Direct-Transmission Models of Dynamical Supersymmetry Breaking

Izawa K.-I., Y. Nomura, K. Tobe,† and T. Yanagida

Department of Physics, University of Tokyo,
Bunkyo-ku, Hongou, Tokyo 113, Japan

Abstract

We systematically construct gauge-mediated supersymmetry(SUSY)-breaking models with direct transmission of SUSY-breaking effects to the standard-model sector. We obtain a natural model with the gravitino mass $m_{3/2}$ smaller than 1 keV as required from the standard cosmology. If all Yukawa coupling constants are of order one, the SUSY-breaking scale $m_{SUSY}$ transmitted into the standard-model sector is given by $m_{SUSY} \simeq 0.1 \frac{\alpha_{i}}{16\pi} \Lambda$ where $\Lambda$ is the original dynamical SUSY-breaking scale. Imposing $m_{SUSY} \simeq (10^{2} - 10^{3})$ GeV, we get $\Lambda \simeq (10^{5} - 10^{6})$ GeV, which yields the gravitino mass $m_{3/2} \simeq (10^{-2} - 1)$ keV.

†Fellow of the Japan Society for the Promotion of Science.
1 Introduction

Low-energy dynamical supersymmetry (SUSY) breaking with gauge mediation is extremely attractive, since it may not only solve various phenomenological problems but also its dynamical nature may provide a natural explanation of the large hierarchy between the electroweak and some higher (say the Planck) scales [1]. Several mechanisms [2, 3, 4, 5] for dynamical SUSY breaking have been discovered and their applications to realistic models have been also proposed [6, 7, 8].

Structures of the proposed models [6, 7, 8] predict a relatively large SUSY-breaking scale $\Lambda > 10^6$ GeV to provide sufficiently large soft masses in the SUSY standard-model sector. On the other hand, the unclosure condition of our universe yields a constraint on the gravitino mass as $m_{3/2} \lesssim 1$ keV [9], which corresponds to the SUSY-breaking scale $\Lambda \lesssim 10^6$ GeV. This is not achieved in the referred models. In fact, a detailed analysis [10] on the models in Ref. [6] has shown that the gravitino is likely to be heavier than 1 keV, which necessitates a late-time entropy production [10, 11] to dilute the gravitino energy density in the universe.

In this paper, we systematically construct gauge-mediated models of low-energy SUSY breaking with the structure of direct transmission (that is, without messenger gauge interactions). We obtain models in which the gravitino mass can be set smaller than 1 keV. The existence of such models suggests that low-energy dynamical SUSY breaking with gauge mediation does not necessarily require complicated non-standard cosmology.

2 Dynamical scale generation

We first discuss a dynamics for scale generation since it is crucial for the dynamical SUSY breaking in our models. We adopt a SUSY SU(2) gauge theory with four doublet chiral superfields $Q_i$, where $i$ is a flavor index ($i = 1, \cdots, 4$). Without a superpotential, this theory has a flavor SU(4)$_F$ symmetry. This SU(4)$_F$ symmetry is explicitly broken down to a global SP(4)$_F$ by a superpotential in our models. We add gauge singlets $Y^a$ ($a = 1, \cdots, 5$) which constitute a five-dimensional representation of SP(4)$_F$ to obtain a
tree-level superpotential

\[ W_Y = \lambda_Y Y^a(QQ)_a, \]  

(1)

where \((QQ)_a\) denote a five-dimensional representation of \(\text{SP}(4)_F\) given by a suitable combination of gauge invariants \(Q_iQ_j\).

An effective superpotential [12] which describes the dynamics of the \(\text{SU}(2)\) gauge interaction may be given by

\[ W_{\text{eff}} = S(V^2 + V_a^2 - \Lambda^4) + \lambda_Y Y^a V_a \]  

(2)

in terms of low-energy degrees of freedom

\[ V \sim (QQ), \quad V_a \sim (QQ)_a, \]  

(3)

where \(S\) is an additional chiral superfield, \(\Lambda\) is a dynamically generated scale, and a gauge invariant \((QQ)\) denotes a singlet of \(\text{SP}(4)_F\) defined by

\[ (QQ) = \frac{1}{2}(Q_1Q_2 + Q_3Q_4). \]  

(4)

The effective superpotential Eq.(2) implies that the singlet \(V \sim (QQ)\) condenses as

\[ \langle V \rangle = \Lambda^2, \]  

(5)

and SUSY is kept unbroken in this unique vacuum. Since the vacuum preserves the flavor \(\text{SP}(4)_F\) symmetry, we have no massless Nambu-Goldstone boson. The absence of flat direction at this stage is crucial for causing dynamical SUSY breaking as seen in the next section.

