INTRODUCTION

ABSTRACT

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Testing the Homogeneous Synchrotron Self-Compton Model for Gamma Ray Production in M87
De Jager 1997 and references therein). Even if cascading
is included in the infrared background (Protheroe and Staney
1993, Entel and Protheroe 1995) observation of γ-rays of
these energies seemed unlikely. However, the infrared back-
ground is not well known and it may be possible to observe
the nearest blazars up to energies somewhat below ∼ 100
TeV where absorption on the cosmic microwave background
will give a sharp cut-off. In our calculations below, we shall
therefore neglect photon-photon pair production on the in-
fra-red background.

The purpose of this paper is to confront the homoge-
nous SSC model with the results of recent observations and,
as possible, to derive the parameters of the emission region in
the jet from which this radiation originates, i.e. its Doppler
factor and magnetic field strength.

2 CONSTRAINTS ON A HOMOGENEOUS SSC MODEL

Let us consider relativistic electrons confined in a “blob”
which moves along the jet with the Doppler factor D and
has magnetic field B. In the homogeneous SSC model the
radii of the emission regions of low energy photons (r1), X-
ray photons (r1), and TeV γ-rays (r2) are the same. This
region is constrained by the variability time scale observed,
e.g. in TeV γ-rays, \( \ell_{\text{var}} \), to

\[
r_1 = r_2 = r_3 \approx 0.5 \ell D_{\text{var}}.
\]

(1)

The differential photon density in the blob frame of low
energy synchrotron photons (phot. MeV^{-1} cm^{-3}) is then given by

\[
\eta_\gamma \approx \frac{4\ell^2 F_\gamma}{c^2 \ell_{\text{var}} D^2},
\]

(2)

where \( d \equiv \sqrt{177 \text{ Mpc}} \) is the distance to Mrk 421 (for \( \ell_0 = 50 \text{ km s^{-1} Mpc}^{-1} \)), \( \ell_0 \equiv D_\ell \) and \( e \) are the
photons energies in the observer’s and the blob rest frames,
and \( c \) is the velocity of light.

The differential photon flux in the optical to X-ray region
observed from Mrk 421 during the 16 May 1994 flare
(phots \text{ cm}^{-2} \text{s}^{-1} \text{s}^{-1} \text{ MeV}^{-1}) can be approximated by a broken
power-law,

\[
F_\gamma \approx \begin{cases}
10^{-3} \ell_\ell < \ell \leq \ell_0, \\
10^{-5} \ell_\ell < \ell \leq \ell_0 \frac{1}{10^{\beta_1 - 3}}
\end{cases}
\]

(3)

where \( \beta_1 = 1.8, \beta_2 = 2.2, b_1 = 6.9 \times 10^{-4} \text{ and } b_2 = 2.8 \times
10^{-6} \text{ are obtained from A 3CA observations, and } \ell_0 = 1.65 \times
10^{20} \text{ MeV is the energy at which a break in the synchrotron
spectrum is observed. Eq. 3 is based on the peak 2–10 \text{ keV
luminosity and spectral index given in Fig. 3 of Takahashi
et al. (1996) (for } \ell > \ell_0\text{), and from Fig. 2 of Macomb et al.
(1995) we estimate the spectrum for } \ell \leq \ell_0. \text{ Note that for at
least } 2 \text{ decades below } 10^{-3} \ell_0 \text{ the spectrum is uncertain.}

The shape of synchrotron spectrum defines the shape of
the electron spectrum in the blob rest frame which can be
approximated by

\[
\frac{dN}{d\gamma} \approx \begin{cases}
\alpha_1 \gamma^{\beta_1} \gamma < \gamma_0, \\
\alpha_2 \gamma^{\beta_2} \gamma > \gamma_0
\end{cases}
\]

(4)

where \( \gamma \) is the Lorentz factor in the blob frame,

\[
\gamma_0 = (2 \ell_0 / D_\ell)^{1/2},
\]

