WIGNER MEMORIAL LECTURE

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Wigner Memorial Lecture

Born on November 17, 1902, Eugene Wigner died on January 1, 1995. Since his death, this is the first Wigner medal award ceremony. It is befitting to commemorate this very great physicist now. Unfortunately his lifelong friend, Edward Teller, has not been able to come, but he did send some recollections which will be read later.

An excellent way, may be the best, to commemorate a scientist is to publish a collection of his works. The first volume of the Complete Scientific Works of Eugene Wigner [1] appeared more than three years ago, the eighth and last volume will appear probably next year. These volumes include unpublished papers from the Manhattan project and many scattered philosophical papers on the nature, the meaning and the role of science. Some of the latter had already been gathered by Wigner in “Symmetries and Reflections” [2].

Wigner’s complete works are so vast and so deep that an eight-day colloquium would be necessary for a real memorial. In a half-hour lecture I can only speak about one domain of his activity and that only briefly. At this XXI International Colloquium on Group Theoretical Methods in Physics, I have of course chosen to speak about Wigner's work on symmetry in physics. He wrote about sixty papers on this topic, but I have time only to speak about some of them. I am sure that every one of you has read, and probably studied, his book on “Group theory”[3].

However one cannot commemorate a scientist without speaking about the man himself. Happily Wigner has done this for us in 1988, and the resulting book The recollections of Eugene P. Wigner as told to Andrew Szanton [4] is marvellous. Reading it, Wigner appears before you, speaking to you, telling you about his life (with delicate discretion regarding his private life), his science, his philosophical and political ideas. Yes, his scientific judgements were sharp, but his legendary politeness was genuine. What struck me most was his immense generosity towards colleagues and younger physicists. One of the greatest surprises of my life was to find my name among the four persons to whom “He wishes to record his deep indebtetness” in the preface of the English edition of his Group Theory book. J. von Neumann had already died. Later V. Bargmann received the first Wigner medal. The only other survivor is Arthur Wightman who has worked tremendously hard editing Wigner's collected works and was his colleague at Princeton University for forty years. It is a pity that Arthur could not give this Memorial Lecture.

For me Wigner has been a model in science: a complete physicist, drawing, when necessary, from his deep mathematical culture. Let me read to you what was written for his sixtieth birthday, by some of his Princeton colleagues [5] : “A characteristic feature of Wigner’s way of working is its down-to-earth quality. There are many young men who, their head bulging with information on Hilbert spaces, have come to him with an idea, only to have him try it out first on $2 \times 2$ matrices. (It usually helped.) Another aspect of this down-to-earth quality is his great respect for, and knowledge of, facts. If one comes to discuss a crystal with him, it is a good bet that he will be able to give off hand its density, structure, thermal conductivity, and the slow neutron cross sections of its elements. Moreover the stuff he is thinking about probably has the right color.”

After graduating in chemical engineering in Berlin and the publication of his first three papers, including his thesis, Wigner returned to Budapest, to work in a Mauthner brothers’ tannery (his father had worked all his life in it and became a director). He had
subscribed to the Zeitschrift für Physik, and was studying it at night, when he could. As he told us in a long, published interview [6], he missed the first Heisenberg paper on quantum mechanics [7], but read and understood immediately the Born and Jordan paper “Zur Quantenmechanik” [8]. In 1925, invited as an assistant by Weissenberg (a well known crystallographer who followed Polanyi’s advice) in Berlin he became a physicist. He continued to publish important work in chemistry with M. Polányi [9] but started to use symmetry in quantum mechanical systems: first the general methodology [10], the parity quantum number [11], the atomic spectra (with von Neumann) [12], the diatomic molecules (with Wittmer) [13], to which must be added the work with Jordan [14] on the second quantization of fermion fields, the introduction of the “Vierbein” for gravitating spinors in general relativity [15], two other papers with von Neumann (one showed the possible existence of a state with discrete eigenvalue inside the interval of a continuous spectrum [16], the other studied the changes of atomic level structure in adiabatic transformations [17]), two papers on statistics and the paper [18] establishing the group theoretical study of the vibrations of molecules; indeed the treatment of the symmetry of this classical linear problem is similar to that for bounded quantum states of finite systems. Such an activity in four years! But they were also exceptional years for physics in general. The Jordan and Wigner paper [14] was submitted on January 26, 1928 (exactly 24 days after Dirac’s paper on his equation); they used Clifford algebra for building the representation of the anticommutation rules (they were very near to finding the equation for the spinning electron!) and proved the uniqueness of the representation (this applies also to the Dirac equation).