## 3 Dynamical SUSY breaking

Let us further introduce a singlet chiral superfield \(Z\) to consider a superpotential for dynamical SUSY breaking [4]:

\[ W_0 = W_Y + \lambda Z(QQ). \]  

(6)
For a relatively large value of the coupling $\lambda_Y$, we again obtain the condensation Eq.(5) with the low-energy effective superpotential approximated by

$$W_{\text{eff}} \simeq \lambda \Lambda^2 Z.$$  

(7)

On the other hand, the effective Kähler potential is expected to take a form

$$K = |Z|^2 - \frac{\eta}{4\Lambda^2}|\lambda Z|^4 + \cdots,$$  

(8)

where $\eta$ is a real constant of order one.

The effective potential for the scalar $Z$ (with the same notation as the superfield) is given by

$$V_{\text{eff}} \simeq |\lambda|^2 \Lambda^4 (1 + \frac{\eta}{\Lambda^2}|\lambda|^4|Z|^2).$$  

(9)

If $\eta > 0$, this implies $\langle Z \rangle = 0$. Otherwise we expect $|\lambda \langle Z \rangle| \sim \Lambda$, since the effective potential is lifted in the large $|Z| (\sim \Lambda)$ region [4, 7, 13]. Anyway, the $F$-component of $Z$ superfield has nonvanishing vacuum-expectation value, $\langle F_Z \rangle \simeq \lambda \Lambda^2$, and thus SUSY is dynamically broken in this model.

In the following analyses, we assume the latter case $|\lambda \langle Z \rangle| \sim \Lambda$, which results in the breakdown of $R$ symmetry.1

## 4 One-singlet model

Let us first consider a realistic model with one singlet $Z$ for SUSY breaking which couples directly to $(QQ)$. It is referred as a ‘multiplier’ singlet, hereafter. We introduce four pairs of massive chiral superfields $d, \bar{d}, l, \bar{l}, d', \bar{d'}, l', \bar{l'}$ which are all singlets under the strong SU(2). We assume that the $d, d'$ and $\bar{d}, \bar{d'}$ transform as the down quark and its antiparticle, respectively, under the standard-model gauge group. The $l, l'$ and $\bar{l}, \bar{l'}$ are assumed to transform as the lepton doublet and its antiparticle, respectively. These fields are referred as messenger quarks and leptons.

1The spontaneous breakdown of the $R$ symmetry produces a Nambu-Goldstone $R$-axion. This $R$-axion is, however, cosmologically harmless, since it acquires a mass from the $R$-breaking constant term in the superpotential which is necessary to set the cosmological constant to zero[14]. Modifications for the case $\langle Z \rangle = 0$ is touched upon in the final section.
The superpotential of the one-singlet model is given by

$$W_1 = W_Y + Z(\lambda QQ + k_d \bar{d}d + k_l \bar{l}l) + m_\phi \bar{d}d + m_\phi \bar{l}l + m_\phi \bar{l}l,$$  \hspace{1cm} (10)

where m’s denote mass parameters.\(^2\) For relatively small values of the couplings \(k_d\) and \(k_l\), we have a SUSY-breaking vacuum with the vacuum-expectation values of the messenger quarks and leptons vanishing. Then the soft SUSY-breaking masses of the messenger quarks and leptons are directly generated by \(\langle F_Z \rangle = \lambda \Lambda^2 \neq 0\) through the couplings \(Z(k_d \bar{d}d + k_l \bar{l}l)\).

The above SUSY-breaking vacuum is the true vacuum as long as the mass parameters \(m_\psi\) are much larger than \(\sqrt{k_\psi F_Z} \neq \sqrt{k_\psi \lambda \Lambda}\) for \(\psi = d, l\). To find the stability condition of our vacuum, we examine the scalar potential

$$V = |\lambda \Lambda^2 + k_d \bar{d}d + k_l \bar{l}l|^2 + |m_d|^2 + |m_l|^2 + |m_\bar{d}d|^2 + |m_\bar{l}l|^2$$

$$+ |k_d Z \bar{d}d + m_\bar{d}d|^2 + |k_d Z \bar{l}l + m_\bar{l}l|^2 + |k_l Z l + m_l l|^2.$$ \hspace{1cm} (11)

The vacuum \(\langle F_Z \rangle \simeq \lambda \Lambda^2, \langle d \rangle = \langle \bar{d} \rangle = \langle l \rangle = \langle \bar{l} \rangle = \langle d' \rangle = \langle \bar{d}' \rangle = \langle l' \rangle = \langle \bar{l}' \rangle = 0\) \hspace{1cm} (12)

is stable when

$$|m_d m_\bar{d}|^2 > |k_d \langle F_Z \rangle|^2,$$

$$|m_l m_\bar{l}|^2 > |k_l \langle F_Z \rangle|^2.$$ \hspace{1cm} (13)

In the following analysis, we restrict ourselves to the parameter region Eq.(13).