(5)

\( \alpha_1 = 2, \alpha_2 = 36, \epsilon_B = m_e c^2 B^2 / \epsilon_{\ell}, \epsilon_{\ell} = 4.414 \times 10^{13} \text{G, } m_e \) is the electron rest mass, \( \alpha_1 = a_2 \gamma_0^2 \gamma_0^2 \) and \( a_2 \) can be obtained from fitting the observations. Note that below
\( 10^{-3} \gamma_0^2 \gamma_0^2 = 0.0325 \text{, the spectrum is uncertain.}

The spectrum of Mrk 421 shows two clear bumps which,
during the outburst stage, extend up to at least \( \sim \text{10 \text{ keV
(Takahashi et al. 1996), and } \sim 8 \text{ TeV (Krennrich et al. 1997).
These multiwavelength observations of Mrk 421 allow us to
define the ratio } \eta \text{ of the power emitted at a } \gamma \text{-ray energy,
} \epsilon_\gamma \text{ at which the emission is due to Compton scattering,
to the power emitted at an energy, } \epsilon, \text{ at which the emission is
due to X-ray synchrotron radiation,

\[
\eta = \left( \frac{dN}{dE_\gamma dt} \right) / (dN/d\epsilon dt)^2 = \left( \frac{dN}{dE_\gamma dt} \right) / (dN/d\epsilon dt)^2,
\]

(6)

where the primed quantities are measured in the blob frame.
For the power at \( \gamma \)-ray energies we adopt the value reported
for the threshold of the Whipple telescope at \( E_\gamma = 0.3 \text{ TeV
(Macomb et al. 1996), and for the power at X-ray synchrotron energies we take the value corresponding to the
peak emission at } \epsilon = \epsilon_\ell \text{ (Takahashi et al. 1996). For these
two energies } \eta \approx 1.2.

The synchrotron spectrum at \( \epsilon' \) in the above formula
(6) can be obtained approximately analytically from the relation

\[
\epsilon' \frac{dN}{d\epsilon' dt} \approx \frac{dN}{d\gamma' dt} b_{\text{syn}} (\gamma'),
\]

(7)

where \( dN/d\gamma' \) is the electron spectrum (Eq. 4). The characteristic
energy of synchrotron photons is given by

\[
\epsilon' \approx 0.5 \epsilon_\ell \gamma_0^2,
\]

(8)

the energy loss rate of electrons is \( b_{\text{syn}} (\gamma') = k U_B \gamma_0^2 \), \( k = 4c \sigma_T / 3 \), \( \sigma_T \) is the Thomson cross section, and \( U_B \approx
2.5 \times 10^6 B^2 \text{ (MeV cm}^{-3}) \text{ is the magnetic field energy density.
The synchrotron spectrum emitted by electrons with power-
law spectral index } \alpha, \text{ multiplied by the square of the photon
energy, is given by

\[
\frac{dN}{dE_{\gamma'} d\epsilon'} \approx \frac{2a k U_B \epsilon_\ell^2}{\epsilon_B^2} \left( \frac{\epsilon_B}{\epsilon_\ell} \right)^{-(\alpha + 1)/2}
\]

(9)

The ICS part of the Eq. 6 cannot be obtained analytically
in the general case because of the complicated form of the
Klein-Nishina cross section, and so we compute this numerically
using

\[
\frac{dN}{dE_{\gamma'} d\epsilon'} \approx E_{\gamma'}^2 \int_{\gamma_{\text{min}}^{\text{s}}}^{\infty} \frac{dN}{d\gamma'} \int_{\gamma_{\text{min}}^{\text{d}}}^{\infty} E_{\gamma'}^2 \frac{dN}{dE_{\gamma'} d\epsilon'} dE_{\gamma'},
\]