“There was a great reluctance among physicists toward accepting group theoretical arguments and the group theoretical point of view” wrote Wigner in the preface of the English edition of his book; he added “the recognition that almost all rules of spectroscopy follow from the symmetry of the problem is the most remarkable result.” The book has had a much bigger impact. It is the first place where, for the rotation group, the matrix elements of the operators which decompose the Kronecker (=tensor) product of two irreducible representations into the direct sum of irreducible representations are computed and systematically studied. As Wigner said much later, in a conference to the American Mathematical Society [19], they “were also called Clebsch-Gordan coefficients (though the reason for this is mysterious to me), or vector coupling coefficients, which is the name I prefer”. In the English version of the book, he presented them in the symmetrical and covariant form called 3-j coefficients, see also [20]. He had already extended them [21] to the “simply reducible groups” [22]. That started a whole industry in theoretical physics; in chapter 27 of the English version of his book, Wigner gave an interesting geometrical interpretation of the 3-j and 6-j (=Racah) symbols and their classical limit.

In his book, Wigner emphasized an original point of view: in quantum mechanics, the action of the symmetry group $G$ on the projective space of rays (of vector states) can be embedded in a linear unitary projective representation of $G$ on the Hilbert space of vector states (he gave the proof even for infinite dimension). As you know, for $SO_3$ this corresponds to using the representations of its universal covering group (i.e. $SU_2$); so it leads to the concept of half-integer spin. In 1937 Wigner solved the same problem for the Poincaré group [23]. That paper is one of the scientific landmarks. Using some
not yet published von Neumann's results, it is the first paper giving complete families of unitary infinite dimensional representations of a non semi-simple, non compact Lie group; moreover this paper also deals with the projective representations! In physics it is the foundation of the quantum relativistic kinematics of elementary particles.

Two German editions of Weyl's book [24] had appeared before the first edition of Wigner's book. These two excellent books are still necessary readings for understanding symmetry in quantum physics. Weyl's book has a wider mathematical point of view (but does not study projective representations) and deals with a broader domain of physics. The emphasis is different in the two books and their overlap is relatively small. A very weak point in Weyl's book is time reversal. In 1932 Wigner wrote the fundamental paper on time reversal in quantum physics [25]. For physicists outside the Wigner's school of thought it took a long time to understand and use this symmetry. One had indeed to tame a new tool necessary for this symmetry: the antiunitary operators. Chapter 26 of the English version of Wigner's book calls corepresentations the extension of unitary representations with antiunitary operators and teaches us how useful they are. The following year Wigner published two simple and elegant papers on antiunitary operators [26]; the projective corepresentations of the full Poincaré group were dealt with in [27].