The standard-model gauginos acquire their masses through loops of the messenger quarks and leptons when \(\langle Z \rangle \neq 0\) (see Figs.1-2 and the Appendix). The gaugino masses are obtained as

$$m_{\tilde{g}_1} = \frac{\alpha_1}{4\pi} \left\{ \frac{2}{3} \frac{k_d \langle F_Z \rangle^2}{m_d m_\bar{d}} \left[ \frac{k_d \langle F_Z \rangle}{\sqrt{m_d m_\bar{d}}} \right] F_d \right\}$$

\(^2\) Dynamical generation of these mass terms will be discussed in the following sections. Mass terms for SUSY-breaking transmission were considered in Ref.[7, 15]. In the course of writing this paper, we received a paper [16] which also treated similar mass terms in SUSY-breaking models.
If the messenger quarks and leptons have a degenerate SUSY-invariant mass \( \bar{C} \), where leptons satisfy the GUT relation at the GUT scale. Does not hold even when all the couplings and mass parameters for messenger quarks and vanishes. Hence the GUT relation among gaugino masses, \( g_\alpha \equiv \frac{5}{3} \alpha_Y \), and \( \bar{g}_3, \bar{g}_2, \) and \( \bar{g}_1 \) are gauginos of the standard-model gauge groups SU(3)_C, SU(2)_L, and U(1)_Y, respectively. The \( \mathcal{F}_\psi \) for \( \psi = d, l \) are defined in the Appendix. Here, we have assumed \( (k_\psi \langle F_Z \rangle / m_\psi m_\psi)^2 \ll 1 \). Notice that the leading term of \( (k_\psi \langle F_Z \rangle / m_\psi m_\psi) \) in Fig.1 vanishes. Hence the GUT relation among gaugino masses, \( m_{\bar{g}_3}/\alpha_1 = m_{\bar{g}_2}/\alpha_2 = m_{\bar{g}_3}/\alpha_3 \), does not hold even when all the couplings and mass parameters for messenger quarks and leptons satisfy the GUT relation at the GUT scale.

The soft SUSY-breaking masses for squarks and sleptons \( \tilde{f} \) in the standard-model sector are generated by two-loop diagrams shown in Fig.3. We obtain them as

\[
m_\tilde{f}^2 = 2 \left[ C_3^{\tilde{f}} \left( \frac{\alpha_3}{4\pi} \right)^2 \Lambda_{(d)}^2 + C_2^{\tilde{f}} \left( \frac{\alpha_2}{4\pi} \right)^2 \Lambda_{(l)}^2 + \frac{3}{5} Y^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \left( \frac{2}{5} \Lambda_{(d)}^2 + \frac{3}{5} \Lambda_{(l)}^2 \right) \right],
\]

where \( C_3^{\tilde{f}} = \frac{4}{3} \) and \( C_2^{\tilde{f}} = \frac{3}{4} \) when \( \tilde{f} \) is in the fundamental representation of SU(3)_C and SU(2)_L, and \( C_1^{\tilde{f}} = 0 \) for the gauge singlets, and \( Y \) denotes the U(1)_Y hypercharge \( (Y \equiv Q - T_3) \). Here the effective scales \( \Lambda^{(\psi)} \) are of order \( k_\psi \langle F_Z \rangle / m_\psi \). For example, the effective scales \( \Lambda^{(\psi)} \) are given by

\[
\Lambda^{(\psi)^2} = \frac{|k_\psi \langle F_Z \rangle|^2}{m_\psi^2},
\]

if the messenger quarks and leptons have a degenerate SUSY-invariant mass \( \bar{m}_\psi \), which is an eigenvalue of the mass matrix

\[
\begin{pmatrix}
  k_\psi \langle Z \rangle & m_\psi \\
  m_\psi & m_\psi
\end{pmatrix}.
\]

