Exotic Leptoquarks from Superstring Derived Models

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Abstract

The H1 and ZEUS collaborations have recently reported a significant excess of $e^+p \rightarrow e^+\text{jet}$ events at high $Q^2$. While there exists insufficient data to conclusively determine the origin of this excess, one possibility is that it is due to a new leptoquark at mass scale around 200 GeV. We examine the type of leptoquark states that exist in superstring derived standard–like models, and show that, while these models may contain the standard leptoquark states which exist in Grand Unified Theories, they also generically contain new and exotic leptoquark states with fractional lepton number, $\pm 1/2$. In contrast to the traditional GUT–type leptoquark states, the couplings of the exotic leptoquarks to the Standard Model states are generated after the breaking of $U(1)_{B-L}$. This important feature of the exotic leptoquark states may result in local discrete symmetries which forbid some of the undesired leptoquark couplings. We examine these couplings in several models and study the phenomenological implications. The flavor symmetries of the superstring models are found to naturally suppress leptoquark flavor changing processes.

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1 Introduction

The H1 [1] and ZEUS [2] collaborations have recently reported an excess of events in high–$Q^2$ $e^+p \rightarrow e^+\text{jet}$ collisions. While it is premature to conclude whether or not this excess arises from physics beyond the Standard Model [3], one of the possible explanations is the existence of a leptoquark state around $O(200 \text{ GeV})$ [4, 5, 6, 7, 8, 9, 10, 11, 12]. Leptoquark states arise generically in the context of Grand Unified Theories, and their properties have been discussed extensively [14]. In this paper we examine the leptoquark states that arise in superstring derived standard–like models. These models give rise to leptoquark states similar to those which exist in Grand Unified Theories, as well as exotic leptoquark states arising from the breaking of the non–Abelian gauge symmetry to the Standard Model gauge group at the string level, rather than at the effective field theory level. As a result, an important property of the exotic “stringy” leptoquark states is that they carry fractional charges under the $U(1)$ generators in the Cartan subalgebra of $SO(10)$, $U(1)_{B−L}$ and $U(1)_{T_{3R}}$. Consequently, while the exotic leptoquarks carry the usual charges under the Standard Model gauge group, they carry “fractional” charge under the $U(1)_{Z'}$ symmetry*, and therefore carry fractional lepton number $\pm 1/2$. For this reason, the couplings of the exotic leptoquarks to the Standard Model states are generated only after the breaking of $U(1)_{Z'}$. This is an important property of the exotic leptoquark states, as it may give rise to local discrete symmetries [15] that can forbid some of the undesired leptoquark couplings not forbidden for regular leptoquarks.

In this paper we study the leptoquark states which exist in the superstring derived standard–like models. We first discuss how the different types of leptoquark states arise in the superstring models. We then study the couplings of the regular and exotic leptoquarks in several specific models, and show that the string models under investigation naturally give rise to symmetries which forbid some of the undesired leptoquark couplings. For example, we find that the flavor symmetries of the models, which arise due to the underlying $Z_2 \times Z_2$ orbifold compactification, forbid flavor non–

* $U(1)_{Z'}$ is the combination of $U(1)_{B−L}$ and $U(1)_{T_{3R}}$ which is orthogonal to the weak–hypercharge $U(1)_Y$. 
diagonal couplings at leading order. We therefore arrive at the pleasing conclusion that stringy symmetries prevent the leptoquark states from inducing unacceptable flavor changing interactions.

2 The superstring standard–like models

In this section, we give a brief overview of the superstring models in the free fermionic formulation. It is important to note that, although we will examine the leptoquark states in some specific models, the types of exotic states that we describe are generic in the free fermionic standard–like models, and similar exotic leptoquarks may in fact arise in general string compactifications. The purpose of providing this brief overview is to emphasize those generic features of the construction responsible for the standard and exotic leptoquark states.

The superstring models that we discuss are constructed in the free fermionic formulation [16]. In this formulation, a model is constructed by choosing a consistent set of boundary condition basis vectors. The basis vectors, $b_k$, span a finite additive group $\Xi = \sum_{k} n_k b_k$, where $n_k = 0, \cdots, N_{zk} - 1$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$ are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The $U(1)$ charges with respect to the unbroken Cartan generators of the four dimensional gauge group, $Q(f)$, are in one to one correspondence with the $U(1)$ currents $f^*f$ for each complex fermion $f$, and are given by:

$$Q(f) = \frac{1}{2} \alpha(f) + F(f),$$

(2.1)

where $\alpha(f)$ is the boundary condition of the world–sheet fermion $f$ in the sector $\alpha$, and $F_\alpha(f)$ is a fermion number operator.

The realistic models in the free fermionic formulation are generated by a basis of boundary condition vectors for all world–sheet fermions [17, 18, 19, 20, 21, 22, 23]. In the models that we examine the basis is constructed in two stages. The first stage consists of the NAHE set [21], which is the set of five boundary condition basis vectors $\{1, S, b_1, b_2, b_3\}$. The gauge group after the NAHE set is $SO(10) \times SO(6)^3 \times$
$E_8$, and possesses $N = 1$ space–time supersymmetry. The right–moving complex fermions $\bar{\psi}^{1, \ldots, 5}$ produce the observable $SO(10)$ symmetry. In addition to the gravity and gauge multiplets, the Neveu–Schwarz sector produces six multiplets in the 10 representation of $SO(10)$, and several $SO(10)$ singlets transforming under the flavor $SO(6)^3$ symmetries. The sectors $b_1$, $b_2$ and $b_3$ produce 48 spinorial 16 of $SO(10)$, sixteen each from the sectors $b_1$, $b_2$ and $b_3$. The free fermionic models correspond to $Z_2 \times Z_2$ orbifold models with nontrivial background fields [24]. The NS sector corresponds to the untwisted sector, and the sectors $b_1$, $b_2$ and $b_3$ to the three twisted sectors of the $Z_2 \times Z_2$ orbifold model.

