Proton spin structure in the rest frame

Petr Závada

Institute of Physics, Academy of Sciences of Czech Republic
Na Slovance 2, CZ-180 40 Prague 8
E-mail: zavada@fzu.cz

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Abstract

It is shown that the quark-parton model in the standard infinite momentum approach overestimates the proton spin structure function $g_1(x)$ in comparison with the approach taking consistently into account the internal motion of quarks described by a spherical phase space in the proton rest frame. Particularly, it is shown the first moment of the spin structure function in the latter approach, assuming only the valence quarks contribution to the proton spin, does not contradict to the experimental data.

1 Introduction

The proton spin problem attracting significant attention during last few years was triggered by the surprising results [1] of the European Muon Collaboration (EMC), which analyzed data on the polarized deep inelastic scattering (DIS). Since that time the hundreds of papers have been devoted to this topic, for the present status see e.g. [2], [3] and the comprehensive overview [4].

The essence of the problem is the following. From the very natural assumption, that proton spin is created by the composition of the spins of three valence quarks being in $s$-state, one can estimate value of the first moment $\Gamma_1^p$ of the spin structure function $g_1^p(x)$

$$\Gamma_1^p = \int_0^1 g_1^p(x) dx \simeq 0.17. \quad (1)$$

In fact, such value was well reproduced in the SLAC experiment preceding the EMC. Nevertheless, the EMC covering also a lower $x$ region, has convincingly shown, the first
moment is considerably lower: $\Gamma_1^p = 0.126 \pm 0.18$. And the latter experiments [6], [7] gave values compatible with the EMC. Such values can hardly correspond with the concept that proton spin is a simple sum of the valence spins. In fact, a global fit [8] to all available data evaluated at a common $Q^2$ in a consistent treatment of higher-order pertubative QCD effects suggests for the spin carried by the quarks the value less than one third the proton spin. So there is a question, what is the proton spin made?

In this paper we discuss the spin structure functions in the approach [9] based on the proton rest frame and make a comparison with the standard approach based on the infinite momentum frame (IMF). We do not attempt to account for all the details important for the complete description of the polarized proton, as e.g. the constrains resulting from axial vector current operators, but we rather try to isolate the net effect of the oversimplified kinematics in the IMF picture. Since this paper should be read together with [9], for convenience we refer to the equations and figures in the previous paper simply with prefix 'P', e.g. see Eq. (P3.41).

2 Spin structure functions

In our paper [9] the master equation (P3.41) has been based on the standard symmetric tensors (P3.33) and (P3.34) corresponding to the unpolarized DIS. After introduction the spin terms into both the tensors (see e.g. [10], Eqs. (33.9), (33.10)) our spin equation reads

$$P_\alpha P_\beta \frac{W_2}{M^2} - g_{\alpha\beta}W_1 + i\epsilon_{\alpha\beta\lambda\sigma}q^\lambda \left[ s^\sigma MG_1 + (Pq s^\sigma - sqP^\sigma) \frac{G_2}{M} \right] + A(P_\alpha q_\beta + P_\beta q_\alpha) + Bq_\alpha q_\beta$$

$$= \frac{P_0}{M} \int \frac{G(p)}{p_0} (2p_\alpha p_\beta - g_{\alpha\beta} pq) \delta((p + q)^2 - m^2) d^3 p$$

$$+ \frac{P_0}{M} \int \frac{H(p)}{p_0} i\epsilon_{\alpha\beta\lambda\sigma}q^\lambda m w^\sigma \delta((p + q)^2 - m^2) d^3 p,$$  \hspace{1cm} (2)

where $G$ and $H$ are related to the polarized quark distributions

$$G(p) = \sum_j e_j^2(h_j^\uparrow(p) + h_j^\downarrow(p)),$$  \hspace{1cm} (3)

$$H(p) = \sum_j e_j^2(h_j^\uparrow(p) - h_j^\downarrow(p))$$  \hspace{1cm} (4)

and the spin fourvectors fulfill

$$s_\mu s^\mu = w_\mu w^\mu = -1, \quad s_\mu P^\mu = w_\mu p^\mu = 0.$$  \hspace{1cm} (5)
The Eq. (2) requires for the spin terms
\[ s^\sigma MG_1 + (Pqs^\sigma - sqP^\sigma) G_2 = \frac{m}{2M^\nu} \int \frac{H(p)}{p_0} w^\sigma \delta \left( \frac{pq}{M^\nu} - x \right) d^3p, \] (6)
where we use for the δ–function the relation (P3.46).