The SUSY-breaking squark and slepton masses are proportional to \( (k_\psi \langle F_Z \rangle / m_\psi m_\psi)^2 \). On the other hand, the gaugino masses have an extra suppression \( (k_\psi \langle F_Z \rangle / m_\psi m_\psi)^2 \) as

\[^3\text{In the present analysis, we only discuss the sfermion masses qualitatively. A more detailed analysis will be given in Ref.[17].}\]
shown in Eqs.(14)-(16) since the leading term of \((k_\psi\langle F_Z \rangle/m_\psi m_\bar{\psi})\) vanishes. Thus, to avoid too low masses for the gauginos, we must take \((k_\psi\langle F_Z \rangle/m_\psi m_\bar{\psi})^2 > 0.1\). It is interesting that this condition is necessary to have a light gravitino with mass less than 1 keV as shown below.

We are now at a point to derive a constraint on the gravitino mass. The conservative constraint comes from the experimental lower bounds\(^4\) on the masses of wino and gluino\(^{18, 19}\)\(^5\)

\[
m_{\tilde{g}_1} \gtrsim 50 \text{ GeV}, \quad m_{\tilde{g}_3} \gtrsim 220 \text{ GeV},
\]

which yield

\[
\left| \frac{k_i \langle F_Z \rangle}{m_im_i} \right|^2 \frac{k_i \langle F_Z \rangle}{\sqrt{m_im_i}} |F_i| \gtrsim 1.9 \times 10^4 \text{ GeV},
\]

\[
\left| \frac{k_d \langle F_Z \rangle}{m_dm_d} \right|^2 \frac{k_d \langle F_Z \rangle}{\sqrt{m_dm_d}} |F_d| \gtrsim 2.3 \times 10^4 \text{ GeV}.
\]

We obtain

\[
\langle F_Z \rangle \gtrsim \frac{3 \times 10^8}{k_i F_i^2} \left( \frac{m_im_i}{k_i \langle F_Z \rangle} \right)^5 \text{ GeV}^2,
\]

\[
\langle F_Z \rangle \gtrsim \frac{5 \times 10^8}{k_d F_d^2} \left( \frac{m_dm_d}{k_d \langle F_Z \rangle} \right)^5 \text{ GeV}^2.
\]

The gravitino mass is given by

\[
m_{3/2} = \frac{\langle F_Z \rangle}{\sqrt{3}M} \sim \frac{0.8}{k_i} \left( \frac{0.1}{F_i} \right)^2 \left( \frac{m_im_i}{k_i \langle F_Z \rangle} \right)^5 \times 10^{-2} \text{ keV}.
\]

\[
m_{3/2} = \frac{\langle F_Z \rangle}{\sqrt{3}M} \sim \frac{1}{k_d} \left( \frac{0.1}{F_d} \right)^2 \left( \frac{m_dm_d}{k_d \langle F_Z \rangle} \right)^5 \times 10^{-2} \text{ keV}.
\]

Since the \(|F_\psi|\) has the maximal value 0.1 (see the Appendix), we see that in the region of

\[
0.2 \lesssim \left( \frac{k_\psi \langle F_Z \rangle}{m_\psi m_\bar{\psi}} \right)^2 \lesssim 1 \quad \text{and} \quad k_\psi \simeq 1 \quad \text{for} \quad \psi = d, l,
\]

the gravitino can be lighter than 1 keV, which is required from the standard cosmology.

\(^4\)These bounds are derived assuming the GUT relation of the gaugino masses. The bound on the gluino mass assumes that the gluino is heavier than all squarks. A more detailed phenomenological analysis on the models in this paper will be given in Ref.[17].

\(^5\)We find in Ref.[17] that even when \((k_\langle F_Z \rangle/m^2)^2 \simeq 1\), the constraint from the right-handed slepton mass is weaker than those from the gaugino masses.
We have found that the gravitino mass can be set smaller than 1 keV if \( m_\psi \) are of order the SUSY-breaking scale \( \Lambda \). In principle, the masses \( m_\psi \) of the messenger quarks and leptons might be considered to arise from dynamics of another strong interaction. In that case, however, it seems accidental to have \( m_\psi \sim \Lambda \). Thus it is natural to consider a model in which the SUSY-breaking dynamics produces simultaneously the mass terms for the messenger quarks and leptons. This possibility will be discussed in section 6.