In the second stage of the basis construction, three additional basis vectors are added to the NAHE set. These three additional basis vectors correspond to “Wilson lines” in the orbifold formulation. They are needed to reduce the number of generations to three, one each from the sectors $b_1$, $b_2$ and $b_3$. At the same time, the additional boundary condition basis vectors break the gauge symmetries of the NAHE set. The $SO(10)$ symmetry is broken to one of its subgroups, either $SU(5) \times U(1)$, $SO(6) \times SO(4)$, or $SU(3) \times SU(2) \times U(1)_{B-L} \times U(1)_{T_{3R}}$. This is achieved by the following assignment of boundary conditions to the set $\bar{\psi}^{1, \ldots, 5}$:

$$b\{\bar{\psi}_{\frac{1}{2}}^{1, \ldots, 5}\} = \{1\ 1\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\} \Rightarrow SU(5) \times U(1),$$  \hspace{1cm} (2.2)

$$b\{\bar{\psi}_{\frac{3}{2}}^{1, \ldots, 5}\} = \{1\ 1\ 1\ 0\ 0\} \Rightarrow SO(6) \times SO(4).$$  \hspace{1cm} (2.3)

To break the $SO(10)$ symmetry to $SU(3) \times SU(2) \times U(1)_C \times U(1)_L \uparrow$, both (2.2) and (2.3) are used, in two separate basis vectors. In the superstring derived standard–like models, the three additional basis vectors beyond the NAHE set are denoted $\{\alpha, \beta, \gamma\}$. The two basis vectors $\alpha$ and $\beta$ break the $SO(10)$ symmetry to $SO(6) \times SO(4)$, while the vector $\gamma$ breaks the $SO(10)$ symmetry to $SU(5) \times U(1)$.

In the models discussed below, the observable gauge group after application of the generalized GSO projections is $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \times U(1)_B \times U(1)_{T_{3R}}$. The hidden $E_8$ gauge group is broken to $SU(5) \times SU(3) \times U(1)^2$, and the flavor $SO(6)$

$^\uparrow U(1)_C = \frac{3}{2}U(1)_{B-L}; U(1)_L = 2U(1)_{T_{3R}}.$
symmetries are broken to $U(1)^3 \times U(1)^3$. The weak hypercharge is given by

$$U(1)_Y = \frac{1}{3} U(1)_C + \frac{1}{2} U(1)_L,$$

while the orthogonal combination is given by

$$U(1)_{Z'} = U(1)_C - U(1)_L.$$  

The massless spectrum of the standard-like models contains three chiral generations from the sectors which are charged under the horizontal symmetries. Each of these consists of a 16 of $SO(10)$, decomposed under the final $SO(10)$ subgroup as

$$e_L^c \equiv [(1, \frac{3}{2}); (1, 1)](1, 1/2, 1); \quad u_L^c \equiv [(3, -1/2); (1, -1)](-2/3, 1/2, -2/3);$$

$$d_L^c \equiv [(3, -1/2); (1, 1)](1/3, -3/2, 1/3); \quad Q \equiv [(3, 1/2); (2, 0)](1/6, 1/2, -2/3, -1/3);$$

$$N_L^c \equiv [(1, -3/2); (1, -1)](0, 5/2, 0); \quad L \equiv [(1, -3/2); (2, 0)](-1/2, -3/2, 0, 1),$$

where we have used the notation

$$[(SU(3)_C \times U(1)_C); (SU(2)_L \times U(1)_L)](Q_Y, Q_{Z'}, Q_e, m),$$

and have written the electric charge of the two components for the doublets.

The matter states from the NS sector and the sectors $b_1$, $b_2$, and $b_3$ transform only under the observable gauge group. In the realistic free fermionic models, there is typically one additional sector that produces matter states transforming only under the observable gauge group. Usually, this sector is a combination of two of the vectors which extend the NAHE set. For example, in the model of Ref. [20], the combination $b_1 + b_2 + \alpha + \beta$ produces one pair of electroweak doublets, one pair of color triplets and five pairs of $SO(10)$ singlets which are charged with respect to the $U(1)$ currents of the observable gauge group. All matter states from the NS and $b_1 + b_2 + \alpha + \beta$ sectors carry Standard Model charges, and are obtained by GSO projection from the 10 and $\overline{10}$ representation of $SO(10)$.

In addition to the states mentioned above transforming solely under the observable gauge group, the sectors $b_j + 2\gamma$ produce matter states that fall into the 16
representation of the hidden $SO(16)$ gauge group decomposed under the final hidden gauge group. The states from the sectors $b_j + 2\gamma$ are $SO(10)$ singlets, but are charged under the flavor $U(1)$ symmetries. The sectors which arise from combinations of $\{b_1, b_2, b_3, \alpha, \beta, \pm \gamma\}$ produce additional massless matter states in vector–like representations. Such states are exotic stringy states and cannot fit into representations of the underlying $SO(10)$ symmetry group of the NAHE set. They result from the breaking of the $SO(10)$ gauge group at the string level via the boundary condition assignment in Eqs. (2.2) and (2.3). These sectors give rise to the exotic leptoquark states that we describe below.

Analysis of the fermion mass terms up to order $N = 8$ reveals the general texture of fermion mass matrices in the superstring standard–like models [24, 25]. The light Higgs doublets are obtained from the NS and $b_1 + b_2 + 2\gamma$ sectors and typically consist of $\bar{h}_1$ or $\bar{h}_2$ and $h_{45}$. The sectors $b_1$ and $b_2$ produce the two heavy generations and the sector $b_3$ produces the lightest generation. This is due to the flavor $U(1)$ charges and because the Higgs pair $h_3$ and $\bar{h}_3$ necessarily get a Planck scale mass [24]. We adopt a notation consistent with this numbering scheme throughout the remainder of the paper, so that, for example, $Q_3$ represents the left–handed up–down quark doublet.

3 Leptoquarks from the superstring models

There are several types of leptoquark states which arise in the free fermionic models. The first type are obtained from the Neveu-Schwarz sector and from the sector $b_1 + b_2 + \alpha + \beta$. At the level of the NAHE set, the NS sector gives rise to vectorial 10 representations obtained by acting on the vacuum with $\bar{\psi}_1, \cdots, \bar{\psi}_5$ and $\bar{\psi}_1, \cdots, \bar{\psi}_5^*$. Thus, these states are in the $5 + \bar{5}$ of $SU(5)$, and produce the color triplets and electroweak doublets

$$D_j \equiv [(3, -1), (1, 0)]_{(-1/3, -1, -1/3)} \quad h_j \equiv [(1, 0), (2, 1)]_{(1/2, -1, (1, 0))} , \quad (3.1)$$
along with the complex conjugate representations. The $B - L$ charge of the color triplets is

$$Q_{B-L} = 2/3Q_C = -2/3.$$ 

These states therefore carry baryon number $1/3$ and lepton number $1$, and are standard leptoquarks, a type of leptoquark identical to the types appearing in $SO(10)$ and $E_6$ models. This should not be particularly surprising, as they are obtained from the 10 vectorial representation of $SO(10)$ by the GSO projections. These leptoquark states from the NS sector can appear in $SU(5) \times U(1)$, $SO(6) \times SU(4)$, and $SU(3) \times SU(2) \times U(1)^2$ type models. In the last two cases, there exists a superstring doublet–triplet splitting mechanism that projects these leptoquark states from the massless spectrum, while the corresponding electroweak doublets remain in the light spectrum [21]. Thus, string models can be constructed in which all leptoquark states from the NS sector are projected out of the massless spectrum.

