Bright Lenses and Optical Depth

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ABSTRACT

In gravitational lensing, the concept of optical depth assumes the lens is dark. Several microlensing detections have now been made where the lens may be bright. Relations are developed between apparent and absolute optical depth in the regime of the apparent and absolute brightness of the lens. An apparent optical depth through bright lenses is always less than the true, absolute optical depth. The greater the intrinsic brightness of the lens, the more likely it will be found nearer the source.

Subject headings: gravitational lensing – dark matter
1. Introduction

Optical depth to gravitational lensing is a measurement of lens surface density (Vietri & Ostriker 1983). More specifically, the optical depth $\tau$ to Schwarzschild lenses is the angular fraction of the sky inside their Einstein radii. The angular Einstein radius has magnitude $\theta_E = \sqrt{2R_S D_{LS}/D_{OL} D_{OS}}$ where $R_S$ is the Schwarzschild radius of the lens, $D$ is angular diameter distance, and subscripts $O$, $L$, and $S$ refer to the observer, lens and source, respectively (see, for example, Refsdal 1964). When the lens is inside an Einstein ring centered on the source, the resulting magnification is greater than $\sim 1.34$.

In the past few years, hundreds of microlensing detections have been claimed by several collaborations actively seeking out such events (Paczynski et al. 1986; Alcock et al. 1993; Aubourg et al. 1993; Udalski et al. 1993). These detections have been converted into estimates of the optical depth to lensing through our Galactic halo and disk (Bennett et al. 1995; Gates, Gyuk, and Turner 1995). Currently much debate exists about the optical depth of our Galactic halo, as well as the implied masses for the lens events (Alcock et al. 1996; Jetzer 1994; Han and Gould 1996; Gould 1996) and the implied mass density of the lenses.

The addition of any unlensed component to a source brightness determination is called “blending” (Griest and Hu 1992; Kamionkowski 1995; Di Stefano & Esin 1995; Buchalter and Kamionkowski 1997). For lenses below about $10^6 M_\odot$, source and lens images are too close to resolve by normal telescopic means (Gould 1992). There are, however, several ways to detect the presence of an unlensed component. Photometrically, a blended light curve has a different shape than a unblended one, even for a single lens (Pratt 1994). Therefore, at least theoretically, the brightness of the unlensed component can be deconvolved (Buchalter, Kamionkowski, and Rich 1996). In practice, however, the photometry needs to be increasingly good to detect an increasingly faint unlensed component (Wozniak and
At least two other methods of detecting an unlensed component exist. An unlensed component is unlikely to have an identical spectrum as the lensed component, so lens amplification will change the relative contribution to the total light: a color change (Kamionkowski 1995). Also, were the lens a binary creating at any time 5 or more source images, a drop below a given minimum brightness would unambiguously indicate the presence of an unlensed component (Witt and Mao 1994). In this paper it is assumed that the amount of blending could not be determined from any of the above methods.

The information content in any unblended microlensing event is contained in only three variables: the maximum amplification $A_{\text{max}}$, the duration $t_{\text{dur}}$, and the time of maximum light $t_{\text{max}}$ (Nemiroff & Wickramasinghe 1994). In theory, optical depth can be computed solely from the distribution of these three parameters - including no information about unlensed components. In practice, an optical depth reported may include a model for source blending (Pratt et al. 1994, Alcock et al. 1995). Such a model may include significant assumptions about the luminosity function of potentially blended sources and specifics of realistic observing programs (Buchalter, Kamionkowski, and Rich 1996). Typically, no model is included for the luminosity function of the potentially blended lenses, however.

This paper studies optical depth and differential optical depth in the regime of the significantly bright lens. Previous work is extended to specifically include relations between the optical depth for dark lenses and the optical depth for bright lenses. The generated relations are also relevant to specific cases of a dark lens and multiple, unresolved, blended, sources. Although exploring a purely theoretical limit, practical application to present microlensing claims will be briefly touched on in Section 3.
2. Apparent Lens Luminosity and Optical Depth

Were a gravitational lens to have apparent luminosity $l_L$ equal to an unlensed source luminosity $l_S$, a measured 'apparent' source light amplification $A_{app}$ of 1.34 would only result from a true or 'absolute' amplification $A_{abs}$ of 1.62. This example shows that the brightness of the lens dilutes the true apparent magnification. More generally,

$$A_{abs} = A_{app}(1 + l_L/l_S) - l_L/l_S.$$  \hspace{1cm} (1)