We note that there is no CP violation in this model. All the coupling constants \( k_d, k_l \) and the mass parameters \( m_\psi (\psi = d, l, \bar{d}, \bar{l}) \) can be taken real without loss of generality. The vacuum-expectation values \( \langle QQ \rangle \) and \( \langle Z \rangle \) are also taken real by phase rotations of the corresponding superfields. Thus only the \( \langle F_Z \rangle \) is a complex quantity and then all the gaugino masses have a common phase coming from the phase of \( \langle F_Z \rangle \). However, this phase can be eliminated by a common rotation of the gauginos.\(^6\)

5 Two-singlet model

Next we consider a realistic model with two ‘multiplier’ singlets \( Z_1 \) and \( Z_2 \) for SUSY breaking. We introduce two pairs of chiral superfields \( d, \bar{d} \) and \( l, \bar{l} \) which are all singlets under the strong SU(2).

We also introduce an additional singlet \( X \) to obtain a superpotential \(^7\)

\[
W_2 = W_Y + Z_1(\lambda_1(QQ) - f_1X^2) + Z_2(\lambda_2(QQ) - f_2X^2) + X(f_dd\bar{d} + f_ll\bar{l}).
\]

(27)

Without loss of generality, we may set \( f_2 = 0 \) by an appropriate redefinition of \( Z_1 \) and \( Z_2 \). Then the superpotential yields a vacuum with \( \langle X \rangle = \sqrt{f_1^{-1}\lambda_1\Lambda} \). The masses of messenger quarks and leptons are given by

\[
m_\psi = f_\psi \langle X \rangle
\]

(28)

for \( \psi = d, l \). Since \( F_{Z_2} = \lambda_2\Lambda^2 \) is nonvanishing, SUSY is broken.

\(^6\)The rotation of the gauginos induces a complex phase in the Yukawa-type gauge couplings of the gauginos. However, such a complex phase is eliminated by a rotation of the sfermions and Higgs fields \( H \) and \( \bar{H} \), since we have no SUSY-breaking trilinear couplings and no SUSY-breaking \( B \) term \( B\mu H\bar{H} \) at the tree-level.

\(^7\)We could construct a model without the additional singlet superfield [7] at the sacrifice of complete naturalness. It may manage to accommodate a light gravitino with \( m_{3/2} \sim 1 \text{ keV} \) in a strong-coupling regime.

7
The soft masses of the messenger quarks and leptons stem from radiative corrections. For example, the diagrams shown in Fig.4 generate an effective Kähler potential of the form

$$\delta \left( \frac{16 \pi^2}{2} \right)^2 |\lambda_1|^2 |\lambda_2|^2 f_1 f_1^* Z_1^* Z_2 (\lambda_1^* Z_1^* + \lambda_2^* Z_2^*) X^* X \frac{\Lambda^6}{f_1^2 f_2 d \bar{d} + |f_1|^2 f_1 \bar{d} \bar{l}},$$

which gives soft mass terms of the form

$$\delta \left( \frac{16 \pi^2}{2} \right)^2 |\lambda_1|^2 |\lambda_2|^2 f_1 f_1^* Z_1^* Z_2 (\lambda_1^* Z_1^* + \lambda_2^* Z_2^*) X^* X \frac{\Lambda^6}{f_1^2 f_2 d \bar{d} + |f_1|^2 f_1 \bar{d} \bar{l}},$$

when $\langle Z_2 \rangle \neq 0$.

Since the induced soft masses for messenger squarks and sleptons are suppressed by loop factors, the gravitino mass is expected to be much larger than 1 keV in this model.

### 6 Three-singlet model

We finally obtain a realistic model with three ‘multiplier’ singlets $Z_1$, $Z_2$, and $Z_3$ for SUSY breaking. The model is a combination of the one- and the two-singlet models discussed in the previous sections. The masses $m_\psi$ of messenger quarks and leptons in the one-singlet model are generated by Yukawa couplings of $X$ introduced in the two-singlet model.

The superpotential in this three-singlet model is given by

$$W_3 = W_Y + Z_1 (\lambda_1 (Q\bar{Q}) + k_{d1} d \bar{d} + k_{l1} l \bar{l} - f_1 X^2) + Z_2 (\lambda_2 (Q\bar{Q}) + k_{d2} d \bar{d} + k_{l2} l \bar{l} - f_2 X^2) + Z_3 (\lambda_3 (Q\bar{Q}) + k_{d3} d \bar{d} + k_{l3} l \bar{l} - f_3 X^2) + X (f_1 d \bar{d} + f_2 d' \bar{d} + f_3 l \bar{l} + f_4 l \bar{l}).$$

(31)