The second type of leptoquark states arises from the sector $b_1 + b_2 + \alpha + \beta$ [26]. These states are similar to those originating from the NS sector, and are obtained by acting on the NS vacuum of the right–moving fermions, $\bar{\psi}_1, \ldots, 5$. The existence of leptoquark states from this sector depends as well on the choice of boundary conditions in the basis vectors $\{\alpha, \beta, \gamma\}$ which extend the NAHE set. For example, in the model of Ref. [20], one such vector–like state is obtained

$$D_{45} \equiv [(3, -1), (1, 0)]_{(-1/3, -1, -1/3)} \quad \bar{D}_{45} \equiv [(\bar{3}, 1), (1, 0)]_{(-1/3, -1, -1/3)},$$

while in the model of Ref. [22], all the leptoquark states from this sector are projected out by the GSO projections. The leptoquark states from this sector are identical to those in $SO(10)$ and $E_6$ models.

There exist additional exotic leptoquark states obtained from sectors which arise due to the breaking of $SO(10)$ to $SU(3) \times SU(2) \times U(1)^2$. These states come from sectors produced from combinations of the NAHE set basis vectors and the additional basis vectors $\{\alpha, \beta, \gamma\}$. Massless states arising from such sectors do not fit into representations of the original $SO(10)$ symmetry, as they carry fractional charges with respect to the unbroken $U(1)$ generators, $U(1)_C$ and $U(1)_L$, of the original
non–Abelian $SO(10)$ Cartan subalgebra. These fractional charges are a result of the boundary conditions in Eqs. (2.3) and (2.2), which break the $SO(10)$ symmetry to $SU(5) \times U(1)$ and $SO(6) \times SO(4)$, respectively. The exotic states from these sectors are therefore classified according to the pattern of symmetry breaking in each sector. Sectors which contain the vector $\alpha$ (or $\beta$, but not $\alpha + \beta$) break the $SO(10)$ symmetry to $SO(6) \times SO(4)$, while sectors that contain the vector $\gamma$ break the $SO(10)$ symmetry to $SU(5) \times U(1)$, and sectors containing both $\alpha$ (or $\beta$) and $\gamma$ break the $SO(10)$ symmetry to $SU(3) \times SU(2) \times U(1)^2$.

Sectors of the last sort, and with vacuum energy $V.E. = -1 + 3/4$ in the right–moving sector, arise frequently in the free fermionic standard–like models. Massless states in these sectors are obtained by acting on the vacuum with a complex fermion with 1/2 or $-1/2$ boundary condition, and with fermionic oscillator 1/4. Such sectors give rise to the exotic leptoquark states with the quantum numbers,

$$[(3, -1/4), (1, -1/2)]_{(-1/3,1/4,-1/3)} ; [(3, 1/4), (1, 1/2)]_{(1/3,-1/4,1/3)} .$$

These states have $Q_{EM} = \mp 1/3$, and therefore have the regular down–type electric charge. The $B − L$ charge and lepton number, however, are†

$$Q_{B−L} = \mp \frac{1}{6} \text{ and } Q_L = \pm \frac{1}{2} .$$

Such exotic states therefore carry fractional lepton number $\pm 1/2$ and, in fact, appear generically in the free fermionic standard–like models. For example, in the model of Ref. [18], such states are obtained from the sectors $b_3 + \alpha \pm \gamma$ and $b_1 + b_2 + b_4 + \alpha \pm \gamma$, in the model of Ref. [20], from the sectors $b_{1,2} + b_3 + \alpha \pm \gamma$ (Table 4), and in the model of Ref. [22], from the sectors $b_{1,2} + b_3 + \beta \pm \gamma$ (Table 5).

We listed above the GUT–type and exotic leptoquark states which exist in the superstring models appearing in the literature to date. Below, we enumerate several additional types of exotic leptoquark states that may appear in the superstring derived models.

†here $Q_L$ is the lepton number.
In addition to the sectors in the additive group with $\pm 1/2$ boundary conditions and with $X_R \cdot X_R = 6$, the superstring models may contain sectors with $\pm 1/2$ boundary conditions and with $X_R \cdot X_R = 4$. Whereas in the former case only one oscillator with $\nu_f = 1/4$ acting on the NS vacuum was needed to get a massless state, in the later case two such operators are needed. The type of states arising from these sectors depends on the boundary conditions of the complex world–sheet fermions $\bar{\psi}^{1 \cdots 5}$.

Two possibilities exist,

$$b\{\bar{\psi}^{1 \cdots 5}\} = \{\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}\}, \quad (3.5)$$

$$b\{\bar{\psi}^{1 \cdots 5}\} = \{\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2}\}, \quad (3.6)$$

with the conjugate sectors, obtained by taking the opposite sign of the $\pm 1/2$ phases, producing the complex conjugate states.

Sectors with these boundary conditions and with $X_R \cdot X_R = 4$ can then give rise to new exotic leptoquark states. In particular, as two oscillators are now required in order to get a massless state, these sectors may give rise to exotic diquarks, rather than single quark states. Sectors of this type may generally exist in the additive group. To construct an actual model that includes a sector of this type, we simply include a sector of the desired form in the basis vectors that define the model. Table 1 gives an example of one such model that gives rise to exotic diquarks from the sector $\gamma$.

We emphasize that this three generation standard–like model should not be regarded as a realistic model, but simply as an example that illustrates the type of sectors giving rise to exotic diquarks, and suggests the manner in which they may arise in concrete string models.

