\text{GeV}^3$ for $f_D = 200$ MeV [41], which is smaller than those in bottom or hyperon decay. It seems that the smallness of $|\psi_{\Lambda c}^D(0)|^2$ is ascribed to the assumption that the $D$ meson wavefunction at the origin squared $|\psi_{Dc}(0)|^2$ is given by $\frac{1}{12}f_D^2 m_D$. We will come back to this point in Sec. IV.

Finally we are in position to estimate the HQET parameter $\lambda_{\text{baryon}}^2$ [see Eq. (2.12)]. Using the baryon masses [37]

\begin{align*}
m_{\Sigma_b} &= 2452.9\text{ MeV}, \\
m_{\Xi'_c} &= 2580.8\text{ MeV}, \\
m_{\Omega_c} &= 2699.9\text{ MeV}, \\
m_{\Sigma_b} &= 5824.2\text{ MeV}, \\
m_{\Xi'_b} &= 5950.9\text{ MeV}, \\
m_{\Omega_b} &= 6068.7\text{ MeV},
\end{align*}

and (2.30)-(2.33) we find

\[
\lambda_{\text{baryon}}^2 = \begin{cases} 
0.055\text{ GeV}^2 & \text{for charmed baryons;} \\
0.041\text{ GeV}^2 & \text{for } \Sigma_b; \\
0.040\text{ GeV}^2 & \text{for } \Xi'_b; \\
0.039\text{ GeV}^2 & \text{for } \Omega_b.
\end{cases} 
\]

(2.37)

It is interesting to note that the large-$N_c$ relation [37]

\[
\lambda_{\text{meson}}^2 \sim N_c \lambda_{\text{baryon}}^2
\]

(2.38)

is fairly satisfied especially for bottom baryons.

### III. LIFETIMES OF BOTTOM HADRONs

Using the formulism described in the last section, semileptonic and nonleptonic widths are calculated in this section. We shall first try to fix the heavy quark pole mass from the measured inclusive semileptonic decay rate. The semileptonic width of the $B$ meson given by (2.5)

\[
\Gamma_{\text{SL}}(B \to X e \bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \eta(x, 0, 0) \left\{ I_0(x, 0, 0) \frac{\langle \bar{B}|\bar{b}|B\rangle}{2m_B} - \frac{1}{m_b^2} I_1(x, 0, 0) \frac{\langle \bar{B}|\bar{b}\sigma \cdot Gb|B\rangle}{2m_B} \right\}
\]

(3.1)

has the salient feature that empirically $\Gamma_{\text{SL}}(B)$ is very insensitive to the choice of $m_b$ as long as $m_b - m_c$, which is free of renormalon ambiguity, is fixed according to Eq. (2.14). Hence, we may use the measured $\Gamma_{\text{SL}}(D)$ to fix $m_c$ to be 1.6 GeV (see Sec. IV below) which in turn
implies $m_b = 5$ GeV, in excellent agreement with the pole mass determined from lattice QCD: $m_b = 5.0 \pm 0.2$ GeV [42]. Since $x = (m_c/m_b)^2 = 0.1024$, the phase-space factors $I_i$ in (2.2) and (2.3) read

$$
I_0(x, 0, 0) = 0.476, \quad I_0(x, x, 0) = 0.147, \quad I_1(x, 0, 0) = 0.649, \\
I_1(x, x, 0) = 0.328, \quad I_2(x, 0, 0) = 0.723, \quad I_2(x, x, 0) = 0.220. \quad (3.2)
$$

From (3.1) we obtain

$$
\Gamma(B \to X e \bar{\nu}) = 4.44 \times 10^{-14}\text{GeV}, \\
\Gamma_{SL}(B) = 2.24 \Gamma(B \to X e \bar{\nu}) = 9.95 \times 10^{-14}\text{GeV}, \quad (3.3)
$$

for $|V_{cb}| = 0.039$, where uses of Eqs. (2.8), (2.9), (2.12) and (2.13) have been made, for example,

$$
\frac{\langle \bar{B} | \bar{b} \sigma \cdot G b | B \rangle}{2m_B} = 6\lambda^2_{\text{meson}} = 0.72 \text{GeV}^2. \quad (3.4)
$$

Since the phase space for the $\tau$ semileptonic decay mode relative to that of the $e$ mode is $0.24 : 1$, this accounts for the factor 2.24 in (3.3). The result (3.3) agrees very well with experiment [40]

$$
\Gamma(B^−/B^0\text{admixture} \to X e \bar{\nu}) = (4.31 \pm 0.17) \times 10^{-14}\text{GeV}. \quad (3.5)
$$

Likewise, we find for bottom-baryon semileptonic decays

$$
\Gamma(\Lambda_b \to X e \bar{\nu}) = \Gamma(\Xi_b \to X e \bar{\nu}) = 4.59 \times 10^{-14}\text{GeV}, \\
\Gamma(\Omega_b \to X e \bar{\nu}) = 4.53 \times 10^{-14}\text{GeV}, \quad (3.6)
$$

and hence

$$
\Gamma_{SL}(\Lambda_b) = \Gamma_{SL}(\Xi_b) = 2.24 \Gamma(\Lambda_b \to X e \bar{\nu}) = 1.027 \times 10^{-13}\text{GeV}, \\
\Gamma_{SL}(\Omega_b) = 2.24 \Gamma(\Omega_b \to X e \bar{\nu}) = 1.014 \times 10^{-13}\text{GeV}. \quad (3.7)
$$

Note that the tiny difference between $\Gamma_{SL}(\Lambda_b)$ and $\Gamma_{SL}(\Omega_b)$ arises from the fact that the chromomagnetic operator contributes to the matrix element of $\Omega_b$ but not to $\Lambda_b$ (or $\Xi_b$) as the light degrees of freedom in the latter are spinless; that is,

$$
\frac{\langle \Lambda_b | \bar{b} \sigma \cdot G b | \Lambda_b \rangle}{2m_{\Lambda_b}} = 0, \quad \frac{\langle \Omega_b | \bar{b} \sigma \cdot G b | \Omega_b \rangle}{2m_{\Omega_b}} = 8\lambda^2_{\text{baryon}} = 0.31 \text{GeV}^2. \quad (3.8)
$$
To compute the nonleptonic decay rate we apply the Wilson coefficient functions

\[ c_1(\mu) = 1.14, \quad c_2(\mu) = -0.31, \]  

which are evaluated at \( \mu = 4.4 \) GeV to the leading logarithmic approximation (see Table XIII of [43]). From Eq. (2.1) the nonleptonic widths of bottom baryons arising from \( b \) quark decay are found to be

\[ \Gamma^{\text{dec}}(B) = 2.216 \times 10^{-13} \text{GeV}, \quad \Gamma^{\text{dec}}(\Omega_b) = 2.217 \times 10^{-13} \text{GeV}, \]

\[ \Gamma^{\text{dec}}(\Lambda_b) = \Gamma^{\text{dec}}(\Xi_b) = 2.220 \times 10^{-13} \text{GeV}. \]  

(3.10)