Without loss of generality, we may set $k_{d1} = k_{l1} = f_2 = 0$ by an appropriate redefinition of $Z_1$, $Z_2$, and $Z_3$. For relatively small values of the couplings $k_{d2}$, $k_{l2}$, $\lambda_3$, $k_{d3}$, $k_{l3}$, and $f_3$, the superpotential yields a vacuum with $\langle X \rangle = \sqrt{f_1^{-1} \lambda_1 \Lambda}$ and the vacuum expectation values of the messenger quarks and leptons vanishing. The masses $m_\psi$ of messenger quarks and leptons in the one-singlet model are given by

$$m_\psi = f_\psi \langle X \rangle$$

(32)

for $\psi = d, l, \bar{d}, \bar{l}$. In this vacuum, the $F$-components of $Z_i$ are given by

$$F_{Z_1} \simeq 0, \quad F_{Z_2} \simeq \lambda_2 \Lambda^2, \quad F_{Z_3} \simeq \lambda_3 \Lambda^2 - f_3 \langle X \rangle^2,$$

(33)
and thus SUSY is broken. The masses of gauginos, squarks, and sleptons are generated as in the one-singlet model in section 4. We should replace \( k_\psi \langle F_Z \rangle \) in Eqs.(14)-(16) by \( k_\psi^2 \langle F_{Z2} \rangle + k_\psi^3 \langle F_{Z3} \rangle \).

If \( k_{d1}/k_{d2} \neq k_{l1}/k_{l2} \), the phases of the three gauginos’ masses are different from one another. Then, the phases of the gauginos’ masses cannot be eliminated by a common rotation of the gaugino fields and thus CP is broken. However, there is no such problem in the GUT models since \( k_{d1}/k_{d2} \simeq k_{l1}/k_{l2} \) holds even at low energies.

We comment on the \( \mu \)-problem[6, 20]. If the superfield \( X \) couples to \( HH \) where \( H \) and \( \bar{H} \) are Higgs fields in the standard model, the SUSY-invariant mass \( \mu \) for Higgs \( H \) and \( \bar{H} \) is generated. To have the desired mass \( \mu \simeq (10^2 - 10^3) \) GeV, we must choose a small coupling constant \( \lambda_h \simeq 10^{-3} \), where \( \lambda_h \) is defined by \( W = \lambda_h XHH \). This is natural in the sense of ’t Hooft. We note that no large \( B \) term \( (B\mu HH) \) is induced since the \( F \)-component of \( X \) is very small. Hence the scale \( \mu \) may originate from the SUSY-breaking scale in the present model.\(^{8}\)

Finally, we should stress that the superpotential Eq.(31) is natural, since it has a global symmetry \( U(1)_R \times U(1)_\chi \), where \( U(1)_R \) is an \( R \) symmetry. That is, the superpotential Eq.(31) is a general one allowed by the global \( U(1)_R \times U(1)_\chi \).\(^{9}\) The charges for chiral superfields are given in Table 1.

### 7 Conclusion

We have constructed gauge-mediated SUSY-breaking models with direct transmission of SUSY-breaking effects to the standard-model sector. In our three-singlet model, the gravitino mass \( m_{3/2} \) is expected to be smaller than 1 keV naturally as required from the standard cosmology: If all the Yukawa coupling constants are of order one, the SUSY-breaking scale \( m_{SUSY} \) transmitted into the standard-model sector is given by \( m_{SUSY} \simeq 0.1 \frac{\alpha_i}{4\pi} \Lambda \). Imposing \( m_{SUSY} \simeq (10^2 - 10^3) \) GeV, we get \( \Lambda \simeq (10^5 - 10^6) \) GeV, which yields the gravitino mass \( m_{3/2} \simeq (10^{-2} - 1) \) keV.

---

\(^{8}\)There has been also proposed an interesting solution to the \( \mu \)-problem in Ref.[21].

\(^{9}\)This global symmetry may forbid mixings between the messenger quarks and the down-type quarks in the standard-model sector. This avoids naturally the flavor-changing neutral current problem[22]. Then there exists the lightest stable particle in the messenger sector[23].
In the present models, we have four gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)$. It is well known that the three gauge coupling constants of the SUSY standard-model gauge groups meet at the GUT scale $\sim 10^{16}$ GeV. It is remarkable that in the three-singlet model, all the four gauge coupling constants meet at the scale $\sim 10^{16}$ GeV as shown in Fig.5. Here, we have assumed that the gauge coupling constant $\tilde{\alpha}_2$ of the strong $SU(2)$ becomes strong ($\tilde{\alpha}_2 / \pi \simeq 1$) at the scale $\Lambda \simeq (10^5 - 10^6)$ GeV.