Now, to be more definite, let us consider a simple scenario assuming the following.
1) To the function \( H \) in Eq. (4) only the valence quarks contribute.
2) In the proton rest frame the valence quarks are in the \( s \)–state and their momenta distributions have the same (spherically symmetric) shape for \( u \) and \( d \) quarks
\[ h_d(p) = \frac{1}{2} h_u(p) = h(p). \] (7)
3) Both the quarks have the same effective mass \( m^2 = p^2 \) in the sense suggested in [9]. In this way it is assumed the effective mass of the valence quark is characterized by the one fixed value, on the end this point will obtain more realistic form.
4) All the three quarks contribute to the proton spin equally
\[ h_d^+ - h_d^- = \frac{1}{2} (h_u^+ - h_u^-) = \Delta h(p_0) = \frac{1}{3} h(p_0), \quad p_0 = \sqrt{m^2 + p_1^2 + p_2^2 + p_3^2}. \] (8)
Since the proton and each of the three quarks have spin one half, spin of two quarks must cancel and spin of the third, remaining, gives the proton spin, so the last equation implies
\[ 3 \int \Delta h(p_0) d^3p = 1. \] (9)
The combination with (4) gives
\[ H(p_0) = \frac{4}{9} \Delta h(p_0) + \frac{1}{9} \Delta h(p_0) = \Delta h(p_0) \] (10)
and
\[ \int H(p_0) d^3p = \frac{1}{3}. \] (11)

Now, let us assume the proton is polarized in the direction of the collision axis (coordinate one), then Eq. (5) requires for the proton at rest
\[ s = (0, 1, 0, 0) \] (12)
and for the quark with fourmomentum \( p \)
\[ w = \left( \frac{p_1}{\sqrt{p_0^2 - p_1^2}}, \frac{p_0}{\sqrt{p_0^2 - p_1^2}}, 0, 0 \right). \] (13)
The contracting of Eq. (6) with \( P_\sigma \) and \( s_\sigma \) (or equivalently, simply taking \( \sigma = 0, 1 \)) gives the equations

\[
q_1 G_2 = \frac{m}{2M_\nu} \int \frac{H(p_0)}{p_0} \frac{p_1}{\sqrt{p_0^2 - p_1^2}} \delta \left( \frac{pq}{M_\nu} - x \right) d^3p,
\]

\( M G_1 + \nu G_2 = \frac{m}{2M_\nu} \int \frac{H(p_0)}{p_0} \frac{p_0}{\sqrt{p_0^2 - p_1^2}} \delta \left( \frac{pq}{M_\nu} - x \right) d^3p. \)

In the next step we apply the approximations from the Eqs. (P2.9) and (P2.14)

\[
q_1 \simeq -\nu, \quad \frac{pq}{M_\nu} \simeq \frac{p_0 + p_1}{M_\nu}.
\]

Let us note, the negative sign in the first relation is connected with the choice of the lepton beam direction giving the Eq. (P2.14). The opposite choice should give

\[
q_1 \simeq +\nu, \quad \frac{pq}{M_\nu} \simeq \frac{p_0 - p_1}{M_\nu}
\]

and one can check the both alternatives result in the equal pairs \( G_1, G_2 \), which reads

\[
2g_1(x) \equiv 2M^2\nu G_1 = m \int \frac{H(p_0)}{p_0} \frac{p_0 + p_1}{\sqrt{p_0^2 - p_1^2}} \delta \left( \frac{p_0 + p_1}{M_\nu} - x \right) d^3p,
\]

\[
2g_2(x) \equiv 2M\nu^2 G_2 = -m \int \frac{H(p_0)}{p_0} \frac{p_1}{\sqrt{p_0^2 - p_1^2}} \delta \left( \frac{p_0 + p_1}{M_\nu} - x \right) d^3p.
\]