We see that the \( b \) quark decay contribution \( \Gamma^{\text{dec}} \) is very similar for bottom hadrons even though the chromomagnetic mass splitting is different among them. Therefore, to \( \mathcal{O}(1/m_b^3) \) we obtain

\[ \frac{\tau(\Lambda_b)}{\tau(B_d)} \approx \frac{\tau(\Xi_b)}{\tau(B_d)} \approx \frac{\tau(\Omega_b)}{\tau(B_d)} = 0.99 + \mathcal{O}(1/m_b^3). \]

(3.11)

We next turn to the spectator effects of order \( 1/m_b^3 \). The Pauli interference in inclusive nonleptonic \( B^- \) decay and the \( W \)-exchange contribution to \( B_d \) can be evaluated from the first and second terms in (2.4):

\[ \Gamma^{\text{ann}}(B_d) = -\Gamma_0 \eta_{\text{spec}} \left\{ (1 - x)^2 (1 + \frac{1}{2} x) \left[ (\frac{1}{3} c_1^2 + 2 c_1 c_2 + N c_2^2) B_1 + 2 c_1^2 \varepsilon_1 \right] \right. 
\]

\[ \left. - (1 - x)^2 (1 + 2 x) \left[ (\frac{1}{3} c_1^2 + 2 c_1 c_2 + N c_2^2) B_2 + 2 c_1^2 \varepsilon_2 \right] \right\}, \]

\[ \Gamma^{\text{int}}(B^-) = \Gamma_0 \eta_{\text{spec}} (1 - x)^2 \left[ (c_1^2 + c_2^2)(B_1 + 6 \varepsilon_1) + 6 c_1 c_2 B_1 \right], \]

(3.12)

with [9]

\[ \Gamma_0 = \frac{G_F^2 m_B^5}{192 \pi^3} |V_{cb} V_{ud}|^2, \quad \eta_{\text{spec}} = 16 \pi^2 f_B^2 m_B \frac{m_B}{m^2}, \]

(3.13)

where we have applied Eqs. (2.15), (2.16) and neglected Cabibbo-suppressed \( W \)-exchange contribution to \( B_d \). As stressed in [9], the coefficients of \( B_i \) in (3.12) are one to two orders of magnitude smaller than that of \( \varepsilon_i \). Therefore, the contributions of \( B_i \) can be safely neglected at least in \( \Gamma^{\text{ann}}(B_d) \). Numerically,

\[ \Gamma^{\text{ann}}(B_d) = (-0.491 \varepsilon_1 + 0.563 \varepsilon_2) \times 10^{-13} \text{GeV}, \]

\[ \Gamma^{\text{int}}(B^-) = (-0.130 B_1 + 1.505 \varepsilon_1) \times 10^{-13} \text{GeV}. \]

(3.14)
Beyond the factorization approximation, $\varepsilon_1$ may receive nonfactorizable contributions. A QCD sum rule estimate gives $\varepsilon_1 \approx -0.15$ and $\varepsilon_2 \approx 0$ [23]. This implies a constructive $W$-exchange to $B_d$ and a destructive Pauli interference to $B^-$.  

As for the spectator effects in nonleptonic decays of bottom baryons we obtain from (2.4) that 

$$
\Gamma_{\text{ann}}(\Lambda_b) = \Gamma_0 \eta_{\text{spec}} r(1 - x)^2 \left( \tilde{B}(c_1^2 + c_2^2) - 2c_1c_2 \right),
$$

$$
\Gamma_{\text{int}}(\Lambda_b) = -\frac{1}{4} \Gamma_0 \eta_{\text{spec}} r \left[ (1 - x)^2(1 + x) + \frac{|V_{cd}|^2}{|V_{ud}|^2} \sqrt{1 - 4x} \right] \left( \tilde{B}c_1^2 - 2c_1c_2 - N_c c_2^2 \right),
$$

$$
\Gamma_{\text{ann}}(\Xi_b^0) = \Gamma_{\text{ann}}(\Lambda_b),
$$

$$
\Gamma_{\text{int}}(\Xi_b^0) = \Gamma_{\text{int}}(\Lambda_b),
$$

$$
\Gamma_{\text{int}}(\Xi_b^-) = -\frac{1}{4} \Gamma_0 \eta_{\text{spec}} r \left[ (1 - x)^2(1 + x) + \sqrt{1 - 4x} \right] \left( \tilde{B}c_1^2 - 2c_1c_2 - N_c c_2^2 \right),
$$

$$
\Gamma_{\text{int}}(\Omega_b^-) = -\frac{1}{6} \Gamma_0 \eta_{\text{spec}} r \sqrt{1 - 4x} (5 - 8x) \left( \tilde{B}c_1^2 - 2c_1c_2 - N_c c_2^2 \right),
$$

(3.15)

where use has been made of (2.28). Note that there is no $W$-exchange contribution to the $\Xi^-_b$ and $\Omega_b$ and that there are two Cabibbo-allowed Pauli interference terms in $\Xi^-_b$ decay, and one Cabibbo-allowed as well as one Cabibbo-suppressed interferences in $\Lambda_b$ decay. It is easily seen that under the valence-quark approximation i.e. $\tilde{B} = 1$, the $W$-exchange contribution $\Gamma_{\text{ann}}$ is proportional to $c_+ = (c_1 - c_2)/2$ as the four-quark operator $O_+ = (\bar{q}_1 q_2)(\bar{q}_3 q_4) + (\bar{q}_1 q_4)(\bar{q}_3 q_2)$ is symmetric in color indices whereas the color wavefunction for a baryon is totally antisymmetric. Writing 

$$
\Gamma_{\text{NL}} = \Gamma_{\text{dec}} + \Gamma_{\text{ann}} + \Gamma_{\text{int}},
$$

(3.16)

the numerical results for nonleptonic inclusive decay rates are 

$$
\Gamma_{\text{NL}}(\Lambda_b) = \left[ 2.220 + (0.042 + 0.058\tilde{B})r \right] \times 10^{-13}\text{GeV},
$$

$$
\Gamma_{\text{NL}}(\Xi_b^0) = \left[ 2.220 + (0.043 + 0.066\tilde{B})r \right] \times 10^{-13}\text{GeV},
$$

$$
\Gamma_{\text{NL}}(\Xi_b^-) = \left[ 2.220 - (0.037 + 0.114\tilde{B})r \right] \times 10^{-13}\text{GeV},
$$

$$
\Gamma_{\text{NL}}(\Omega_b^-) = \left[ 2.217 - (0.043 + 0.133\tilde{B})r \right] \times 10^{-13}\text{GeV},
$$

(3.17)

where for later convenience we have normalized the parameter $r$ in (3.17) to $r_{A_b}$ [see (2.34)]; that is, we have taken into account SU(3) breaking effect for $r$. Note that $\varepsilon_i$, $B_i$ in (3.14) and $\tilde{B}$, $r$ in (3.17) are all renormalized at $\mu = 4.4$ GeV.