Let us return to 1930: after well-know papers with Weisskopf on linewidth in spectra, Wigner wrote important papers on chemical physics and conceived his quantum mechanical phase space distribution function [28] (one of his great inventions). In 1932, when the neutron was discovered, Wigner immediately studied this particle [29] and the nuclear forces, without ignoring other important problems, e.g. studying with Jordan and von Neumann the role of Jordan algebras in quantum mechanics and the determination of the exceptional Jordan algebras [30]. Since Wigner worked for thirty more years in nuclear physics, it is outside the scope of this lecture to review his related work. I shall mention only three papers. In 1937 the one [31] on supermultiplets; it is a formal $SU_4$ invariance treating spins and isospins on an equal footing. This abstract approximate symmetry was useful for the study of light nuclei. In the sixties it was a surprise to discover that it was also valid for heavier nuclei; and in 1964 the symmetry was extended to $SU_6$ independently by Sakita, Güresy and Radicati, Gell-Mann, followed by many physicists. The title of the second one [32], with Eisenbud, summarizes perfectly the content of this basic paper: "Invariant forms of interaction between nuclear particles". The third paper I choose, [33], is an excellent review of isospin, very rich in physics, one which appeared at the right time, viz. that of the 1958 Feynman, Gell-Mann paper [34].

From 1932 on, Wigner was not only pioneering in nuclear physics, but also in solid state physics! In this domain, his work and those of his three students Seitz, Herring, Bardeen (in chronological order) have had a tremendous impact. The first paper is on metallic sodium [35]; this crystal contains one atom per fundamental cell of the body centered cubic space group $G = Im3m$. The authors consider, for the first time in solid state physics, the Voronoï cell (generally called, because of that paper, Wigner-Seitz cell by physicists); they do note that for any $G$-invariant function and on any face of the cell, the normal component of the gradient vanishes and that the cell can be inscribed in a sphere (i.e. its 24 vertices lie on a sphere). Then they show that it is a good physical approximation to replace the computation of the valence electron wave function in the
crystal by the corresponding boundary problem inside this sphere, for the Schrödinger equation with the Coulomb potential of the ion \( Na^+ \). They solve the problem including the effect of Pauli principle and obtain good values for the size of the cell, the binding energy by atom (yielding the heat of vaporisation) and the compressibility coefficient. This paper is a great classic. In the papers [36],[37] two predictions were made; respectively, the existence of electron crystals (they were first observed in 1981 on the surface of superfluid helium) and the existence of a metallic hydrogen phase at a pressure greater than .25 millions of atmospheres (it may have been discovered this year at Livermore). In his thesis Seitz constructed the unitary irreducible representations of the space groups; for each space group \( G \) there is a continuous infinity of such representations, labelled by the wave vectors \( \vec{k} \) of the Brillouin zone and a finite valued index (labelling the different irreducible, "allowed" representations of the little space group \( G_k \)).

The 1936 paper [38], with Bouckaert and Smoluchowsky, is one of the most often quoted; indeed it is the fundamental paper for the application of group theory to the quantum physics of crystals although he solved only some aspects of the very general problem it described. Let us quote from the introduction:

"Thus far the group theory of the Brillouin zone is not different from the group theory of any other system. But while in atoms, molecules, etc., the characteristic values of (1) [= the eigenvalues of the Schrödinger equation for bound states] are well separated, the characteristic values of (1) for a crystal form a continuous manifold... [for instance the energy \( E \) is a function over the Brillouin zone.].... Thus a certain topology for the representations must exist and it will be shown that part of this topology is independent of the special Brillouin zone.... "

And the last sentence of the introduction is:

"The investigation of the “topology” of representations will be essentially the subject of this paper, from the mathematical point of view.” Indeed, after laying the method for studying the symmetry of energy bands, the authors showed how to obtain the compatibility conditions. The paper announced Herring’s thesis [39] which studied the contacts between band branches imposed by time reversal invariance as well as the possible accidental degeneracies. This grandiose program has been forgotten by many solid state physicists: those of them who use only the finite Born-von Karman groups, with a finite set of points for the Brillouin zone (passing to the continuous limit, when it can be done, is very deceptive because the topology is different). J. Zak and I are presently completing this aspect of Wigner’s program: its predictions are rich and harmonious.