(10)

where \( \gamma_{\text{min}}^{\text{s}} \approx \gamma_0 \text{, } E_{\text{min}}^{\text{s}} = E_{\text{min}} / \epsilon_\ell \gamma_0^2 \), \( \gamma_{\text{min}}^{\text{d}} \approx \gamma_0 \text{, } E_{\text{min}}^{\text{d}} = E_{\text{min}} / \epsilon_\ell \gamma_0^2 \), \( E_{\text{min}} = E_0 / D \), and \( dN(\gamma', E_{\gamma'}) / dE_{\gamma'} d\epsilon' \) is the ICS spectrum
(by Eq. 2.48, in Blumenthal & Gould 1970) produced by electrons with Lorentz factor \( \gamma' \) which scatter synchrotron
photons in the blob having the spectrum given by Eqs. 2 and
3.

Having determined the spectra in Eq. 6, we can now
investigate the parameter space (magnetic field strength in
the blob, B, and Doppler factor, D) for the homogeneous
SSC model which is consistent with the value of } \eta \approx 1.2.
In Figs. 1(a) and 1(b) we show the allowed value of B as a
function of D (thick full curves) for the case of outbursts as
reported by the Whipple Observatory which varied on (a) a

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\[ t_{\text{cool}} \leq t_{\text{var}} D. \]  

The cooling time scale for synchrotron losses of electrons with Lorentz factor \( \gamma_k \), which contribute mainly to synchrotron photons at the peak of the spectrum, is given by

\[ t_{\text{cool}}^{\text{syn}} = \frac{m_e c^2}{k_B \theta \gamma_k}, \] 

Eqs. (5), (11), and (12) allow us to place a lower limit on the magnetic field in the blob

\[ B > 15.1 \times 10^{23} \eta^{-1/3} \varepsilon_0^{-1/3} D^{-1/3}. \]  

Figure 1.

The parameter space \((B, D)\) allowed by the homogeneous SSC model for variability in Mrk 421 on a time scale \( t_{\text{var}} \) of (a) 1 day and (b) 15 min. The thick full curves show the condition for \( \eta = 1.2 \), and the thin full curves labelled by value of \( \eta \) show the condition for other values of \( \eta \). The other curves give allowed ranges for: efficient cooling of electrons during the flare time scale by synchrotron radiation (Eq. 13) – area above dot-dash curve labelled ‘Synch’; efficient cooling by ICS (Eq. 16) – area below dotted curve labelled ‘ICS’; escape of 8 TeV \( \gamma \)-rays – area to right of long-dashed curve; escape of 50 TeV \( \gamma \)-rays – area to right of short-dashed curve. The shaded area is the allowed region for the parameters for a spectrum extending to 8 TeV. The marginal values of \((B, D)\) which just fulfill the condition \( \eta = 1.2 \) are marked by squares and labelled (i) to (iv).

2.2 Absorption of gamma-rays in the blob radiation

The observation of \( \gamma \)-ray flares with a spectrum extending up to \( \sim 8 \) TeV, or even 50 TeV, allows us to place a lower limit on the Doppler factor of the blob under the assumptions of the homogeneous SSC model. Using the observed soft photon spectrum of Mrk 421 (Eq. 3) we can compute the optical depth \( \tau(E_\gamma, D) \) for \( \gamma \)-ray photons with energy \( E_\gamma \) for \( e^k \) pair production inside the blob.
principle produce flares with $\eta = 1.2$ as required. We note that the values of $(B, D)$ used in earlier modeling of the Mrk 421 spectrum (Inoue & Takahara 1996, Mastichiadis & Kirk 1997, Stecker, De Jager & Salamon 1996) are generally consistent with the parameter space derived by us. In order to determine if the broad band spectrum expected in the homogeneous SSC model is consistent with the $\gamma$-ray observations during flaring, we compute the synchrotron and IC spectra for four example parameters $(B, D)$ from the allowed region indicated by points $(i)$ to $(iv)$ in Figs. 1(a) and 1(b). The calculated spectra are shown in Fig. 2. Note that in each case, the lowest energy we predict corresponds to $\gamma^* = 0.032\gamma_i$ (see Eq. 4 and comments below) which depends on the magnetic field. For the 4 cases this gives $E_i^*= \approx 36$ GeV $(i)$, 120 GeV $(ii)$, 19 GeV $(iii)$, and 47 GeV $(iv)$, for the minimum energies for which we can predict the $\gamma$-ray spectrum with any confidence in the homogeneous SSC model.