Before proceeding, it is worth emphasizing the difference between the $W$-exchange contributions in the inclusive nonleptonic decays of the $B$ meson and the bottom baryon. It is
conventionally argued that $W$-exchange in heavy meson decay is suppressed by helicity and color mismatch. For example, $W$-exchange in $B$ decay is helicity suppressed by a factor of $16\pi^2(f_B/m_B)^2$ relative to the heavy quark decay amplitude. By contrast, $W$-exchange in baryon decay is neither helicity nor color suppressed. The diquark $Qq$ system in the heavy baryon can have a spin 0 configuration and the decay of a spin 0 (not spin 1!) state into two quarks is not subject to helicity suppression.

Since $\tilde{B}$ is of order unity and $r \sim 0.60$, it is evident from (3.17) and (3.10) that the bottom baryon lifetimes follow the pattern (see also Table I below)

$$\tau(\Omega_b^-) \simeq \tau(\Xi_b^-) > \tau(\Lambda_b^0) \simeq \tau(\Xi_b^0).$$

(3.18)

This pattern originates from the fact that while $\Lambda_b$, $\Xi_b^0$, $\Xi_b^-$, $\Omega_b$ all receive contributions from destructive Pauli interference, only $\Lambda_b$ and $\Xi_b^0$ have $W$-exchange and that $\Gamma_{\text{int}}$ is most large in $\Omega_b$ due to the presence of two valence $s$ quarks in its quark content. We shall see shortly that this lifetime pattern is dramatically modified when the $b$ quark mass is replaced by the bottom baryon mass in nonleptonic widths.

It follows from (3.3), (3.7), (3.10), (3.14) and (3.17) that

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.99 - 0.15\varepsilon_1 + 0.17\varepsilon_2 - (0.013 + 0.018\tilde{B})r,$$

(3.19)

which is a model-independent result. This is consistent with the result (1.6) obtained in [9] with $\varepsilon_i$, $\tilde{B}$, $r$ renormalized at $\mu = 4.85$ GeV and with $f_B = 200$ MeV. As stated in the Introduction, $\varepsilon_1$ and $\varepsilon_2$ obey the constraint $\varepsilon_1 \approx 0.3\varepsilon_2$, then it is quite difficult, if not impossible, to accommodate the experimental value of $\tau(\Lambda_b)/\tau(B_d)$ [cf. (1.5)] without invoking too large value of $r$ and/or $\tilde{B}$. We will argue below that the contribution of $-0.15\varepsilon_1 + \cdots - 0.018\tilde{B}r$ in (3.19) is at most of order 6%.

---

3It had been claimed that soft gluon emission from the initial quark line or soft gluon content in the initial wavefunction can vitiate both helicity and color suppression [44]. The net effect is that the factor $f_B/m_B$ is effectively replaced by $f_B/m_q$, where $m_q$ is the constituent quark mass of the antiquark in the $\bar{B}$ meson [45]. As a consequence, contributions of $W$-exchange will exhibit powerlike $(m_B/m_q)^2$ enhancement and this renders the treatment of the heavy quark expansion for $W$-exchange invalid. This issue was resolved by Bigi and Uraltsev [45] who showed that such powerlike enhancement does not arise for fully inclusive transitions and the soft gluon effect merely amounts to renormalizing the coefficients of 4-quark operators.
Irrespective of the above-mentioned lifetime ratio problem, there exists another serious difficulty, namely the predicted absolute decay width of the $B$ or $\Lambda_b$ hadron based on the heavy quark expansion [see (3.3), (3.7), (3.10) and (3.17)] is too small compared to the experimental values [11]:

$$\Gamma(B_d) = \left(4.246^{+0.094}_{-0.125}\right) \times 10^{-13}\text{GeV}, \quad \tau(B_d) = (1.55 \pm 0.04)\text{ps},$$

$$\Gamma(B^-) = \left(3.965^{+0.098}_{-0.093}\right) \times 10^{-13}\text{GeV}, \quad \tau(B^-) = (1.66 \pm 0.04)\text{ps},$$

$$\Gamma(\Lambda_b) = \left(5.351^{+0.422}_{-0.365}\right) \times 10^{-13}\text{GeV}, \quad \tau(\Lambda_b) = (1.23 \pm 0.09)\text{ps}. \quad (3.20)$$

Obviously, even if the destructive contribution $\Gamma^{\text{int}}(B^-)$ is not taken into account, the result $\Gamma^{\text{dec}}(B) + \Gamma_{\text{SL}}(B) = 3.211 \times 10^{-13}\text{GeV}$ is too small by about 20% to account for the observed decay rate of $B^-$. 4 To compute the decay widths of bottom baryons, we have to specify the values of $\tilde{B}$ and $r$. Since $\tilde{B} = 1$ in the valence-quark approximation and since the wavefunction squared ratio $r$ is evaluated using the quark model, it is reasonable to assume that the NQM and the valence-quark approximation are most reliable when the baryon matrix elements are evaluated at a typical hadronic scale $\mu_{\text{had}}$. As shown in [9], the parameters $\tilde{B}$ and $r$ renormalized at two different scales are related via the renormalization group equation to be

$$\tilde{B}(\mu)r(\mu) = \tilde{B}(\mu_{\text{had}})r(\mu_{\text{had}}),$$

4The problem with the absolute total decay width $\Gamma(B)$ of the $B$ meson is intimately related to the problem with the $B$-meson semileptonic branching ratio $B_{\text{SL}}$. The theoretical prediction for $B_{\text{SL}}$ is in general above 12.5% [46], while experimentally $B_{\text{SL}} = (10.23 \pm 0.39)\%$ [47]. In our case we obtain $B_{\text{SL}} > 1.38\%$. Several scenarios have been put forward in the past to resolve the discrepancy between theory and experiment for $B_{\text{SL}}$ or $\Gamma(B)$. Here we mention two of the possibilities. (i) Since the theoretical results depend on the scale $\mu$ to renormalize $\alpha_s(\mu)$ and the Wilson coefficients $c_{1,2}(\mu)$, one may choose a low renormalization scale, $\mu/m_b \sim 0.3 - 0.5$, to accommodate the data [9]. Local duality holds in this scenario. (ii) Next-to-leading order QCD radiative corrections to nonleptonic decay will increase the rate for $b \to c\bar{c}s$ substantially and decrease $B_{\text{SL}}$ [48,49]. Using the result of [48], we find that the QCD effect will bring $B_{\text{SL}}$ down by 1% and hence $B_{\text{SL}} > 12.7\%$. It was suggested in [50,16] that a failure of local duality in the $b \to c\bar{c}s$ channel, which has smaller energy release than that in $b \to c\bar{u}d$, will further enhance $\Gamma(B)$ and suppress $B_{\text{SL}}$. However, this explanation encounters a problem: The charm counting $n_c$ will increase and become as large as 1.30 [48], which is too large compared to the experimental value $n_c = 1.12 \pm 0.05$ [47]. One way out of this difficulty for $n_c$ is proposed in [51] that a sizeable fraction of $b \to c\bar{c}s$ transitions can be seen as charmless $b \to s$ processes. In the present paper we will not pursue any of the aforementioned possibilities as none of them can explain the lifetime difference between $\Lambda_b$ and $B_d$. The recipe we are going to discuss below [see (3.23)] will solve all the problems with $B_{\text{SL}}$, $\Gamma(B)$, $n_c$ and $\tau(\Lambda_b)/\tau(B_d)$. 20
\[
\tilde{B}(\mu) = \frac{\tilde{B}(\mu_{\text{had}})}{\frac{1}{N_c}(\kappa - 1)\tilde{B}(\mu_{\text{had}})},
\]

