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Wavelet Analysis of Cosmic Ray Jets

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Wavelet analysis is applied to the high multiplicity cosmic ray jets to search significant peaks in the pseudo-rapidity distributions.

1. Introduction

As the colliding energy per nucleon goes up to several hundred TeV, the number density of produced particles in the pseudo-rapidity space becomes very high, and new states of matter are expected to be formed. In such events, how to measure the strength of fluctuations in the pseudo-rapidity distributions, and whether some signals are hidden or not are very important and interesting problems.

In the wavelet transform, function (data) is expanded by self-similar wavepackets[1]. Therefore characteristics of local fluctuations can be extracted from the data.

We have reported that the wavelet transform is useful to search a spike-like event in a pseudo-rapidity distribution $dn/d\eta$, and that the fluctuations in $dn/d\eta$ can be classified by the wavelet spectra [2].

In this paper, wavelet analysis is also applied to $dn/d\eta$ of high multiplicity (gamma ray) events, the JF1afl[3] and the Texas Lone Star[4], observed in the cosmic ray experiments.

2. Wavelet Transform

Wavelets are constructed from dilation and translation of a scaling function $\phi(x)$. It is constructed by an iteration equation,

$$\phi_i(x) = \sum_{k=0}^{N-1} \sqrt{2} c_k \phi_{i-1}(2x - k) \quad i = 1, 2, \dots, (1)$$

from a primary scaling function,

$$\phi_0(x) = \begin{cases} 1 & \text{for } 0 \leq x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

In (1), N is a even number, and c_k ($k = 0, 1, \dots, N-1$) are constants. The iteration is continued until $\phi_i(x)$ becomes indistinguishable from $\phi_{i-1}(x)$, and $\phi(x)$ is defined by this $\phi_i(x)$.

If N is fixed at 4, the coefficients are given as

$$\begin{aligned} \sqrt{2} c_0 &= \frac{1 + \sqrt{3}}{4}, & \sqrt{2} c_1 &= \frac{3 + \sqrt{3}}{4}, \\ \sqrt{2} c_2 &= \frac{3 - \sqrt{3}}{4}, & \sqrt{2} c_3 &= \frac{1 - \sqrt{3}}{4}. \end{aligned}$$

The mother wavelet $W(x)$ is given by

$$W(x) = \sum_{k=0}^{N-1} (-1)^{k+1} \sqrt{2} c_{N-1-k} \phi(2x - k). \quad (2)$$

The j -th level wavelet ($j = 0, 1, \dots$) is defined from the mother wavelet as

$$\psi_{j,k}(x) = 2^{\frac{j}{2}} W(2^j x - k), \quad k = 0, 1, \dots, 2^j - 1. \quad (3)$$

The wavelets and the scaling function satisfy the following normalization and orthogonal conditions;

$$\begin{aligned} \int_{-\infty}^{\infty} \phi(x) \phi(x) dx &= 1, \\ \int_{-\infty}^{\infty} \phi(x) \psi_{j,k}(x) dx &= 0, \\ \int_{-\infty}^{\infty} \psi_{j,k}(x) \psi_{r,s}(x) dx &= \delta_{j,r} \delta_{k,s}. \end{aligned} \quad (4)$$

The additional conditions are imposed to the wavelet to fix the coefficients c_k 's;

$$\int_{-\infty}^{\infty} x^k W(x) dx = 0, \quad k = 0, 1, \dots, \frac{N}{2} - 1. \quad (5)$$

In the following, function $f(x)$ is assumed to be defined in the region for $0 \leq x < 1$, and to be periodic outside the defined region. Wavelets are redefined as $\sum_l \psi_{j,k}(x+l)$, which are also expressed by $\psi_{j,k}(x)$. In this case, the scaling function coincides with the primary scaling function, and the scaling function and the wavelets also satisfy (4). Then, function $f(x)$ is expanded as

$$f(x) = a_0 \phi(x) + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} \alpha_{j,k} \psi_{j,k}(x).$$

Let $f(x)$ be given by discrete values,

$$f_i = f(\Delta/2 + \Delta \cdot (i-1)) \quad i = 1, 2, \dots, 2^r,$$

where $\Delta \cdot 2^r = 1$. By the use of column matrices,

$$F = \begin{pmatrix} f_1 \\ \vdots \\ f_{2^r} \end{pmatrix}, A_j = \begin{pmatrix} \alpha_{j,0} \\ \vdots \\ \alpha_{j,2^j-1} \end{pmatrix},$$

the wavelet transform is given by

$$\begin{aligned} a_0 &= 2^{-\frac{r}{2}} L_1 L_2 \cdots L_r F, \\ A_j &= 2^{-\frac{j}{2}} H_{j+1} L_{j+2} \cdots L_r F, \\ &\quad j = 0, 1, \dots, r-2, \\ A_{r-1} &= 2^{-\frac{r}{2}} H_r F. \end{aligned} \quad (6)$$

In (6), L_j and H_j are $2^{j-1} \times 2^j$ matrices. The i -th row of L_j is expressed as

$$\begin{pmatrix} \cdots & 0 & l_{2(i-1)+1} & \cdots & l_{2(i-1)+N} & 0 & \cdots \end{pmatrix} \\ = \begin{pmatrix} \cdots & 0 & c_0 & \cdots & c_{N-1} & 0 & \cdots \end{pmatrix}.$$

If $2(i-1) + k > 2^j$, element $l_{2(i-1)+k}$ is added to the $(2(i-1) + k - 2^j)$ -th element in each row.