Figure 1 gives a plot of $A_{abs}/A_{app}$ versus $l_L/l_S$. Note that for small $l_L/l_S$ (dark lenses), the apparent amplification $A_{app}$ is a good measure of absolute amplification $A_{abs}$. For apparently very bright lenses ($l_L >> l_S$), however, absolute amplifications may be much greater than the apparent amplifications, by as much as the factor $l_L/l_S$. This relation holds even when subscript $L$ is interpreted as labeling an unresolved source component.

When the apparent lens amplification differs from the absolute lens amplification, the apparent optical depth $\tau_{app}$ differs from the absolute optical depth $\tau_{abs}$. Here absolute optical depth $\tau_{abs}$ corresponds to the probability of absolute lens amplification above 1.34, whereas apparent optical depth $\tau_{app}$ corresponds to the probability of apparent lens amplification above 1.34. To compute $\tau_{abs}$ from the measured $\tau_{app}$ and $l_L/l_S$, one can consider the case where $A_{app}$ is 1.34. Given $A_{app}$ and a lens distance $D_L$, the lens must be inside some radius $b < b_{app}$ to generate $A > A_{app}$. The relation between $b$ and $A$ is given in Nemiroff (1989):

$$b = \sqrt{4R_SD_{OL}D_{LS}\Phi/D_{OS}},$$  \hspace{1cm} (2)

where $\Phi = \sqrt{A^2/(A^2 - 1)} - 1$. Now Figure 1 shows that for any $l_L/l_S$, $A_{abs} > A_{app}$, so a more perfect lens and source alignment must exist to create the higher true $A_{abs}$. (In other words, the bright lens must working harder to be seen over its own brightness.) So the corresponding radius $b_{abs}$ can be computed from equation (2), which generates $A_{abs} = 1.34$, is necessarily less than $b_{app}$. For any lens and source distance, $b \propto \sqrt{\Phi}$, and since $\tau \propto b^2$, ...
\[ \tau \propto \Phi(A). \] Therefore,

\[ \tau_{\text{abs}} = \tau_{\text{app}} \Phi_{\text{abs}} / \Phi_{\text{app}}. \tag{3} \]

For large \( A, \Phi \sim 1/(2A) \) so \( \tau_{\text{abs}} \sim \tau_{\text{app}} A_{\text{app}} / A_{\text{abs}}. \) For large \( A \) and a very bright lens \( (l_L \gg l_S), A_{\text{abs}} / A_{\text{app}} \sim l_L / l_S \) from equation (1), so then \( \tau_{\text{abs}} \sim \tau_{\text{app}} l_L / l_S. \)

The line marked “Apparent” in Figure 2 shows the relation between \( \tau_{\text{abs}} / \tau_{\text{app}} \) and apparent relative luminosity of the lens: \( l_L / l_S. \) As with the relationship between \( A_{\text{abs}} / A_{\text{app}} \) and \( l_L / l_S, \) for small \( l_L / l_S \) (dark lenses), the apparent optical depth \( \tau_{\text{app}} \) is a good measure of absolute optical depth \( \tau_{\text{abs}}. \) For apparently very bright lenses \( l_L \gg l_S, \) however, absolute optical depth may be much greater than the apparent optical depth.

### 3. Absolute Lens Luminosity and Optical Depth

How is optical depth affected by a field of bright lenses all with the same absolute luminosity \( L_L \)? Given both \( L_L \) and the absolute source luminosity \( L_S, \) one must also know the relative lens and source distances to determine the relationship between \( A_{\text{abs}} \) and \( A_{\text{app}}. \) Given equation (1) and the \( L \propto l/D^2, \) it is clear that

\[ A_{\text{abs}} = A_{\text{app}} \left( 1 + \frac{L_L D_{\text{OS}}^2}{L_S D_{\text{OL}}^2} \right) - \frac{L_L D_{\text{OS}}^2}{L_S D_{\text{OL}}^2}. \tag{4} \]