(3.21)

with

\[
\kappa = \left( \frac{\alpha_s(\mu_{\text{had}})}{\alpha_s(\mu)} \right)^{3N_c/2\beta_0} = \sqrt{\frac{\alpha_s(\mu_{\text{had}})}{\alpha_s(\mu)}}
\]

(3.22)

and \(\beta_0 = \frac{11}{3}N_c - \frac{5}{3}n_f\). Choosing \(\alpha_s(\mu_{\text{had}}) = 0.5\) and \(\mu = 4.4\ \text{GeV}\), we obtain \(\tilde{B}(\mu) = 0.59\tilde{B}(\mu_{\text{had}}) \simeq 0.59\) and \(r(\mu) \simeq 1.7r(\mu_{\text{had}})\). Using \(r(\mu_{\text{had}}) = 0.61\) [see (2.34)], the calculated decay rates of bottom baryons are summarized in Table I. It is evident that the predicted \(\Lambda_b\) lifetime is too large by 8 standard deviations.

### Table I. Various contributions to the decay rates (in units of \(10^{-13}\ \text{GeV}\)) of bottom baryons.

<table>
<thead>
<tr>
<th>(\Lambda_0^b)</th>
<th>(\Gamma_{\text{dec}})</th>
<th>(\Gamma_{\text{ann}})</th>
<th>(\Gamma_{\text{int}})</th>
<th>(\Gamma_{SL})</th>
<th>(\Gamma_{\text{tot}})</th>
<th>(\tau(10^{-12}\ \text{s}))</th>
<th>(\tau_{\text{expt}}(10^{-12}\ \text{s}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda_0^b)</td>
<td>2.220 (\pm) 0.145</td>
<td>0.064 (\pm) 0.051</td>
<td>1.027 (\pm) 0.051</td>
<td>1.027 (\pm) 0.051</td>
<td>3.327 (\pm) 0.051</td>
<td>1.98 (\pm) 0.051</td>
<td>1.23 (\pm) 0.09</td>
</tr>
<tr>
<td>(\Xi_0^b)</td>
<td>2.220 (\pm) 0.138</td>
<td>0.051 (\pm) 0.051</td>
<td>1.027 (\pm) 0.051</td>
<td>1.027 (\pm) 0.051</td>
<td>3.334 (\pm) 0.051</td>
<td>1.97 (\pm) 0.051</td>
<td></td>
</tr>
<tr>
<td>(\Xi^-)</td>
<td>2.220 (\pm) 0.110</td>
<td>0.077 (\pm) 0.051</td>
<td>1.027 (\pm) 0.051</td>
<td>1.027 (\pm) 0.051</td>
<td>3.137 (\pm) 0.051</td>
<td>2.10 (\pm) 0.051</td>
<td></td>
</tr>
<tr>
<td>(\Omega^-)</td>
<td>2.217 (\pm) 0.127</td>
<td>0.077 (\pm) 0.051</td>
<td>1.014 (\pm) 0.051</td>
<td>1.014 (\pm) 0.051</td>
<td>3.104 (\pm) 0.051</td>
<td>2.12 (\pm) 0.051</td>
<td></td>
</tr>
</tbody>
</table>

It has been advocated in [14] that, unlike the semileptonic inclusive case, since OPE cannot be rigorously justified for nonleptonic inclusive decays, the failure of explaining the observed lifetime ratio \(\tau(\Lambda_b)/\tau(B_d)\) implies that the assumption of local duality is not correct for nonleptonic inclusive widths. It is further suggested in [14] that corrections of order \(1/m_Q\) should be present and this amounts to replacing the heavy quark mass by the mass of the decaying hadron in the \(m_Q^5\) factor in front of all nonleptonic widths. In the following we shall see that the ansatz

\[
\Gamma_{\text{NL}} \rightarrow \Gamma_{\text{NL}} \left( \frac{m_{H_b}}{m_b} \right)^5
\]

(3.23)

will not only solve the short \(\Lambda_b\) lifetime problem but also provide the correct absolute decay rates for bottom hadrons.

Employing the hadron masses

\[
m_{B_d} = 5279.2 \pm 1.8 \ \text{MeV} \ \text{[40]}, \quad m_{B^-} = 5278.9 \pm 1.8 \ \text{MeV} \ \text{[40]},
\]

\[
m_{\Lambda_b} = 5621 \pm 5 \ \text{MeV} \ \text{[52]},
\]

we obtain
\[
\Gamma_{\text{tot}}(A_b) = \left[ 3.986 + (0.075 + 0.105\tilde{B})r \right] \times 10^{-13}\text{GeV} + \Gamma_{\text{SL}}(A_b), \\
\Gamma_{\text{tot}}(B_d) = \left[ 2.908 + (-0.644\varepsilon_1 + 0.739\varepsilon_2) \right] \times 10^{-13}\text{GeV} + \Gamma_{\text{SL}}(B), \\
\Gamma_{\text{tot}}(B^-) = \left[ 2.907 + (-0.171B_1 + 1.974\varepsilon_1) \right] \times 10^{-13}\text{GeV} + \Gamma_{\text{SL}}(B),
\]

(3.25)

with \(\Gamma_{\text{SL}}(A_b)\) and \(\Gamma_{\text{SL}}(B)\) being given by (3.3) and (3.7), respectively. Consequently,

\[
\frac{\tau(A_b)}{\tau(B_d)} = 0.78 - 0.13\varepsilon_1 + 0.15\varepsilon_2 - (0.015 + 0.021\tilde{B})r.
\]

(3.26)

Comparing this with Eq. (3.19) we see that the main effect of including linear \(1/m_b\) corrections is to shift the central value of the lifetime ratio from 0.99 to 0.78. Moreover, the experimental value \(\tau(A_b)/\tau(B_d) = 0.79 \pm 0.06\) [11] indicates that the remaining contribution \(-0.13\varepsilon_1 + \cdots\) in (3.26) is at most \(\pm 6\%\). It is also evident from (3.25) that the discrepancy between theory and experiment for the absolute decay width of \(B\) mesons is greatly improved.