So far we have assumed spontaneous breakdown of $R$ symmetry in the models. If $\langle Z \rangle = 0$, we need to introduce $R$-breaking mass terms such as $m d \bar{d} + m' l \bar{l}$ to generate the standard-model gaugino masses. These mass terms might be induced through the $R$ symmetry breaking which is necessary for the cosmological constant to be vanishing [14].
Appendix

In this Appendix, we evaluate the standard-model gaugino masses in our SUSY-breaking models. The superpotential which relates to the mass terms of messenger fields $\psi, \bar{\psi}, \psi', \bar{\psi}'$ for $\psi = d, l$ is represented as

$$W = \sum_{\psi=d,l} (\bar{\psi}, \bar{\psi}') M^{(\psi)} \begin{pmatrix} \psi \\ \psi' \end{pmatrix},$$  \hspace{1cm} (34)

where the mass matrix $M^{(\psi)}$ is given by

$$M^{(\psi)} = \begin{pmatrix} m_1^{(\psi)} & m_3^{(\psi)} \\ m_2^{(\psi)} & 0 \end{pmatrix},$$  \hspace{1cm} (35)

In the one-singlet model, the mass parameters $m_i^{(\psi)}$ are given by

$$m_1^{(\psi)} = k_\psi \langle Z \rangle,$$  \hspace{1cm} (36)

$$m_2^{(\psi)} = m_\psi,$$  \hspace{1cm} (37)

$$m_3^{(\psi)} = m_{\bar{\psi}},$$  \hspace{1cm} (38)

and in the three-singlet model, they are given by

$$m_1^{(\psi)} = k_{\psi_2} \langle Z_2 \rangle + k_{\psi_3} \langle Z_3 \rangle,$$  \hspace{1cm} (39)

$$m_2^{(\psi)} = f_\psi \langle X \rangle,$$  \hspace{1cm} (40)

$$m_3^{(\psi)} = f_{\bar{\psi}} \langle X \rangle.$$  \hspace{1cm} (41)

The soft SUSY-breaking mass terms of the messenger fields are given by

$$\mathcal{L}_{soft} = \sum_{\psi=d,l} F^{(\psi)} \bar{\psi} \bar{\psi},$$  \hspace{1cm} (42)

where

$$F^{(\psi)} = k_\psi \langle F_Z \rangle$$  \hspace{1cm} (43)

in the one-singlet model and

$$F^{(\psi)} = k_{\psi_2} \langle F_{Z_2} \rangle + k_{\psi_3} \langle F_{Z_3} \rangle$$  \hspace{1cm} (44)
in the three-singlet model. Then the standard-model gauginos acquire their masses through loops of the messenger quarks and leptons. Their masses of order \( F^{(\psi)}/m^{(\psi)} \) are given by (see Fig.1)

\[
m_{\tilde{g}_3} = \frac{\alpha_3}{4\pi} F^{(d)} \left( M^{(d)^{-1}} \right)_{11},
\]

\[
m_{\tilde{g}_2} = \frac{\alpha_2}{4\pi} F^{(l)} \left( M^{(l)^{-1}} \right)_{11},
\]

\[
m_{\tilde{g}_1} = \frac{\alpha_1}{4\pi} \left\{ \frac{2}{5} F^{(d)} \left( M^{(d)^{-1}} \right)_{11} + \frac{3}{5} F^{(l)} \left( M^{(l)^{-1}} \right)_{11} \right\},
\]

where the masses \( m_{\tilde{g}_i} (i = 1, \ldots, 3) \) denote bino, wino, and gluino masses, respectively, and we have adopted the SU(5) GUT normalization of the U(1)\(_Y\) gauge coupling \( \alpha_1 \equiv \frac{5}{3} \alpha_Y \).

Because of \( (M^{(\psi)^{-1}})_{11} = 0 \), the above contributions vanish. However, the contributions of higher powers of \( F^{(\psi)}/m^{(\psi)2} \) do not vanish in general: We now work in a basis where the supersymmetric masses \( M^{(\psi)} \) are diagonalized as

\[
O_{\psi} M^{(\psi)} O^{\dagger}_{\psi} = \begin{pmatrix} m_{\psi_1} & 0 \\ 0 & m_{\psi_2} \end{pmatrix}.
\]