Let us remark the integration of Eqs. (14) and (19) over \( x \) gives on r.h.s. the integral

\[
\int \frac{H(p_0)}{p_0} \frac{p_1}{\sqrt{p_0^2 - p_1^2}} d^3p = 0,
\]

which is zero due to spherical symmetry. Therefore in this approach the first moment of \( g_2(x) \) is zero as well. In the next we shall pay attention particularly to the function \( g_1 \), which can be rewritten

\[
2g_1(x) = \frac{x_0}{3} \int h(p_0) \frac{M}{p_0} \sqrt{\frac{p_0 + p_1}{p_0 - p_1}} \delta \left( \frac{p_0 + p_1}{M_\nu} - x \right) d^3p, \quad x_0 = \frac{m}{M_\nu}.
\]

What our assumptions 1)-4) do mean in the language of the standard IMF approach? In [9] (end of section III.B) we have shown our approach is equivalent to the standard one for the static quarks described by the distribution function \( h(p_0) \) sharply peaked around
\( m \). In such a case the last equation for \( p_0 \approx m, p_1 \approx 0 \) after combining with (4) and (P3.1) gives

\[
2g_1(x) = \int \sum_j e_j^2(h_j^+(p_0) - h_j^-(p_0)) \delta \left( \frac{p_0 + p_1}{M} - x \right) d^3p = \int \sum_j e_j^2(f_j^+(x) - f_j^-(x)), \tag{22}
\]

where \( f_j(x) \) are corresponding distribution functions in the IMF, so in this limiting case our spin equation (21) is also identical with the standard one, see Eq. (33.14) in [10]. On the other hand the last equation can be in our simplified scenario rewritten

\[
2g_1(x) = \frac{1}{3} \int h(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) d^3p = \frac{1}{3} f(x) = \frac{F_{2\text{val}}(x)}{3x}. \tag{23}
\]

This relation could be roughly expected in the standard IMF approach and correspondingly

\[
\Gamma_{IMF} \equiv \int g_1(x) dx = \frac{1}{6} \int f(x) dx = \frac{1}{6}. \tag{24}
\]

The Eqs. (21) and (23) are equivalent for the static quarks, but how they differ for the non static ones? In accordance with (P3.54) let us denote

\[
V_j^-(x) \equiv \int h(p_0) \left( \frac{p_0}{M} \right)^j \delta \left(\frac{p_0 + p_1}{M} - x \right) d^3p,
\]

then (P3.52) and (23) give

\[
2g_1(x) = \frac{xV_{-1}(x)}{3}, \quad \Gamma_{IMF} = \frac{1}{6} \int xV_{-1}(x) dx. \tag{26}
\]

Now, let us calculate the corresponding integral from our rest frame equation (21)

\[
\Gamma_{lab} = \frac{x_0}{6} \int \int h(p_0) \frac{M}{p_0} \sqrt{\frac{p_0 + p_1}{p_0 - p_1}} \delta \left( \frac{p_0 + p_1}{M} - x \right) d^3p dx. \tag{27}
\]

Due to the \( \delta - \) function, the square root term in the integral can be rewritten

\[
\sqrt{\frac{p_0 + p_1}{p_0 - p_1}} = \sqrt{\frac{Mx}{2p_0} - Mx} = \sqrt{\frac{Mx}{2p_0}} \left( 1 - \frac{Mx}{2p_0} \right)^{-\frac{1}{2}} = \left( \frac{Mx}{2p_0} \right)^{\frac{1}{2}} \sum_{j=0}^{\infty} -\frac{1}{2} (-1)^j \left( \frac{Mx}{2p_0} \right)^j
\]

and with the using of (25) the integral correspondingly

\[
\Gamma_{lab} = \frac{x_0}{6} \int \sum_{j=0}^{\infty} -\frac{1}{2} (-1)^j V_{-j-3/2}(x) \left( \frac{x}{2} \right)^{j+1/2} dx. \tag{29}
\]
The integration by parts combined with the relations (P3.56) gives
\[
\int V_{-j-3/2}(x) \left(\frac{x}{2}\right)^{j+1/2} dx = \int V_{-j-3/2}(x) \frac{2(x/2)^{j+3/2}}{j + 3/2} dx = \int V'_0(x) \left(\frac{x}{2} + \frac{x_0^2}{2x}\right)^{-j-3/2} \frac{2(x/2)^{j+3/2}}{j + 3/2} dx
\]
\[
= \int V'_0(x) \frac{2}{j + 3/2} \left(\frac{1}{1 + x_0^2/x^2}\right)^{j+3/2} dx
\]
\[
= \int V'_0(x) 2 \left(\frac{1}{1 + x_0^2/x^2}\right)^{j+1/2} \frac{2x_0^2/x^3}{(1 + x_0^2/x^2)^2} dx.
\]