Now a dark lens must fall into an ellipsoidal detection volume to create an absolute amplification of a given source by an amount \( A_{\text{abs}} \) (Nemiroff 1989, Griest 1991, Nemiroff 1991). But what volume must a bright lens fall into to create an apparent amplification of an amount \( A_{\text{app}}? \) At each relative lens and source distance, equations (2) and (4) were solved computationally to generate three such canonical volumes, which are shown in Figure 3. Here, the plotted cross-sectional boundaries surround the volume a lens of intrinsic brightness \( L_L / L_S = 1/100, 1, \) and 100 must fall into to create an apparent amplification greater than 1.34.
Inspection of Figure 3 leads to several conclusions. First, it indicates that $V$ is a monotonically decreasing function of $L_L/L_S$. In other words, the brighter the lens, the smaller the detection volume, the “harder” it is to detect the lens over its own brightness.

Next, the shape of each detection volume shows that, in general, $dV/dD_{OL}$ has a maximum nearer the source than the observer. Therefore, although the bright lens could be at any distance between the observer and the source and still be detected, the single most detectable bright lens placement is closer to the source than the lens. This is in direct contrast to dark lenses, which are most likely to be detected precisely midway between the observer and the source.

Additionally, most of the space in the detection volume is nearer the source. Therefore, for uniformly distributed lenses, brighter lenses are more likely to be detected closer to the source than the observer. This is again in contrast to dark lenses, which have symmetric detection probability about the midway point between the lens and the source.

The relative change in the shapes of the detection volumes as absolute lens brightness changes indicates that the maximum of $d/dL_L(dV/dD_{OL})$ increases with $D_{OL}$. In other words, for any spatial distribution of lenses, increasingly (absolute) brighter lenses are increasingly more likely to be detected nearer the source.

For a lens of a given absolute luminosity, it is, of course, more probable to detect a low amplification event than a high amplification event. This is shown graphically in Figure 4 for $A_{app} = 0.1, 1.34, \text{and } 10$. To generate this plot, equations (2) and (4) were again solved computationally, but this time for constant $L_L/L_S$ (instead of for $A_{app} = 1.34$). Inspection of this plot shows, first, that $V$ is a decreasing function of $A_{app}$, so that increasingly higher amplitudes yield increasingly smaller detection volumes and are therefore increasingly less likely to be detected. Although the shape of each $A_{app}$ detection volume is not exactly a scaled version of other $A_{app}$ detection volumes, the difference is only slight. Therefore,
although high $A_{app}$ events have a slightly larger fraction of their volume at higher $D_{OL}$, the relative likely placement between the observer and the source of a lens causing a high $A_{app}$ is approximately the same as with a low $A_{app}$ event.

It should be noted that the actual probability of detecting a lens at any distance from the observer is directly proportional to $n_L(D_{OL})$, the density of lenses at that distance. However, for any lens distribution, relatively lower $A_{app}$ events will be found, on average, shifted relatively closer to the source.

The relationship between $\tau_{abs}$ and $\tau_{app}$ for a field of intrinsically bright lenses is defined similarly as for a field of apparently bright lenses. Given a relative lens and source distance, one again must find the relative radii $b_{app}$ and $b_{abs}$ the lens be placed from the observer-source axis to create the canonical $A_{app} = 1.34$ and $A_{abs} = 1.34$ magnification that defines optical depth. From integrating $\pi b^2$ for all lens positions near the observer-source axis, one finds the volume of the respective detection volumes. If the lens density is constant between the observer and the source, these volumes are directly proportional to $\tau$ (Nemiroff 1989).

The line marked “Absolute” in Figure 2 shows the relation between $\tau_{abs}/\tau_{app}$ and absolute relative luminosity of the lens: $L_L/L_S$. As expected, for small $L_L/L_S$ (dark lenses), the apparent optical depth $\tau_{app}$ is a good measure of absolute optical depth $\tau_{abs}$. For absolutely very bright lenses $L_L \gg L_S$, however, absolute optical depth may be much greater than the apparent optical depth. Because of the divergence between $A_{abs}$ and $A_{app}$ at small $D_{OL}$, the discrepancy between $\tau_{abs}$ and $\tau_{app}$ is greater at a given $L_L/L_S$ than at the same level of $l_L/l_S$. 
4. Practical Applications