The most dramatic effect due to the ansatz (3.23) occurs in the lifetime pattern of bottom baryons. Employing the bottom-baryon masses (2.36), (3.24) and \(m_{\Xi_b} = 5803.7 \pm 7.1\) MeV, \(^5\) some large enhancement to various nonleptonic contributions to the decay widths of bottom baryons is shown in Table II. We see that the improved \(\Lambda_b\) lifetime is in agreement with experiment and the new hierarchy of bottom-baryon lifetimes emerges as

\[
\tau(\Lambda^0_b) > \tau(\Xi^-_b) > \tau(\Xi^0_b) > \tau(\Omega^-_b),
\]

(3.27)

which is drastically different from the previous one: The longest-lived \(\Omega_b\) among bottom baryons in the conventional OPE now becomes shortest-lived. Needless to say, it is of great importance to measure the hierarchy of bottom-baryon lifetimes in order to test the ansatz (3.23). The branching ratios of semileptonic inclusive decays are calculated from Table II to be:

\[
B(\Lambda_b \to Xe\bar{\nu}) = 8.9\%, \quad B(\Xi^0_b \to Xe\bar{\nu}) = 7.8\%, \\
B(\Xi^-_b \to Xe\bar{\nu}) = 8.4\%, \quad B(\Omega_b \to Xe\bar{\nu}) = 6.9\%.
\]

(3.28)

Since serious and precise measurements of the hierarchy of lifetimes of bottom baryons

\(^5\)We have used the CDF mass of the \(\Lambda_b\) [see (3.24)] to update the \(\Xi_b\) mass prediction given in [37].
may not be available in the very near future,\(^6\) it is thus important to carry out more precise measurement of the \(B_s\) lifetime. An application of the prescription (3.23) will modify the prediction\(^9\)

\[
\frac{\tau(B_s)}{\tau(B_d)} = 1 \pm \mathcal{O}(1\%) \quad (3.29)
\]

to\(^{14}\)

\[
\frac{\tau(B_s)}{\tau(B_d)} = 0.938 \quad (3.30)
\]

for the average \(B_s\) lifetime. The current world average is \(\tau(B_s)/\tau(B_d) = 0.98 \pm 0.05\)\(^{11}\).

Table II. Various contributions to the decay rates (in units of \(10^{-13} \text{ GeV}\)) of bottom baryons.

<table>
<thead>
<tr>
<th>(B)</th>
<th>(\Gamma_{\text{dec}})</th>
<th>(\Gamma_{\text{ann}})</th>
<th>(\Gamma_{\text{int}})</th>
<th>(\Gamma_{\text{SL}})</th>
<th>(\Gamma_{\text{tot}})</th>
<th>(\tau(10^{-12}s))</th>
<th>(\tau_{\text{exp}}(10^{-12}s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda_b^0)</td>
<td>3.986</td>
<td>0.260</td>
<td>-0.116</td>
<td>1.027</td>
<td>5.157</td>
<td>1.28</td>
<td>1.23 \pm 0.09</td>
</tr>
<tr>
<td>(\Xi_b^0)</td>
<td>4.678</td>
<td>0.290</td>
<td>-0.107</td>
<td>1.027</td>
<td>5.888</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>(\Xi_b^-)</td>
<td>4.678</td>
<td>-0.231</td>
<td>1.027</td>
<td>5.474</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Omega_b^-)</td>
<td>5.840</td>
<td>-0.335</td>
<td>1.014</td>
<td>6.519</td>
<td>1.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

IV. LIFETIMES OF CHARMED BARYONS

In Sec. III we see that a replacement of the heavy quark mass with the decaying hadron mass in the \(m_Q^5\) factor in front of nonleptonic widths provides a much better description of the lifetimes of the \(\Lambda_b\) baryon and \(B\) mesons. It is claimed in\(^{14}\) that a much better fit to the charmed hadron lifetimes is also achieved if \(\Gamma_{\text{NL}}\) for charm decay approximately scales with the fifth power of charmed hadron masses, apart from corrections of order \(1/m_c^2\). We will carefully examine the applicability of this recipe in this section. For a theoretical overview of charmed baryon lifetimes, the reader is referred to the review of Blok and Shifman\(^{2}\).

We begin with the semileptonic inclusive decay of the \(D\) meson:

\[
\Gamma(D \to X e\nu) = \frac{G_F^2 m_e^5}{192 \pi^3} |V_{cs}|^2 \eta(x, 0, 0) \left\{ I_0(x, 0, 0) \frac{\langle D|\bar{c}c|D\rangle}{2m_D} - \frac{1}{m_c^2} I_1(x, 0, 0) \frac{\langle D|\bar{c}\sigma \cdot Gc|D\rangle}{2m_D} \right\} \quad (4.1)
\]

\(^6\)The current LEP results for the lifetime of \(\Xi_b\) are \((1.35^{+0.37}_{-0.28}^{+0.15})\) ps by ALEPH\(^{53}\) and \((1.5^{+0.7}_{-0.4} \pm 0.3)\) ps by DELPHI\(^{54}\). The average is \(\tau(\Xi_b) = (1.39^{+0.34}_{-0.28})\) ps. Evidently, the uncertainty is still too large to have a meaningful test on the prediction (3.27).
We find that the experimental values for $D^+$ and $D^0$ semileptonic widths [40] can be fitted by the quark pole mass $m_c = 1.6$ GeV. Taking $m_s = 170$ MeV, we then have $x = (m_s/m_c)^2 = 0.0113$ and

$$I_0(x,0,0) = 0.9166, \quad I_1(x,0,0) = 0.9556, \quad I_2(x,0,0) = 0.9665$$ \hspace{1cm} (4.2)

for charm decay. Repeating the same exercise for charmed baryons, we obtain the charmed-baryon semileptonic decay rates

$$\Gamma(\Lambda_c \to Xe\bar{\nu}) = \Gamma(\Xi_c \to Xe\bar{\nu}) = 1.533 \times 10^{-13}\text{GeV},$$

$$\Gamma(\Omega_c \to Xe\bar{\nu}) = 1.308 \times 10^{-13}\text{GeV},$$ \hspace{1cm} (4.3)

which are larger than that of the $D$ meson:

$$\Gamma(D \to Xe\bar{\nu}) = 1.090 \times 10^{-13}\text{GeV}.$$ \hspace{1cm} (4.4)

The prediction (4.3) for the $\Lambda_c$ baryon is in good agreement with experiment

$$\Gamma(\Lambda_c \to Xe\bar{\nu})_{\text{expt}} = (1.438 \pm 0.543) \times 10^{-13}\text{GeV}. $$ \hspace{1cm} (4.5)

For charmed baryons $\Xi_c$ and $\Omega_c$, there is an additional contribution to the semileptonic width coming from the Pauli interference of the $s$ quark [27]. From (2.5) we obtain

$$\Gamma^{\text{int}}(\Xi_c \to Xe\bar{\nu}) = \frac{1}{4} \Gamma'_0 \eta_{\text{spec}} r_{\Xi_c} (1 - x^2)(1 + x),$$

$$\Gamma^{\text{int}}(\Omega_c \to Xe\bar{\nu}) = \frac{1}{6} \Gamma'_0 \eta_{\text{spec}} r_{\Omega_c} (1 - x^2)(5 + x),$$ \hspace{1cm} (4.6)

where we have applied (2.28) for charmed baryon matrix elements, $\Gamma'_0 = \Gamma_0/|V_{ud}|^2$ and

$$\Gamma_0 = \frac{G_F m_c^5}{192\pi^3 |V_{cs}V_{ud}|^2}, \quad \eta_{\text{spec}} = 16\pi^2 \frac{f_D^2 m_D}{m_c^3}. $$ \hspace{1cm} (4.7)

We shall see later that, depending on the parameter $r$, the spectator effect in semileptonic decay of $\Xi_c$ and $\Omega_c$ can be very significant, in particular for the latter.