If we denote \(t \equiv x_0^2/x^2\) and \(z \equiv 1/(1 + t^2)\) then (29) can be rewritten
\[
\Gamma_{lab} = \frac{1}{6} \int V_0(x) 4t^3 z^{5/2} \sum_{j=0}^\infty -\frac{1}{2}(j-1)jz^j dx = \frac{1}{6} \int V_0(x) 4t^3 z^2 \sqrt{\frac{z}{1-z}} dx,
\]
which implies
\[
\Gamma_{lab} = \frac{1}{6} \int_{x_0^2}^1 \frac{4x_0^2/x^2}{(1 + x_0^2/x^2)^2} V_0(x) dx. \quad (30)
\]

Simultaneously, since
\[
\int_{x_0^2}^1 V_0(x) dx = - \int_{x_0^2}^1 xV'_0(x) dx = - \int_{x_0^2}^1 xV'_{-1}(x) \left(\frac{x}{2} + \frac{x_0^2}{2x}\right) dx
\]
\[
= - \int_{x_0^2}^1 V'_{-1}(x) \left(\frac{x^2}{2} + \frac{x_0^2}{2}\right) dx = \int_{x_0^2}^1 V_{-1}(x) dx,
\]
the integral (26) can be rewritten
\[
\Gamma_{IMF} = \frac{1}{6} \int_{x_0^2}^1 V_0(x) dx. \quad (31)
\]

Let us express the last integral as
\[
\int_{x_0^2}^1 V_0(x) dx = \int_{x_0^2}^{x_0} V_0(x) dx + \int_{x_0}^1 V_0(x) dx
\]
and modify the first integral on r.h.s. using substitution \(y = x_0^2/x\)
\[
\int_{x_0^2}^{x_0} V_0(x) dx = \int_{x_0}^1 V_0 \left(\frac{x_0^2}{y}\right) \frac{x_0^2}{y^2} dy.
\]

Now let us recall the general shape of the functions (25) obeying Eq. (P3.24), which implies
\[
V_0 \left(\frac{x_0^2}{y}\right) = V_0(y),
\]
therefore instead of (31) one can write

\[ \Gamma_{IMF} = \frac{1}{6} \int_{x_0}^{1} V_0(x) \left( \frac{x^2 + x_0^2}{x^2} \right) dx. \tag{32} \]

Similar modification of Eq. (30) gives

\[ \Gamma_{lab} = \frac{1}{6} \int_{x_0}^{1} V_0(x) \left( \frac{4x_0^2}{x^2 + x_0^2} \right) dx. \tag{33} \]

Obviously, both the integrals are equal for \( V_0 \) sharply peaked around \( x = x_0 \), but generally, for non static quarks

\[ \Gamma_{lab} < \Gamma_{IMF}. \tag{34} \]

What can our result (34) mean quantitatively for the more realistic scenario? In our discussion in [9] we have suggested the real structure functions could be rather some superposition of our idealized ones based on the single values of the effective mass \( x_0 = m/M \). That means all the relations involving the functions \( V_j(x) \equiv V_j(x, x_0) \) should be integrated over some distribution of the effective masses \( \mu(x_0) \). But at first, let us try to guess \( V_0 \), at least in the vicinity of \( x_0 \), which is important for the integrals (32) and (33).

According to the Eq. (P3.20) for \( x > x_0 \) one can write

\[ xV'_0(x) = -\frac{M}{2} P(p_0), \quad p_0 = \frac{M}{2} \left( x + \frac{x_0^2}{x} \right). \tag{35} \]

Now, for \( p_0 \) close to \( m \) let us parameterize the energy distribution by

\[ P(p_0) = \frac{\alpha \exp(\alpha)}{m} \exp \left( -\alpha \frac{p_0}{m} \right), \tag{36} \]

which fulfills the normalization

\[ \int_{m}^{\infty} P(p_0) dp_0 = 1. \tag{37} \]