There are several types of exotic diquarks that may arise from these types of string sectors. Sectors with the boundary conditions of Eq. (3.5) may give rise to exotic diquarks with quantum numbers

$$[\left(3, -\frac{1}{4}\right), \left(2, -\frac{1}{2}\right)]_{(-1/3,1/4,(1/6, -5/6)} \quad [\left(\bar{3}, \frac{1}{4}\right), \left(2, \frac{1}{2}\right)]_{(1/3,-1/4,(1/6,5/6)} \quad (3.7)$$

while sectors with the boundary conditions of Eq. (3.6) may give rise to exotic
diquarks with quantum numbers

\[ [(3, -\frac{1}{4}), (2, \frac{1}{2})], (1/6, -3/4, 2/3, -1/3) \quad ; \quad [(3, \frac{1}{4}), (2, \frac{1}{2})], (-1/6, 3/4, -2/3, 1/3) \quad . \]  

(3.8)

The states of the first type produce fractionally charged baryons, and therefore cannot exist in a realistic low energy spectrum, while the states of the second type are exotic diquarks with standard \( SO(10) \) weak hypercharge and “fractional” \( U(1)_Z \) charge.

In general, we may anticipate the presence of additional types of exotic leptoquarks and diquarks in other string models not utilizing the NAHE set, and in which the weak hypercharge does not have the standard \( SO(10) \) embedding. This can only be investigated in specific models. We can, however, place some generic constraints. For example, there is an upper limit on the weak–hypercharge of the exotic leptoquarks in models in which the color and weak non–Abelian groups are obtained at level one affine lie algebras. This follows from the constraint that the conformal dimension of the massless states is \( h = \bar{h} = 1 \). The contribution to the conformal dimension due to the \( SU(3) \times SU(2) \times U(1)^2 \) charges is given by

\[
\frac{C(R_3)}{k_3 + 3} + \frac{C(R_2)}{k_2 + 2} + \frac{Q_Y^2}{k_1} \leq 1 ,
\]

(3.9)

where \( C(R_i) \) and \( k_i \) are the quadratic Casimir of the \( R_i \) representation and the Kac–moody level of the group \( SU(i) \), respectively. Models with \( k_3 = k_2 = 1 \) and \( k_Y = 5/3 \) cannot give an exotic diquark with \( Q_Y \geq 5/6 \). If a diquark with \( Q_Y \geq 5/6 \) is observed at low energies, therefore, it will exclude all level one string models, with \( SO(10) \) embedding of the weak hypercharge. In a general string model in which the group factors are not produced by free fermions or bosons, but rather by higher level conformal field theories, additional types of exotic leptoquarks and diquarks may be possible. As we increase the level of the corresponding group factors, states with larger charges can be obtained. For example, at level \( k = 2 \) with, \( k_1 = k_2 = 2 \), \( k_Y = 10/3 \) states with \( Q_Y \leq 11/6 \) are in principle permissible. Perturbative gauge coupling unification places strong constraints on this possibility, however, as we discuss below.
4 Interactions

To study the phenomenology of the leptoquark states arising in the superstring derived models, we examine the interaction terms with the Standard Model states in the models of Refs. [20] and [22]. The cubic level and higher order non–renormalizable terms in the superpotential are obtained by calculating correlators between vertex operators, $A_N \sim \langle V^f_1 V^f_2 V^b_3 \cdots V^b_N \rangle$, where $V^f_i$ ($V^b_i$) are the fermionic (bosonic) vertex operators corresponding to different fields. The non–vanishing terms are obtained by applying the rules of Ref. [28]. As the free fermionic standard–like models contain an anomalous $U(1)$ symmetry, some Standard Model singlets in the massless string spectrum must acquire a VEV near the string scale which cancels the $D$–term equation of the anomalous $U(1)$. In this process, some of the higher order non–renormalizable terms become renormalizable operators in the effective low energy field theory.

First, as the leptoquark states arise in vector–like representations, mass terms for the leptoquark states are expected to arise from cubic level or higher order terms in the superpotential. For example, in the model of Ref. [20], we find at cubic level the mass terms

$$\xi_3 D_{45} \bar{D}_{45}, \xi_1 H_{21}H_{22},$$

(4.1)

where $\xi_1$ and $\xi_3$ are singlets under the entire four dimensional gauge group so that their VEVs are not constrained by the $D$–term constraints. In this example, the leptoquark states can therefore remain light at this level, at least in principle. Of course, other phenomenological constraints may require $\xi_1$ or $\xi_3$ to have non–vanishing VEVs; one has yet to examine whether a fully realistic solution allows either of these leptoquarks to remain light. In the model of Ref. [22], which contains only exotic leptoquark states, we find the cubic level mass terms

$$\xi_1 D_1 \bar{D}_1, \xi_2 D_2 \bar{D}_2.$$

(4.2)

Here, $\xi_1$ and $\xi_2$ are again singlets of the entire four dimensional gauge group. Finally, in the model of Ref. [18], which also contains only exotic leptoquark states, there

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§here the notation of Ref. [20] is used
¶here the notation of Ref. [22] is used
are no cubic level mass terms for the exotic leptoquarks. One finds in this model a potential mass term at the quintic order\footnote{here the notation of Ref. \cite{18} is used}
\begin{equation}
H_{33} H_{40} H_{31} H_{38} \Phi_{23},
\end{equation}
where \( H_{33} \) and \( H_{40} \) are the exotic leptoquark states, \( H_{31} \) and \( H_{38} \) are Standard Model singlets charged under \( U(1)_Z \), and \( \Phi_{23} \) is an \( SO(10) \) singlet. Giving a VEV to these Standard–Model singlets makes the exotic quark triplets heavy.