We now turn to the nonleptonic inclusive decays of charmed hadrons. It is well known that the longer lifetime of $D^+$ relative to $D^0$ comes mainly from the destructive Pauli interference in $D^+$ decay [55,3]. However, it is also known that, depending on the parameters $B_1$ and especially $\varepsilon_1$, the Pauli interference $\Gamma^{\text{int}}(D^+)$ in analog to $\Gamma^{\text{int}}(B^-)$ given by (3.12) can be easily overestimated and may even overtake the $c$ quark decay rate $\Gamma^{\text{dec}}$ so that the resulting
nonleptonic width becomes negative! This certainly does not make sense. It has been
discussed in great length by Chernyak [23] as how to circumvent the difficulty with the lifetime
of \( D^+ \). We shall not address this issue in the present work and instead focus on the lifetimes
of charmed baryons. Our purpose is to apply the ansatz similar to (3.23) and see if a better
description of charmed baryon lifetimes can be achieved.

In addition to the destructive Pauli interference \( \Gamma^\text{int} \), there exists another Pauli interference
term \( \Gamma^\text{int}_+ \) in charmed baryon decay which arises from the constructive interference between
the \( s \) quark produced in the \( c \) quark decay and the spectator \( s \) quark in the charmed baryon.
Since the expressions of \( \Gamma^\text{ann} \) and \( \Gamma^\text{int}_+ \) for charmed baryons are similar to (3.15) for bottom
baryon decays, here we will only write down the expressions for \( \Gamma^\text{int}_+ \) described by the last term
in Eq. (2.4):

\[
\Gamma^\text{int}_+(\Xi^+_c) = -\frac{1}{4} \Gamma_0 \eta^\text{spec} \rho_{\Xi^+_c}(1 - x^2)(1 + x) \left( \tilde{B} c_2^2 - 2 c_1 c_2 - N_c c_1^2 \right),
\]

\[
\Gamma^\text{int}_+(\Omega_c) = -\frac{1}{6} \Gamma_0 \eta^\text{spec} \rho_{\Omega_c}(1 - x^2)(5 + x) \left( \tilde{B} c_2^2 - 2 c_1 c_2 - N_c c_1^2 \right).
\]

(4.8)

It is easily seen that (4.8) is reduced to (4.6) when \( c_1 = 1, c_2 = 0, N_c = 1 \) and \( V_{ud} = 1 \). The
\( \Xi^+_c \) and \( \Omega_c \) baryons also receive contributions from Cabibbo-suppressed \( W \)-exchange:

\[
\Gamma^\text{ann}(\Xi^+_c) = |V_{us}/V_{ud}|^2 \Gamma_0 \eta^\text{spec} \rho_{\Xi^+_c}(1 - x^2) \left( \tilde{B} (c_1^2 + c_2^2) - 2 c_1 c_2 \right),
\]

\[
\Gamma^\text{ann}(\Omega_c) = 6|V_{us}/V_{ud}|^2 \Gamma_0 \eta^\text{spec} \rho_{\Omega_c}(1 - x^2) \left( \tilde{B} (c_1^2 + c_2^2) - 2 c_1 c_2 \right).
\]

(4.9)

The \( \Omega_c \) matrix element [see Eq. (2.18)]

\[
\langle \Omega_c | (\bar{c}s)(\bar{s}c) | \Omega_c \rangle = -6|\psi^{\Omega_c}_{cs}(0)|^2(2m_{\Omega_c})
\]

(4.10)

accounts for the factor 6 in Eq. (4.9).

To proceed we employ the Wilson coefficients

\[
c_1(\mu) = 1.35, \quad c_2(\mu) = -0.64
\]

(4.11)
evaluated at the scale \( \mu = 1.25 \text{ GeV} \). From Eqs. (3.21) and (3.22) we obtain \( \tilde{B}(\mu) \simeq 0.74 \tilde{B}(\mu_{\text{had}}) \simeq 0.74 \) and \( r(\mu) \simeq 1.36 r(\mu_{\text{had}}) \). Repeating the same exercise as the bottom
baryon case, the results of calculations are exhibited in Table III. We see that the lifetime
pattern

\[
\tau(\Xi^+_c) > \tau(\Lambda^+_c) > \tau(\Xi^0_c) > \tau(\Omega^0_c)
\]

(4.12)
is in accordance with experiment. It is evident that when spectator effects in semileptonic decay are included, as shown in parentheses in Table III, the discrepancy between theory and experiment is improved. This lifetime hierarchy (4.12) is qualitatively understandable. The $\Xi^+_c$ baryon is longest-lived among charmed baryons because of the smallness of $W$-exchange and partial cancellation between constructive and destructive Pauli interferences, while $\Omega_c$ is shortest-lived due to the presence of two $s$ quarks in the $\Omega_c$ that renders the contribution of $\Gamma^\text{int}_+$ largely enhanced. It is also clear from Table III that, although the qualitative feature of the lifetime pattern is comprehensive, the quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor.

Table III. Various contributions to the decay rates (in units of $10^{-12}$ GeV) of charmed baryons. When spectator effects in semileptonic decay are included, the predictions are shown in parentheses. Experimental values are taken from [40].

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma^\text{dec}$</th>
<th>$\Gamma^\text{ann}$</th>
<th>$\Gamma^\text{int}$</th>
<th>$\Gamma^\text{int}_+$</th>
<th>$\Gamma_{\text{SL}}$</th>
<th>$\Gamma^\text{tot}$</th>
<th>$\tau(10^{-13}\text{s})$</th>
<th>$\tau^\text{expt}(10^{-13}\text{s})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda^+_c$</td>
<td>0.903</td>
<td>0.858</td>
<td>-0.238</td>
<td>0.306</td>
<td>1.829</td>
<td>3.60</td>
<td>2.06 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>$\Xi^+_c$</td>
<td>0.903</td>
<td>0.042</td>
<td>-0.226</td>
<td>0.423</td>
<td>0.306(0.498)</td>
<td>1.447(1.639)</td>
<td>4.55(4.02)</td>
<td>3.5_{-0.4}^{+0.7}</td>
</tr>
<tr>
<td>$\Xi^0_c$</td>
<td>0.903</td>
<td>0.817</td>
<td>0.423</td>
<td>0.306(0.498)</td>
<td>2.448(2.640)</td>
<td>2.69(2.49)</td>
<td>0.98_{-0.23}^{+0.23}</td>
<td></td>
</tr>
<tr>
<td>$\Omega^0_c$</td>
<td>0.968</td>
<td>0.224</td>
<td>1.256</td>
<td>0.262(0.772)</td>
<td>2.710(3.220)</td>
<td>2.43(2.04)</td>
<td>0.64 ± 0.20</td>
<td></td>
</tr>
</tbody>
</table>