Obviously, the distribution (36) means the average quark kinetic energy equals to \( m/\alpha \). Inserting (36) into (35) gives

\[ V'_0(x) = -\frac{\alpha \exp(\alpha)}{2x_0 x} \exp \left( -\alpha \frac{p_0}{m} \left( x + \frac{x_0^2}{x} \right) \right). \tag{38} \]

Let us note, for \( |y| \ll 1 \)

\[ (1 + y)^a \approx \exp(a y), \]

therefore if we substitute the exponential function in (38) by

\[ \exp \left( -\frac{\alpha}{2} \left[ \frac{x}{x_0} + \frac{x_0}{x} \right] \right) \sim \left( 1 - x \right) \left( 1 - \frac{x_0^2}{x^2} \right)^{\alpha/2x_0} \equiv f(x, x_0), \quad x_0^2 \leq x \leq 1, \tag{39} \]
the resulting $V_0(x)$ will coincide with (38) in a vicinity of $x_0$, but moreover will obey the global kinematical constraint outlined in Fig. (P2). The ratio of integrals (33) and (32) calculated by parts with the use of Eqs. (38) and (39) gives

$$R_s(\alpha,x_0) \equiv \frac{\Gamma_{lab}}{\Gamma_{IMF}} = \frac{\int_{x_0}^{1} x_0/x (\arctan[x/x_0] - \pi/4) f(x,x_0)dx}{\int_{x_0}^{1} (1-x_0^2/x^2) f(x,x_0)dx},$$  \hspace{1cm} (40)

the results of the numerical computing are plotted in the Fig. 1. What do these curves mean? There are the two limiting cases:

a) The quarks are massive and static, i.e. $\alpha \to \infty$, then $R_s \to 1$. It is the scenario in which both the approaches are equivalent.

b) Both, the quark effective mass and $\alpha \to 0$, but the quark energy $\langle E_{kin} \rangle = m/\alpha > 0$, then $R_s \to 0$. It is due to the fact that the massless fermions having spin oriented always parallel to their momentum cannot contribute to spin structure function of the system with the spherical phase space.

Obviously, the real case could be somewhere between both the extremes, i.e. $\alpha$ and $m$ should be the finite, positive quantities. The combination of Eqs. (40) and (24) gives

$$\Gamma_{lab} = \frac{1}{6} R_s(\alpha,x_0).$$  \hspace{1cm} (41)

The comparison with the experimental value $\Gamma_{exp} \simeq 0.13$ implies $R_s \simeq 0.78$, which according to the Fig. 1 corresponds to $\alpha \simeq 2$. Let us note, this result depends on $x_0$ rather
slightly, therefore irrespective of the unknown distribution of effective masses $\mu(x_0)$ we can conclude the following. If we accept the quarks have on an average (over effective masses distribution) the mean kinetic energy roughly equal to one half of the corresponding effective mass, then within our approach, the experimental value $\Gamma_{\text{exp}}$ is compatible with the assumption that whole proton spin is carried by the valence quarks.

3 Summary and conclusion

We have calculated the first moment $\Gamma_1$ of the proton spin structure function in the approach which takes consistently into account the internal motion of the quarks described by a spherical phase space. Simultaneously we have done a comparison with the corresponding quantity deduced from the standard IMF approach and came to the conclusion that the latter gives a greater value $\Gamma_1$. This difference is due to the fact, that the standard approach is based on the approximation (P3.36), which effectively suppresses the internal motion of quarks. On the other hand, in our approach, the total quark energy is shared between the effective mass and the kinetic energy, and correspondingly the resulting formula correctly reflects the mass dependence of the structure function: $\Gamma_1$ continuously vanishes for massless quarks controlled by a spherical phase space. Let us note, the quark intrinsic motion has been shown to reduce $\Gamma_1$ also in some another approaches [11]-[15].

Finally, we came to the conclusion that our $\Gamma_1$ calculated only from the valence quarks contribution is compatible with the experimental data - provided that their kinetic energies are on an average roughly equal to one half of their effective mass. The application of the constrains due to axial vector current operators on the spin contribution from different flavors can somewhat change the parameter $\alpha$ to achieve an agreement with the data. This question is studied and will be discussed in a next paper.

The corrections on $\Gamma_1$ suggested in this paper together with the corrections on the distribution functions (P3.59) should be taken into account for interpretation of the experimental data. At the same time it is obvious the distribution of effective masses $\mu(x_0)$ is a quantity requiring further study.

References