Although this paper is geared toward a better and more general theoretical understanding of bright lenses, some of the above results may be applied to specific recent claims that our Galaxy is composed of a significant fraction of dark lenses. Recently reported results from the MACHO and EROS microlensing collaborations indicate that much of our Galactic halo is composed of lenses with mass on the order of a fraction of a solar mass (Jetzer 1994; Alcock et al. 1996; Aubourg et al. 1996). Main sequence stars of this mass in our Galaxy would normally be bright, and there is some debate as to whether they would be detectable in HST surveys (Flynn, Gould, and Bahcall 1996). Claims that might be considered by some to be truly extraordinary have become a standard interpretation of these results: that these lenses are dark, implying a whole new class of astronomical objects. Were these lenses bright, then perhaps recent microlensing results could be interpreted as more conventional objects.

Additionally, most microlensing detections are being made toward the Galactic bulge (Udalski et al. 1993, Alcock et al. 1996). In this regime, a common lens is indeed a bright star, and hence the optical depths in this direction too - which are significantly higher than originally expected (Paczynski et al. 1994) – might be affected by better understanding of lens brightness.

This study might also give insight on how bright lenses could be and still fall within measured statistical determinations of measured $A_{max}$, $t_{dur}$ and $t_o$. From inspection of Figure 2, it is evident that were the apparent brightness of the lenses 1/10th of the source, the true optical depth would be a factor of 1.1 higher than the published estimates. This plot also holds for the case of dark lenses and bright unresolved (blended) sources labeled “L.” Were the absolute luminosity of the lenses 1/10th of the source, the true optical depth would be a factor of about 1.6 higher than the published estimates.
One could turn this argument around and show that a limit on $l_L/l_S$ by other means (light curve shape, for example) would more clearly indicate that a new class of dark lens must be invoked to explain the optical depth inferred in our Galactic halo, even without considering separate searches for the lenses (by HST, for example).

Alternatively, the above results could be used to bolster a more conventional interpretation and distance to the lenses. Were the microlenses actually at the upper limit of possible brightness, the above results indicate that it is more likely at least some of the lenses are nearer the source than previously thought. Given any lens brightness, the likelihood is increased that the microlensing events of source stars in the LMC are caused by bright lenses in the LMC itself, and that stars in the Galactic bulge are caused by bright lenses in the bulge itself.

Lastly, as blending is modeled in the optical depth fits of at least one of the microlensing search groups (Alcock et al. 1995), there is the possibility that systematic corrections might apply to account for unknown attributes of the luminosity function of the lenses (or blended sources) in published models. It is hoped that this work gives insight on the possible magnitude of such corrections.

5. **Summary and Conclusions**

Estimates of optical depth that do not include lens brightness always underestimate the true optical depth. For a field of identical objects which act as both the lenses and sources, for example, the measured optical depth will be only about 1/6th of the true optical depth. Given that practically all optical depth estimates assume a dark lens, true optical depth must be at least slightly higher than the published estimates.

Not only does the optical depth change with lens brightness, but the differential
optical depth changes as well. The resulting detection volume is not symmetric between the observer and the source and has more space nearer the source. One consequence is that besides being increasingly difficult to detect intrinsically brighter lenses, it is increasingly difficult to detect them nearer the observer. In other words, the brighter the lens, the more likely it will be detected nearer the source.
REFERENCES


Bennett, D. et al. 1993, B.A.A.S., 187, # 47.07


Pratt, M. 1994, BAAS, 185, 1706


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Fig. 1.— A plot of absolute versus apparent amplification for three values of the relative apparent brightness of the lens.

Fig. 2.— Plots of ‘absolute’ optical depth over ‘apparent’ optical depth for different levels of both apparent and absolute brightness of the lens, relative to the source. Apparent optical depth is derived assuming the lens is dark. In actuality, a bright lens demands a higher ‘absolute’ or true optical depth to exist.

Fig. 3.— Detection volume boundaries for different levels of the absolute luminosity of the lens, relative to the absolute luminosity of the source. For a bright lens to be detected, it must fall inside the detection volume boundary. Note that the shape of the detection volume becomes more asymmetric as lens brightness increases, meaning that bright lenses are more likely detected nearer the source.

Fig. 4.— Detection volume boundaries for different amplifications, given a lens and source of equal absolute luminosity.