We next turn superpotential terms involving both the leptoquarks and the Standard Model states. The potential leptoquark interaction terms are
\begin{align}
LQ \bar{D}, & \quad u_L^e e_L^c D, \quad d_L^c N_L^c D, \quad (4.4) \\
QQD, & \quad u_L^e d_L^c \bar{D}, \quad (4.5) \\
QDh, & \quad (4.6) \\
\bar{D} D u_L^c, & \quad (4.7)
\end{align}
and \( D \bar{D} \phi \). Severe constraints on the couplings of the leptoquarks to the Standard Model states are imposed by proton longevity. If the couplings in Eq. (4.4) and Eq. (4.5) are not sufficiently suppressed, proton decay is induced by leptoquark exchange. For concreteness, we examine in detail the couplings in the models of Ref. \cite{20} and Ref. \cite{22}. In the case of the standard GUT type leptoquarks from the Neveu–Schwarz sector and the sector \( b_1 + b_2 + \alpha + \beta \), such dangerous couplings are indeed expected. In general, if massless triplets from the Neveu–Schwarz sector or the sector \( b_1 + b_2 + \alpha + \beta \) exist in the massless spectrum, then the terms in Eqs. (4.4,4.5) are obtained either at the cubic level of the superpotential, or from higher order nonrenormalizable terms. For example, in the model of Table 2 (the massless spectrum and quantum numbers are given in Ref. \cite{21}), we obtain at the cubic level,
\begin{align}
& u_1^e e_1^c D_1, \quad d_1^c N_1^c D_1, \quad u_2^e e_2^c D_2, \quad d_2^c N_2^c D_1, \\
& D_1 D_2 \Phi_{12}, \quad \bar{D}_1 D_2 \Phi_{12}, \quad (4.8)
\end{align}
while in the model of Ref. [20] we obtain at the quartic order,

\[ u_c^1 e_c^1 D_{45} \bar{\Phi}_1^+ , \quad u_c^2 e_c^2 D_{45} \bar{\Phi}_2^+ , \quad d_c^1 N_c^1 D_{45} \Phi_1^+ , \quad d_c^2 N_c^2 D_{45} \bar{\Phi}_2^- , \]

\[ Q_1 Q_1 D_{45} \Phi_1^+ , \quad Q_2 Q_2 D_{45} \bar{\Phi}_2^- . \]

(4.9)

At higher orders, additional terms will appear. We observe that leptoquark states from the NS sector, or from a sector of the type of \( b_1 + b_2 + \alpha + \beta \), will generally have the undesirable couplings with the Standard Model states. Such couplings are induced from higher order terms by the VEVs of the Standard Model singlets which cancel the anomalous \( U(1) \) \( D \)-term equation. We therefore anticipate that a leptoquark from either of these sectors is not likely to provide a realistic possibility for a light leptoquark state.

We turn now to the exotic leptoquark states. Because of the fractional lepton number of these exotic leptoquarks, direct couplings to the Standard Model states are impossible. Coupling to the Standard Model states can occur only through higher order terms containing a Standard Model singlet field with fractional lepton number \( \pm 1/2 \). Such additional fields generally exist in superstring standard-like models. The coupling of the exotic leptoquarks then depends on these extra fields in specific models, together with a pattern of VEVs which preserves supersymmetry at the Planck scale. The important aspect of the fractional charges of the exotic leptoquarks is that they may give rise to residual local discrete symmetries that forbid the dangerous coupling of the exotic leptoquarks to the Standard Model states.

In the context of the model of Ref. [22], the coupling of the exotic leptoquarks to the Standard Model states has been examined in detail in Refs. [29, 15]. It was shown that, in this model, the interaction terms of the exotic leptoquarks with the Standard Model states vanish to all orders of nonrenormalizable terms for the following reason. The interaction terms take the forms \( f_i f_j D \phi^n \) and \( f_i D D \phi^n \), where \( f_i \) and \( f_j \) are the Standard Model states from the sectors \( b_1 , b_2 \) and \( b_3 \), and \( D \) represents the exotic leptoquark. The product of fields, \( \phi^n \), is a product of Standard Model singlets that ensures invariance of the interaction terms under all \( U(1) \) symmetries and the string selection rules. If all the fields \( \phi \) in the string \( \phi^n \) get VEVs, then the coefficients of
the operators in Eqs. (4.5) and (4.4) are of the order \((\phi/M)^n\), where \(M \sim 10^{18}\text{GeV}\). Because of the fractional charge of the exotic leptoquarks under \(U(1)_{Z'}\), none of the interaction terms in Eqs. (4.5) and (4.4) are invariant under \(U(1)_{Z'}\). The total \(U(1)_{Z'}\) charge of each of these interaction terms is a multiple of \(\pm(2n + 1)/4\). Thus, for these terms to be allowed, the string \(\phi^n\) must break \(U(1)_{Z'}\) and must have a total \(U(1)_{Z'}\) charge in multiple of \(\pm(2n + 1)/4\). The string of Standard Model singlets must therefore contain a field which carries fractional \(U(1)_{Z'}\) charge \(\pm(2n + 1)/4\). In the model of Ref. [22], the only Standard Model singlets with fractional \(U(1)_{Z'}\) charge transform as triplets of the hidden \(SU(3)_H\) gauge group. As these fields transform in vector–like representations, invariance under the symmetries of the hidden sector guarantees that there is a residual \(Z_4\) discrete symmetry which forbids the coupling of the exotic leptoquarks to the Standard Model states to all orders of nonrenormalizable terms. While this symmetry ensures that the exotic leptoquark states do not cause problems with proton decay, it also forbids their generation at \(e^\pm p\) colliders.

Next we turn to the model of Ref. [20]. In this model, we find at the quartic order of the superpotential

\[
\begin{align*}
u_2^c d_2^c H_{21} H_{26} \ , \ & Q_2 L_2 H_{21} H_{26} , \ (4.10)
\end{align*}
\]

at the quintic order,

\[
\begin{align*}
L_3 Q_3 H_{21} H_{18} \Phi_{45} \ , \ & L_3 Q_3 H_{21} H_{24} \xi_2 \ , \ & d_3^c u_3^c H_{21} H_{18} \Phi_{45} \ , \ & d_3^c u_3^c H_{21} H_{24} \xi_2 , \ (4.11)
\end{align*}
\]

and at order \(N = 6\) we find for example,

\[
\begin{align*}
L_1 Q_1 H_{21} H_{24} \Phi_{13} \xi_1 \ , \ & L_1^c u_1^c H_{21} H_{24} \Phi_{13} \xi_1 \\
Q_1 Q_1 H_{22} H_{17} \phi_1^+ \xi_1 \ , \ & d_1^c N_1^c H_{22} H_{17} \Phi_1^+ \xi_1 \\
u_1^c e_1^c H_{22} H_{17} \Phi_1^c \xi_1 \ , \ & u_1^c e_1^c H_{22} H_{17} \Phi_1^c \xi_1 \\
Q_2 Q_2 H_{22} H_{23} \Phi_2^c \Phi_{45} \ , \ & d_2^c N_2^c H_{22} H_{23} \Phi_2^c \Phi_{45} , \ (4.12)
\end{align*}
\]

plus additional terms of the generic form \(f_i f_i H H(\partial W_3/\partial \eta_i)\) which vanish by the cubic level \(F\)-flatness constraints. The important lesson to draw from this model is that couplings of the exotic leptoquarks to Standard Model states are generated from
nonrenormalizable terms by VEVs which break the $U(1)_{Z'}$ gauge group. Another observation is that the exotic leptoquark couplings are flavor diagonal.