In August 1939, just before the start of the big crisis that we call “second world war”, Szilárd and Wigner suggested to Einstein to write a letter to president Roosevelt about the possibility of making nuclear bombs. After two mathematical papers on groups [40], [21] and a few others in nuclear physics (including [32]), there are between 1942 and 1946 no scientific publications from the outstanding theoretical physicist whom we commemorate. During that time he was an exceptional engineer planning in detail a new kind of industrial plants: the neutron chain reactors. He had to fight with “professional engineers” as it is clear in [1]V (a typical incident is told by Wigner in [6]). The basic physics of that work is given in [3]c. After the first nuclear bomb test at Alamogordo, Szilárd and Wigner started a petition for using the bomb only in an uninhabited place of our planet.
At the end of the war Wigner came back to Princeton. From that time on symmetry in physics was no longer one of his main interests in physics. Thus it is very unfair to him to restrict a memorial lecture to this subject. However, I cannot end the lecture here without quoting some later important and very original works of Wigner in this field. The literature on relativistic wave equations was very extensive but partly unsatisfactory without the 1947 contribution of Bargmann and Wigner [41]. At the same time Wigner [42] considered the equations for particles with infinite spin (corresponding to some unitary representations of the Poincaré group which seem pathological for physics); I wish also to mention a subsequent paper [43] with a similar title, which is a good review of relativistic wave equations and contains some approach to general relativity. The relations between the latter theory and quantum mechanics are also studied in [44] and will be one of the subjects of reflections by the older Wigner. Ref [41] led Newton and Wigner to clarify very much in [45] the problem of localisability of relativistic particles in quantum theory.

In 1952, the “three W’s paper” (Wick, Wightman, Wigner) [46] introduced what we call now the superselection rule, a new concept which shocked many physicists at that time, but that cannot be ignored when we are interested in the foundation of quantum theory. I cannot refrain from giving you an excerpt from the footnote 9 of that paper: That $C$ is an exact symmetry property is moreover still far from proved. The disturbing possibility remains that $C$ and $P$ are both only approximate and $CP$ is the only exact symmetry law. A remarkable prophecy which was realized within four years! However a tiny violation of $CP$ was found in 1964....In a little known work, Wigner proved the conservation with time of superselection rules. It was popularized later by the same trio [47]. I wish also to mention the contribution of Wigner [48] to what has become known as “Bell’s inequalities” and if I had to answer the impossible question (already asked me here) “which Wigner paper is a summary of his teaching on symmetry in physics?”, I would suggest ref. [49]. The domain of Wigner’s activity I reviewed, corresponds mainly to vol. I of the “Collected works”; this volume also contains two analyses by B. Judd and G. Mackey.

It is impossible for me to end this lecture without mentioning the creation by Wigner of a new tool for studying many physical phenomena: Random matrices [50]. A whole industry in theoretical physics has been built on the use of them in different domains.

Wigner has been one of the outstanding physicists of modern times. His influence is tremendous and will last. Many of us here, owe him much scientifically. I suggest to conclude this memorial lecture by a moment of silence that each of us may use for his own commemoration.

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[23] E. Wigner, On unitary representations of the inhomogeneous Lorentz group, Annals Math. 40 (1939) 149–204. Wigner quotes the less complete and less rigourous paper of E. Majorana (N. Cim. 9 (1932) 335) on the representations of the Lie algebra of the Poincaré group. It is remarkable that part of this Wigner’s paper is purely algebraic. I was very interested by the beginning of the “Acknowledgement. The subject of this paper was suggested to me as early as 1928 by P.A.M. Dirac who realized even at that date the connection of representations with quantum mechanical equations. I am greatly indebted to him also for mainly fruitful conversations about this subject, especially during the years 1934/35, the outgrowth of which the present paper is”. I had the occasion to talk at length with each of them in the early seventies, but neither could remember these conversations. In [6] Wigner told that his paper was refused by Amer. J. Math. but in 1980 he received a telephone call from this review congratulating him to have published one of the 25 most often quoted papers in mathematics since the beginning of the century!