In order to have a better quantitative description of nonleptonic inclusive decays of charmed baryons, we shall follow [14] to assume that $\Gamma^\text{NL}$ scales with $m_{H_c}^5$ instead of $m_c^5$:

$$
\Gamma^\text{NL} = \Gamma^\text{NL}(\Lambda_c) : \Gamma^\text{NL}(\Xi^+_c) : \Gamma^\text{NL}(\Xi^0_c) : \Gamma^\text{NL}(\Omega_c)
= \Gamma^\text{NL}(\Lambda_c) \left( \frac{m_{\Lambda_c}}{m_c} \right)^5 : \Gamma^\text{NL}(\Xi^+_c) \left( \frac{m_{\Xi^+_c}}{m_c} \right)^5 : \Gamma^\text{NL}(\Xi^0_c) \left( \frac{m_{\Xi^0_c}}{m_c} \right)^5 : \Gamma^\text{NL}(\Omega_c) \left( \frac{m_{\Omega_c}}{m_c} \right)^5,
$$

(4.13)

where $\Gamma^\text{NL}_c$ is the nonleptonic decay rate calculated in the framework of the heavy quark expansion and it has the form

$$
\Gamma^\text{NL}_c = \frac{C_F^2 m_c^5}{192\pi^3} [a + b/m_c^2 + c/m_c^3 + O(1/m_c^4)].
$$

(4.14)

To compute the absolute decay width, we introduce a parameter $\lambda$ so that

$$
\Gamma^\text{NL} = \lambda \Gamma^\text{NL}_c \left( \frac{m_{H_c}}{m_c} \right)^5.
$$

(4.15)

Unlike the ansatz (3.23) for bottom hadrons, it will become clear shortly that $\lambda$ is much less than unity for charmed hadrons. Applying the prescription (4.15), treating $\lambda$, $r$, $\tilde{B}$ as free parameters and fitting them to the data of charmed baryon lifetimes [40], we find
\( \lambda = 0.18, \quad r = 1.72, \quad \tilde{B} = 1.46, \quad \) (4.16)

where \( r \) and \( \tilde{B} \) are renormalized at \( \mu = 1.25 \text{ GeV} \). The numerical results are summarized in Table IV. Contrary to the previous case, a perfect agreement with experiment will be achieved if spectator effects in semileptonic decay are not included.

Table IV. Same as Table III except that the ansatz (4.15) has been applied to enhance the nonleptonic \( c \) quark decay and spectator effects.

<table>
<thead>
<tr>
<th>( \Lambda^+ )</th>
<th>( \Gamma_{\text{dec}} )</th>
<th>( \Gamma_{\text{ann}} )</th>
<th>( \Gamma_{\text{int}}^- )</th>
<th>( \Gamma_{\text{int}}^+ )</th>
<th>( \Gamma_{\text{SL}} )</th>
<th>( \Gamma_{\text{tot}} )</th>
<th>( \tau(10^{-13} \text{s}) )</th>
<th>( \tau_{\text{expt}}(10^{-13} \text{s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Xi^+ )</td>
<td>0.960</td>
<td>2.753</td>
<td>-0.884</td>
<td>0.306</td>
<td>3.136</td>
<td>2.10</td>
<td>2.06 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>1.404</td>
<td>0.195</td>
<td>-1.231</td>
<td>1.227</td>
<td>0.306(0.837)</td>
<td>1.902(2.432)</td>
<td>3.46(2.70)</td>
<td>3.5±0.7</td>
</tr>
<tr>
<td>( \Omega^0 )</td>
<td>1.415</td>
<td>3.868</td>
<td>1.238</td>
<td>0.306(0.837)</td>
<td>6.828(7.358)</td>
<td>0.96(0.89)</td>
<td>0.98±0.23</td>
<td></td>
</tr>
</tbody>
</table>

Let us examine the fitted parameters (4.16) in more detail. The value \( r = 1.72 \) is fairly reasonable as it implies \( |\psi_{\Lambda^c}(0)|^2 = 1.1 \times 10^{-2} \text{GeV}^3 \), which is consistent with those of hyperons and bottom baryons. Then, does it mean that our previous estimate of \( r \) for charmed baryons [see (2.34)] is too small? In our opinion, the enhancement of \( |\psi_{\Lambda^c}(0)|^2 \) is likely due to the fact that \( |\psi_{\Xi^0}(0)|^2 \) is not simply equal to \( \frac{y}{12} f_D^2 m_D \) with \( y = 1 \) and \( f_D \approx 200 \text{ MeV} \). We conjecture that a more realistic value of \( y \) is probably close to 3 for charmed baryons and to unity for bottom baryons.

As for the parameter \( \tilde{B}(\mu) \), it is expected to be less than unity if the valence-quark approximation is believed to be valid at a lower hadronic scale. Therefore, it is not clear to us why \( \tilde{B}(\mu) \) is larger than unity and what is its implication. The smallness of \( \lambda \) is attributed to the fact that the inclusive nonleptonic decays of charmed baryons are not dominated by the \( c \) quark decay. Spectator effects of \( W \)-exchange and Pauli interference terms are expected to be of order

\[
16\pi^2(\Lambda_{QCD}/m_c)^3 \sim 0.5 - 0.7, \quad (4.17)
\]

where the factor \( 16\pi^2 \) is a two-body phase-space enhancement relative to the three-body phase space of heavy quark decay. Realistic calculations (see Tables III and IV) indicate that spectator contributions are comparable to and even dominate over the \( c \) quark decay mechanism. This implies that the charmed quark is not heavy enough (i.e. the energy release is not sufficiently large) to make a sensible and meaningful heavy quark expansion. For bottom
hadrons, we see in Sec. III that at least for the \( \Lambda_b \) baryon and \( B \) mesons, the nonleptonic decay rate is approximated by

\[
\Gamma_{NL}(H_b) \approx \Gamma_{\text{dec}} \left( \frac{m_{H_b}}{m_b} \right)^5,
\]

where \( \Gamma_{\text{dec}} \) is the heavy quark decay rate. However, we find for charmed baryons that \( \Gamma_{\text{dec}}(m_{H_c}/m_c)^5 \) are 5.36, 7.84, 7.84, 13.24 (in units of \( 10^{-12}\text{GeV} \)), respectively, for \( \Lambda_c, \Xi_c^+, \Xi_c^0 \) and \( \Omega_c \), where \( \Gamma_{\text{dec}} \) is taken from Table III. Therefore, even in the absence of \( 1/m_c^2 \) and \( 1/m_c^3 \) corrections or even the heavy quark expansion converges, the scaled nonleptonic \( c \)-quark decay rate \( \Gamma_{\text{dec}}(m_{H_c}/m_c)^5 \) already exceeds the experimental decay widths: 3.20, 1.88, 7.72, 10.28 (in units of \( 10^{-12}\text{GeV} \)) \[40\]; that is,

\[
\Gamma_{\text{dec}} \left( \frac{m_{H_c}}{m_c} \right)^5 > \Gamma_{\text{tot}}(H_c),
\]

except for the \( \Xi_c^0 \). The presence of large spectator contributions (see Tables III and IV) will make the discrepancy between theory and experiment for decay widths even much worse. Hence, we have to introduce a parameter \( \lambda \ll 1 \) to suppress the absolute rates. However, since \( \lambda \) is an entirely unknown parameter in theory, the recipe of scaling \( \Gamma_{NL} \) with the fifth power of charmed hadron mass is \textit{ad hoc} and does not have the predictive power for the absolute decay widths. We conclude that, although the ansatz (4.13) provides a much better description of lifetime \textit{ratios} for charmed baryons (apart from the annoying parameter \( \tilde{B} \)), the prescription (4.15) appears unnatural and unpredictive for describing the \textit{absolute} inclusive decay rates of charmed baryons due to the presence of the unknown parameter \( \lambda \). Since the heavy quark expansion converges very badly, local duality is thus not testable in inclusive nonleptonic charm decay.