5 Phenomenology

In this section, we discuss the phenomenological implications of the standard and exotic leptoquark states appearing in the superstring derived standard–like models. The couplings of exotic leptoquarks to the standard model states are typically quite constrained by low energy phenomenology.

We make several simple observations with regard to interaction of the exotic leptoquarks with the Standard Model states. Although our observations are made primarily for the model of Ref. [20], they are in fact much more general.

The first comment concerns leptoquark induced proton decay. Here we note that in the model of Ref. [22] the problem is solved entirely. This model contains only exotic leptoquarks, since the “regular” leptoquark states from the Neveu–Schwarz and $b_1 + b_2 + \alpha + \beta$ sectors are removed by the GSO projections. Second, the spectrum, charges, and symmetries are such that all interaction terms of the exotic leptoquarks vanish identically. The exotic leptoquark states therefore lead to no conflict with proton lifetime constraints. Clearly, however, the exotic leptoquark states can neither account for the anomalous HERA events.

We next turn to the model of Ref. [20]. Here, all color triplets from the NS sector are removed by GSO projection. The model does contain one “regular” leptoquark from the sector $b_1 + b_2 + \alpha + \beta$ and one exotic leptoquark state from the sector $b_{1,2} + b_3 + \alpha \pm \gamma$, however. We expect that the “regular” leptoquark states do have interaction terms with the Standard Model states appearing at successive orders. Interaction terms of the states from the sector $b_1 + b_2 + \alpha + \beta$ with the Standard Model states do not appear at the cubic level of the superpotential because of the flavor symmetries, but may arise in higher order non–renormalizable terms. Although the higher order terms are expected to be suppressed by powers of $(\langle \phi \rangle / M)^n$, this suppression, in general, cannot make the dangerous couplings sufficiently small. For
this reason, it is not expected that such “regular” leptoquark states can be interpreted as light leptoquarks.

Next, we study the couplings of the exotic leptoquark states in the model of Ref. [20]. We note that such couplings generally arise from higher order terms in this model. A novel feature of the exotic leptoquark couplings is that such couplings can arise only due to a VEV breaking the $U(1)_{B-L}$ symmetry. Thus, the magnitude of the coupling of the exotic leptoquarks to the Standard Model states is tied to the $U(1)_{B-L}$ scale. This is a welcome feature, as the rate of proton decay inducing processes can be sufficiently small, even for a light leptoquark, coupled as it is to the scale of $U(1)_{Z'}$ breaking. Furthermore, upon examining the couplings in Eqs. (4.10,4.11,4.12), we note that the induced couplings depend on the specific choice of fields that breaks the $U(1)_{B-L}$ symmetry. Therefore, for a specific pattern of such VEVs compatible with the anomalous $U(1)$ D–term cancelation mechanism, it is generally possible to allow the $U(1)_{B-L}$ breaking scale to occur at a relatively high scale, while utilizing the freedom in the choice of fields along with the flavor symmetries to avoid conflict with the proton lifetime. The question still remaining, however, concerns the possibility of suppressing proton decay while allowing for a large lepton or baryon number violating coupling, but not for both. Although this does not happen in the model of Ref. [20] where, as can be seen from a quick examination of Eq. (4.11), the product $< H_{18} \Phi_{45} >$ determines the magnitude of both the B and L violating couplings, we claim that such a situation may in general be possible. As evidence to support our claim, we note that the superstring standard–like models occasionally give rise to custodial symmetries that distinguish between the lepton violating and baryon violating operators [32]. In summary, while the models of both Refs. [22] and [20] can sufficiently suppress the couplings of the exotic leptoquarks and avoid problems with proton decay, neither of these models allows a large lepton number violating coupling of the exotic leptoquark while suppressing the baryon violating coupling. We anticipate, however, that there exists a slight modification, or perhaps some synthesis of these models, that admits such a possibility.

We next discuss the suppression of flavor violating couplings in the model. We
note that the couplings of the regular and exotic leptoquark states are flavor diagonal in the first few orders. Flavor mixing terms are therefore suppressed by several powers of $\langle \phi \rangle / M$. This suppression in fact arises due to the flavor symmetries of the superstring derived models, and is a direct consequence of the fact that the Standard Model states from the sectors $b_j$ each carry charges with respect to $(U(1)_L ; U(1)_{L_j+3})$ and $(U(1)_R ; U(1)_{R_j+3})$, $(j = 1, 2, 3)$. Thus, the states from each sector $b_j$ are charged with respect to different pairs of $U(1)$ symmetries. This charge assignment is a reflection of the underlying $Z_2 \times Z_2$ orbifold compactification in which each of the twisted sectors lies along an orthogonal plane. The states from the NS sector, the sector $b_1 + b_2 + \alpha + \beta$, and the sectors which give rise to the exotic leptoquarks, however, are neutral with respect to $U(1)_{L_{j+3}}$ and $U(1)_{R_{j+3}}$. In order to form a gauge invariant flavor mixing leptoquark coupling, therefore, we need to utilize additional fields with half integral charges with respect to $U(1)_{L_{j+3}}$ and $U(1)_{R_{j+3}}$. In the free fermionic models which are based on the NAHE set, the only available fields are those from the sector $b_j + 2\gamma$. Furthermore, since these sectors preserve also the underlying $Z_2 \times Z_2$ orbifold structure, a potential mixing term must contain at least two such fields from two different sectors. This is the reason for the suppression of the flavor mixing terms. Indeed, in the model of Ref. [20] we find that two such terms for the exotic leptoquarks first appear at order $N = 7$,

\[ d_3 N_2 H_{21} H_{17} \phi_{45} \bar{V}_2 V_3 \quad d_2 N_3 H_{21} H_{17} \phi_{45} \bar{V}_2 V_3 \quad (5.1) \]

At higher orders, $N = 8, ...$ additional terms of this type are expected to appear. Note that, in addition to the suppression by the VEV which breaks $U(1)_{B-L}$, these terms have an additional $(\langle \phi \rangle / M)^3$ suppression factor. We conclude that the non–diagonal leptoquark couplings are naturally suppressed in free fermionic models in which each of the generations is obtained from a different twisted sector of the $Z_2 \times Z_2$ orbifold.