Matrix H_j is obtained if c_k ($k = 0, 1, \dots, N-1$) in L_j is replaced by $(-1)^{k+1} c_{N-1-k}$.

From the conditions (4) and (5), matrices L_j and H_j satisfy

$$\begin{aligned} L_j \cdot {}^t L_j &= I_j, \quad L_j \cdot {}^t H_j = 0, \\ H_j \cdot {}^t L_j &= 0, \quad H_j \cdot {}^t H_j = I_j, \end{aligned} \quad (7)$$

where I_j is the j -th rank unit matrix.

The inverse wavelet transform is given from (6) and (7) as

$$F = F^\phi + \sum_{j=0}^{r-1} F^j,$$

$$\begin{aligned} {}^t F^\phi &= 2^{\frac{r}{2}} a_0 L_1 L_2 \cdots L_r, \\ {}^t F^j &= 2^{\frac{j}{2}} A_j H_{j+1} L_{j+2} \cdots L_r, \\ &\quad j = 0, 1, \dots, r-2, \\ {}^t F^{r-1} &= 2^{\frac{r}{2}} A_{r-1} H_r. \end{aligned} \quad (8)$$

Then, we have the identity from (7) and (8);

$$\begin{aligned} {}^t F F 2^{-r/2} &= \sum_{j=1}^{2^r} f_j^2 2^{-r/2} = a_0^2 + \sum_{j=0}^{r-1} E_j, \\ E_j &= {}^t A_j A_j = \sum_{k=0}^{2^j-1} \alpha_{j,k}^2, \end{aligned} \quad (9)$$

where E_j is the j -th level wavelet spectrum.

3. Analysis of the Data

In Fig.1, the pseudo-rapidity distribution $dn/d\eta$ of the JF1af1 [3] is shown.

Figure 1. $dn/d\eta$ of the JF1af1, where the average is subtracted.

In order to search significant peaks in the event, we construct the distribution from $dn/d\eta$ of the JF1af1 with the wavelet coefficients $\alpha_{j,k}$ within the region

$$|\alpha_{j,k} - \mu| > \alpha \sigma,$$

where μ is the average and σ is the standard deviation of the wavelet coefficients. If α is chosen

as 1.96, it corresponds to the confidence level of 0.95.

In Fig.2, $dn/d\eta$ constructed from the JF1afl with $\alpha = 1.96$ is shown. In the analysis, the wavelet with $N=4$ (D4 wavelet) is used. Peaks at $\eta \simeq 9.5$ and $\eta \simeq 10.2$ appear. We also analyze $dE_t/d\eta$ of JF1afl event by the D4 wavelet, and obtain the peak only at $\eta \simeq 10.2$. This peak also appears in both of $dn/d\eta$ and $dE_t/d\eta$, when the wavelet with $N=10$ (D10 wavelet) is used.

Figure 2. Pseudo-rapidity distribution constructed from the JF1afl with $\alpha = 1.96$.

In Fig.3, the wavelet spectra obtained from $dn/d\eta$ ($7.7 \leq \eta < 10.9$) by the D4 wavelet are shown in log-log scale. The spectra are rather flat. The number n of data points is $n = 2^5$; namely $r = 5$ in (9). Coefficient a_0 in (9) is equal to the average of the data;

$$a_0 = \bar{f} = \frac{1}{n} \sum_{j=0}^n f_j.$$

Then, calculated results should satisfy the identity,

$$\frac{1}{n} \sum_{j=1}^n (f_j - \bar{f})^2 = \sum_{j=0}^{r-1} E_j.$$

Both $dn/d\eta$ and $dE_t/d\eta$ of the Texas Lone Star are also analyzed. However, significant peaks do not appear in those distributions.

Figure 3. wavelet spectra of the JF1afl.

4. Summary

Pseudo-rapidity distributions $dn/d\eta$ of two high multiplicity gamma ray events are analyzed by the wavelet transform. The sharp peak at $\eta \simeq 10.2$ is extracted from $dn/d\eta$ and $dE_t/d\eta$ of the JF1afl with D4 and also with D10 wavelets. However, in the Texas lone star no significant peaks are extracted.

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