\[\text{V. DISCUSSIONS AND CONCLUSIONS}\]

We have analyzed the lifetimes of bottom and charmed hadrons within the framework of the heavy quark expansion. Especial attention is paid to the nonperturbative parameter \( \lambda_{baryon}^{2} \) and four-quark matrix elements for baryons. We found that the large-\( N_c \) relation \( \lambda_{meson}^{2} \sim N_c \lambda_{baryon}^{2} \) is satisfactorily obeyed by bottom hadrons. We have followed [9] to parametrize the four-quark matrix elements in a model-independent way. Baryon matrix elements are evaluated using the NQM and the bag model. The bag-model estimate for bottom-baryon matrix elements is smaller than that of the NQM by a factor of \( \sim 3 \). The hadronic parameter \( r \)
defined in Eq. (2.27) is estimated in the NQM to be in the range 0.53 to 0.61 for both bottom and charmed baryons. Spectator effects in inclusive nonleptonic decays are then studied in detail. The main results of our analysis are as follows.

1. Using the charmed quark pole mass fixed from the measured semileptonic decay widths of $D^+$ and $D^0$, we have calculated $1/m_Q^2$ nonperturbative corrections to the semileptonic inclusive widths for other heavy hadrons. We found that while $\Gamma_{SL}(B)$ is very close to $\Gamma_{SL}(\Lambda_b)$, $\Gamma_{SL}(D)$ is smaller than $\Gamma_{SL}(\Lambda_c)$. The predicted semileptonic decay rates for the $B$ meson and the $\Lambda_c$ baryon are in good agreement with experiment. This implies that global duality is valid for inclusive semileptonic decay. For charmed baryons $\Xi_c$ and $\Omega_c$, there is an additional contribution to the semileptonic width coming from the constructive Pauli interference of the $s$ quark. This interference effect is sizeable for the $\Xi_c$ and becomes overwhelming for the $\Omega_c$.

2. The lifetime pattern of the bottom baryons is predicted to be $\tau(\Omega_b) \simeq \tau(\Xi_b^-) > \tau(\Lambda_b) \simeq \tau(\Xi_b^0)$. Spectator effects due to $W$-exchange and destructive Pauli interference account for their lifetime differences. The model-independent expression in the OPE for $\tau(\Lambda_b)/\tau(B_d)$ is given by (3.19), which is difficult to accommodate the data without invoking unnaturally too large values of hadronic parameters. Irrespective of the short $\Lambda_b$ lifetime problem, the calculated absolute decay width of the charged $B^-$ meson is at least 20% too small compared to experiment. Since the predicted $\Gamma_{SL}(B)$ agrees with data, the deficit of the $B$ meson decay rate is blamed on the nonleptonic width.

3. Unlike the semileptonic decays, the heavy quark expansion in inclusive nonleptonic decay cannot be justified by analytic continuation into the complex plane and local duality has to be assumed in order to apply the OPE directly in the physical region. The shorter lifetime of the $\Lambda_b$ relative to that of the $B_d$ meson suggests a significant violation of quark-hadron local duality. The simple ansatz that $\Gamma_{NL} \rightarrow \Gamma_{NL}(m_{H_b}/m_b)^5$ not only solves the lifetime ratio problem but also provides the correct absolute decay widths for the $\Lambda_b$ baryon and the $B$ meson. The hierarchy of bottom baryon lifetimes is modified to $\tau(\Lambda_b) > \tau(\Xi_b^-) > \tau(\Xi_b^0) > \tau(\Omega_b)$: The longest-lived $\Omega_b$ among bottom baryons in the OPE approach now becomes shortest-lived. This ansatz can be tested by measuring the $\Xi_b$ lifetime in the near
future. More precise measurement of the $B_s$ lifetime provides another quick and direct test of local duality.

4. The lifetime hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c) > \tau(\Xi_c^0) > \tau(\Omega_c)$ is qualitatively understandable in the OPE approach but not quantitatively. Apart from an annoying feature with the parameter $\tilde{B}$, a better description of inclusive decays of charmed baryons is achieved by scaling $\Gamma_{\text{NL}}$ with $m_{H_c}^5$ instead of $m_c^5$. Contrary to the bottom case, a small parameter $\lambda \ll 1$ has to be introduced, namely $\Gamma_{\text{NL}} \rightarrow \lambda\Gamma_{\text{NL}}(m_{H_c}/m_c)^5$, otherwise absolute decay widths of charmed baryons will be largely overestimated. Since $\lambda$ is an entirely unknown parameter in theory, it renders the above prescription unnatural and less predictive. As the heavy quark expansion in charm decay converges very badly, it is meaningless to test local duality in nonleptonic inclusive decay of charmed hadrons.

We conclude that the recipe of allowing the presence of linear $1/m_Q$ corrections by scaling the nonleptonic decay widths with the fifth power of the hadron mass is operative in the bottom family but becomes unnatural in charm decay. Can this prescription be justified in a more fundamental way? It is interesting to note that a PQCD-based factorization formalism has been developed for inclusive semileptonic $B$ meson decay [21]. This approach is formulated directly in terms of meson-level kinematics. Quark-hadron duality can be tested by comparing results obtained from quark-level kinematics and those from meson kinematics. The validity of global duality has been demonstrated in the general kinematic region up to $O(1/m_Q^2)$; $1/m_Q$ corrections to inclusive semileptonic widths are indeed nontrivially canceled out. When this factorization approach is generalized to nonleptonic decays and to heavy baryons, it is natural to expect that $\Gamma_{\text{NL}}(B)/\Gamma_{\text{NL}}(\Lambda_b) \approx (m_B/m_{\Lambda_b})^5$ if local duality is violated. Since the application of PQCD and hence the factorization scheme of [21] to charm decay is very marginal due to the fact that the charmed hadron scale is not sufficiently large, the scaling behavior of $\Gamma_{\text{NL}}$ with $m_{H_Q}^5$ occurred in the bottom decay is no longer anticipated in inclusive nonleptonic decays of charmed hadrons.

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REFERENCES