We now comment on the possibility of the existence of exotic leptoquarks that preserve the family numbers. First of all, due to the suppression of the successive orders of nonrenormalizable terms, we generally do not expect these couplings to be in conflict with experimental constraints. As we discuss below, the experimental constraints typically require that the product of two operators be smaller than some
phenomenological limit. As in the string, the separate operators arise a different orders. It is in fact very natural that, while one of the operators is relatively large, the other is sufficiently suppressed so as to avoid conflict with observations.

Let us however discuss briefly the interesting possibility of producing several exotic leptoquark states which carry a family number. In the model of Ref. [20], this is not the case, as there exists a single exotic leptoquark pair that couples to all three generations. However, again we anticipate that such a model may exist for the following reason. In the superstring derived models, a combination of the sectors $\alpha$, $\beta$ and $\gamma$ occasionally gives rise to additional space–time vector bosons which enhance the gauge group. This combination, when added to the vectors $b_j$ ($j = 1, 2, 3$), gives rise to the sectors that produce the additional states that must fill the representations of the enhanced symmetry. Thus, this situation arises because the vector combination which enhances the gauge symmetry preserves the symmetry of the NAHE set. In the vector combination that enhances the gauge symmetry, the left–moving vacuum vanishes. Thus, if it is possible to construct a model with vector combination $X$, with $X_L \cdot X_L = 4$, and whose addition to each of the basis vectors $b_1$, $b_2$ and $b_3$ produces a vacuum with $(b_j + X)_R \cdot (b_j + X)_R = 6$ for ($j = 1, 2, 3$), then this type of model would produce exotic leptoquarks from each sector $b_j + X$ that couple diagonally to the states from the sectors $b_j$. Again, we expect that such a model may exist. It is expected, however, that if such a model exists, higher order terms will mix the family leptoquarks with the different families. Such higher order terms are likely sufficiently suppressed to avoid conflict with experimental constraints, however.

Let us consider a model which contains the exotic leptoquark couplings of Ref. [20], but lacks couplings between the regular leptoquarks and the standard model states. This property may arise, for instance, from additional $U(1)$ symmetries in the effective theory, and is a generic possibility in such superstring derived standard–like models. In such a situation, we need concern ourselves only with the couplings of Eq. (4.10) at lowest order in the superpotential. We expect that the standard model singlet $H_{26}$ will develop a VEV somewhat below the string scale, and define
the dimensionless parameter $x_{26}$ appropriately:

$$x_{26} = \frac{<H_{26}>}{M_{\text{string}}}.$$  \hspace{1cm} (5.2)

Even though the operators of Eq. (4.10) couple only to second generation quarks and leptons, proton decay occurring at one loop level places stringent bounds on their couplings. Denoting their couplings respectively as $\lambda''_{22}$ and $\lambda'_{22}$, one finds for leptoquark masses below 1TeV the constraint [33]

$$|\lambda''_{22} \lambda'_{22} x_{26}^2| < 10^{-9}.$$  \hspace{1cm} (5.3)

In the absence of the lepton number violating coupling $\lambda'_{22}$, cosmological arguments place strong upper limits on $\lambda''_{22}$ [34],

$$|\lambda''_{22} x_{26}| < 10^{-7},$$  \hspace{1cm} (5.4)

although there is some suggestion of model dependence to these determinations [35]. Conversely, in the absence of $\lambda''_{22}$, deep inelastic experiments involving muon neutrinos demand [36]

$$|\lambda'_{22} x_{26}| < 0.22 \left( M_{LQ}/100 GeV \right),$$  \hspace{1cm} (5.5)

where $M_{LQ}$ is the mass of the leptoquark. Due to the appearance of $x_{26}$, which could be significantly less than unity, these relations are satisfied much more naturally for the case of exotic leptoquarks than they are for their more traditional integer lepton number cousins.

In the more general case of a model possessing generic lepton number violating interactions

$$\lambda'_{ij} L_i Q_j H \phi_a^n(i, j),$$  \hspace{1cm} (5.6)

but no baryon number violating interactions, the phenomenological constraints on the couplings $\lambda'_{ij}$ are listed below in Table 3 [36] [37]. In Eq. (5.6), $H$ is a generic exotic leptoquark, and $\phi_a^n(i, j)$ is a string of $n$ standard model singlets necessary to give the appropriate charge to the composite operator. This product of singlets develops expectation value $x_a^n(i, j)$ in the low energy effective theory.
6 Gauge Coupling Unification

In this section, we comment on the effect of low energy leptoquarks and diquarks on gauge coupling unification. It is well known that string theory predicts unification of the gauge couplings at a scale of the order

\[ M_{\text{string}} \approx g_{\text{string}} \times 5 \times 10^{17} \text{ GeV} \, . \]  (6.1)

where \( g_{\text{string}} \approx 0.8 \) at the unification scale. If one assumes that the matter content above the electroweak scale consist only of the MSSM states, then the couplings are seen to intersect at a scale of the order

\[ M_{\text{MSSM}} \approx 2 \times 10^{16} \text{ GeV} \, . \]  (6.2)

Thus, approximately a factor of 20 separates the two scales. A priori one would expect that, in extrapolation of the couplings over fifteen orders of magnitude, this small discrepancy would have many possible resolutions. Surprisingly, however, the problem is not easily resolved. In Ref. [38, 39] a detailed analysis of string scale gauge coupling unification was performed. All possible perturbative corrections to the gauge couplings were taken into account, including string threshold corrections, light SUSY thresholds, enhanced intermediate gauge symmetry, modified weak hypercharge normalizations and intermediate matter thresholds. It was shown that only the existence of intermediate matter thresholds, beyond the MSSM spectrum, can potentially resolve the problem. Existence of additional color triplets in the desert, between the electroweak scale and the Planck scale, has in fact been proposed for some time as a possible solution to the problem of string scale gauge coupling unification [22]. Possible numerical scenarios were presented in Ref. [38], and include the possibility of having a leptoquark with weak hypercharge \(-1/3\) near the experimental limit, provided that additional color and weak thresholds exist at a higher scales. Thus, the existence of light leptoquarks with the appropriate weak hypercharge assignment is desired from the point of view of superstring unification.