For energies between 0.8 TeV and 8 TeV corresponding to observations made by the Whipple observatory during recent flaring of Mrk 421 (Krennrich et al. 1997), our predictions of the spectral index in the homogeneous SSC model range from 2.65 for 1 day variability and case $(i)$, to 2.85 for 15 minute variability and case $(iv)$. The results obtained during flaring by Krennrich et al. (1997) up to $\approx 8$ TeV are consistent with the spectrum of Mohanty et al. (1993) taken during a quiescent state where the spectral index was $\sim 2.25 \pm 0.19 \pm 0.3$ between 0.4 - 4 TeV. Given the error bars, this is just consistent with the spectral index of 2.65 predicted for 1 day variability and case $(i)$. However, we note that the calculated spectrum shows a break close to $\sim 1$ TeV which should be seen in the Whipple observations.

In the case of a flare varying on a 15 min time scale, it seems that the spectra obtained in terms of the homogeneous SSC model are not consistent with the relatively flat spectrum of the Whipple observations. The lower sensitivity HEGRA Cherenkov observations report a very steep spectrum above $\sim 1$ TeV (spectral index $3.6 \pm 1$) during the Dec. 94 - May 95 monitoring (Petry et al. 1996). However these observations refer not to outburst emission, but rather to quiescent emission since the spectrum is integrated over a long period.

In conclusion, detailed spectral measurements in the energy range above 0.3 TeV combined with the observations in the optical-X-ray range should allow one to determine precisely the parameters of the emission region (relativistic blob) and in general answer the question of the applicability of the homogeneous SSC model for $\gamma$-ray production in blazars. We note also that the absorption and synchrotron cooling conditions do not allow flares with 1 day variability having $\eta > 200$ (8 TeV) or $\eta > 60$ (50 TeV) - see the thin solid curves in Fig. 1(a). Similarly, for 15 minute variability $\eta > 70$ (8 TeV) or $\eta > 20$ (50 TeV) are not allowed (see Fig. 1b). Observation of such huge $\gamma$-ray outbursts without accompanying X-ray outbursts would be inconsistent with the homogeneous SSC model.

3 DISCUSSION AND CONCLUSION

Inspection of the Figs. 1(a) and 1(b) shows that for some values of $(B, D)$, i.e. the region of the thick full line inside the shaded area, the homogeneous SSC model can in principle produce flares with $\eta = 1.2$ as required. We note that the values of $(B, D)$ used in earlier modeling of the Mrk 421 spectrum (Inoue & Takahara 1996, Mastichiadis & Kirk 1997, Stecker, De Jager & Salamon 1996) are generally consistent with the parameter space derived by us. In order to determine if the broad band spectrum expected in the homogeneous SSC model is consistent with the $\gamma$-ray observations during flaring, we compute the synchrotron and IC spectra for four example parameters $(B, D)$ from the allowed region indicated by points $(i)$ to $(iv)$ in Figs. 1(a) and 1(b). The calculated spectra are shown in Fig. 2. Note that in each case, the lowest energy we predict corresponds to $\gamma^* = 0.032\gamma_i$ (see Eq. 4 and comments below) which depends on the magnetic field. For the 4 cases this gives $E_i^*= \approx 36$ GeV $(i)$, 120 GeV $(ii)$, 19 GeV $(iii)$, and 47 GeV $(iv)$, for the minimum energies for which we can predict the $\gamma$-ray spectrum with any confidence in the homogeneous SSC model.

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