The constraint of perturbative gauge coupling unification, in tandem with the precision data on \( \sin^2 \theta_W(M_Z) \) and \( \alpha_s(M_Z) \), places significant restrictions on the
masses and charges of some diquark states, however. For instance, requiring that the $U(1)_Y = 7/6$ diquarks present in some models not destroy unification restricts their masses to lie quite close to the unification scale, not less than $0.6M_X$ with two representations of diquarks, and not less than $0.2M_X$ with one (Note, however, that the stringy exotic diquarks mentioned in Eq. (3.8) above have the much more modest $U(1)_Y$ charges $\frac{1}{6}$ and $\frac{1}{3}$, respectively, and are therefore not so restricted). Conversely, taking $k_2 = k_3 = 1$, we find that diquarks with masses in the TeV range are completely excluded by unification for any value of $k_1$. Neither does allowing higher levels for $k_2$ and $k_3$ alleviate the problem. In fact, for all allowed values of $k_1$, $k_2$, and $k_3$, the presence of $U(1)_Y = 7/6$ diquarks at the TeV scale is completely inconsistent with gauge coupling unification. At any rate, such comments should only be viewed as indicative of the problem, as the enormous contribution of these diquarks to the running of $\alpha_1$ actually causes it to encounter its Landau pole in the desert. With two representations of such diquarks, this occurs near $10^{12}$ GeV, while for one representation, the catastrophe is delayed until $10^{16}$ GeV. Each of these cases is of course inconsistent with unification.

7 Conclusions

Recent HERA data shows deviation from the Standard Model expectations. As statistics for the HERA data are still minimal, it is clearly premature to conclude whether this anomaly is a signal of new physics or not. Nevertheless, a possible explanation for the excess of events is the presence of a new leptoquark state around 200 GeV.

In this paper, we studied the different types of leptoquark states appearing in superstring derived models. String models often give rise to leptoquark and diquark states that are similar to those that exist in GUTs. However, a notable difference is that, in string models, leptoquark states can exist without the need for an enhanced non–Abelian gauge symmetry. Also interesting is the fact that the spectrum of string models is quite constrained. For instance, the existence of a diquark with $Q_Y >$
5/6 will exclude all level one superstring models with $SO(10)$ embedding of the weak hypercharge. More interestingly, however, we have shown that superstring models generically give rise to exotic leptoquark states that lack a standard GUT correspondence. These exotic leptoquarks arise due to the breaking of the non-Abelian gauge symmetries at the string level rather than at the level of the effective four dimensional field theory. Moreover, such exotic stringy leptoquarks possess interesting properties. For example, their couplings to the Standard Model states are generated only after the breaking of the $U(1)_{\text{Z'}}$ gauge symmetry. Furthermore, flavor symmetries that arise from the string models provide sufficient suppression to avoid conflict with experimental data. Finally, the presence of an exotic leptoquark at 200 GeV resolves the string-scale gauge coupling unification problem.

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References


Table 1: A three generation $SU(3) \times SU(2) \times U(1)^2$ model. The choice of generalized GSO coefficients is:

$$c^{(b_1,b_2,\alpha,\beta,\gamma)} = -c^{(b_2,\alpha)} = c^{(1,b_j,\gamma)} = -c^{(1,\bar{b}_1,\bar{b}_2)} = c^{(\gamma,\bar{b}_3)} = -1$$

($j = 1, 2, 3$), with the others specified by modular invariance and space–time supersymmetry. This model contains two leptoquarks pairs, $D_1, \bar{D}_1, D_2, \bar{D}_2$, from the Neveu–Schwarz sector.
Table 2: A three generation $SU(3) \times SU(2) \times U(1)^2$ model. The choice of generalized GSO coefficients is: $c(\beta_j) = c(\alpha_1) = c(\beta_1) = c(\gamma_1) = -c(\gamma_{\alpha,\beta}) = 1$ ($j = 1, 2, 3$), with the others specified by modular invariance and space–time supersymmetry. The sector $\gamma$ has the desired form $\gamma_L = \gamma_R = 4$ and give rise to exotic diquarks.

<table>
<thead>
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<tr>
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<td>0.22</td>
<td>$\nu_\mu$ Deep Inelastic Scattering</td>
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<tr>
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<td>0.22</td>
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</tr>
<tr>
<td>13</td>
<td>0.84</td>
<td>$\tau \to \pi\nu$</td>
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Table 3: Experimental constraints on lepton number violating interactions of exotic leptoquarks in models without baryon number violating operators and without couplings of regular leptoquarks to standard model states.
<table>
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<tr>
<th>$F$</th>
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<td>$\frac{1}{4}$</td>
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Table 4: The exotic massless states from the sectors $b_1 + b_3 + \alpha \pm \gamma + (I)$ and $b_2 + b_3 + \alpha \pm \gamma + (I)$, in the model of Ref. [20].
<table>
<thead>
<tr>
<th>$F$</th>
<th>SEC</th>
<th>$SU(3)_C \times SU(2)_L$</th>
<th>$Q_C$</th>
<th>$Q_L$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
<th>$Q_5$</th>
<th>$Q_6$</th>
<th>$SU(5)_H \times SU(3)_H$</th>
<th>$Q_7$</th>
<th>$Q_8$</th>
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</thead>
<tbody>
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<td>$0$</td>
<td>$0$</td>
<td>$(1, 1)$</td>
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<tr>
<td>$\bar{D}_1$</td>
<td>$\beta \pm \gamma^+$</td>
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<td>$-\frac{1}{2}$</td>
<td>$-\frac{1}{4}$</td>
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<td>$\frac{1}{4}$</td>
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<td>$0$</td>
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<tr>
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<td>$-\frac{1}{2}$</td>
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<tr>
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<tr>
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<td>$(1, 3)$</td>
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</tr>
</tbody>
</table>

Table 5: The exotic massless states from the sectors $b_1 + b_3 + \alpha \pm \gamma + (I)$ and $b_2 + b_3 + \alpha \pm \gamma + (I)$, in the model of Ref. [22].