Here the mass eigenstates are given by

\[
\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = O_{\psi} \begin{pmatrix} \psi \\ \psi' \end{pmatrix} = \begin{pmatrix} \cos \theta_{\psi} & -\sin \theta_{\psi} \\ \sin \theta_{\psi} & \cos \theta_{\psi} \end{pmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix},
\]

\[
\begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} = O_{\phi_{\psi}} \begin{pmatrix} \bar{\psi} \\ \bar{\psi}' \end{pmatrix} = \begin{pmatrix} \cos \phi_{\psi} & -\sin \phi_{\psi} \\ \sin \phi_{\psi} & \cos \phi_{\psi} \end{pmatrix} \begin{pmatrix} \bar{\psi} \\ \bar{\psi}' \end{pmatrix},
\]

where we have taken the mass matrices \( M^{(\psi)} \) to be real, which is always possible. Then, for example, the contribution of order \( (F^{(\psi)}/m^{(\psi)})(F^{(\psi)}/m^{(\psi)2})^2 \) to the gaugino masses, which is shown in Fig.2, is represented by

\[
m_{\tilde{g}_3} = \frac{\alpha_3}{4\pi} \left| \frac{F^{(d)}}{m_2^{(d)} m_3^{(d)}} \right|^2 \frac{F^{(d)}}{\sqrt{m_2^{(d)} m_3^{(d)}}} \mathcal{F}_d,
\]

\[
m_{\tilde{g}_2} = \frac{\alpha_2}{4\pi} \left| \frac{F^{(l)}}{m_2^{(l)} m_3^{(l)}} \right|^2 \frac{F^{(l)}}{\sqrt{m_2^{(l)} m_3^{(l)}}} \mathcal{F}_l,
\]

\[
m_{\tilde{g}_1} = \frac{\alpha_1}{4\pi} \left\{ \frac{2}{5} \left| \frac{F^{(d)}}{m_2^{(d)} m_3^{(d)}} \right|^2 \frac{F^{(d)}}{\sqrt{m_2^{(d)} m_3^{(d)}}} \mathcal{F}_d + \frac{3}{5} \left| \frac{F^{(l)}}{m_2^{(l)} m_3^{(l)}} \right|^2 \frac{F^{(l)}}{\sqrt{m_2^{(l)} m_3^{(l)}}} \mathcal{F}_l \right\}.
\]

Here, the \( \mathcal{F}_\psi \) for \( \psi = d, l \) are defined by

\[
\mathcal{F}_\psi \equiv \mathcal{F}(\tan^2 \theta_{\psi}, \tan^2 \phi_{\psi}),
\]

\[\]
where

\[
\mathcal{F}(a, b) = \frac{(ab)^{\frac{1}{4}}}{6(1 - ab)^4(1 + a)^{\frac{3}{2}}(1 + b)^{\frac{3}{2}}} \left\{ 2(a + b)(-1 + 8ab - 8a^3b^3 + a^4b^4 + 12a^2b^2 \ln(ab)) \\
-1 - ab - 64a^2b^2 + 64a^3b^3 + a^4b^4 + a^5b^5 - 36a^2b^2(1 + ab) \ln(ab) \right\}.
\]  

(55)

This function \( \mathcal{F}(a, b) \) has the maximal value 0.1 at \( a \simeq 3 \) and \( b \simeq 3 \). Eqs.(51)-(53) imply that the so-called GUT relation of the gaugino masses does not hold in general.
References


   1.


Table 1: $U(1)_R \times U(1)_\chi$ charges for chiral superfields. Here, $\psi = d, l, \bar{d}, \bar{l}$ and $i = 1, 2, 3.$

<table>
<thead>
<tr>
<th></th>
<th>$Q, \psi, X$</th>
<th>$Z_i, \psi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(1)_R$</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$U(1)_\chi$</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>
Figure 1: Diagram contributing to the gaugino masses where the single soft SUSY-breaking mass $F^{(\psi)}$ is inserted. This contribution vanishes as shown in the Appendix.

Figure 2: Diagram contributing to the gaugino masses where the three $F^{(\psi)}$'s are inserted.

Figure 3: Typical two-loop diagram contributing to the sfermion masses.

Figure 4: Typical diagram generating the effective Kähler potential which contributes to the soft SUSY-breaking masses of the messenger squarks and sleptons.

Figure 5: Renormalization group flow of the coupling constants of $SU(3)_C$, $SU(2)_L$, $U(1)_Y$, and the strong $SU(2)$ gauge groups. Here, the mass of messenger squarks and sleptons is taken to be $(10^5 - 10^6)$ GeV and we assume that the gauge coupling constant $\tilde{\alpha}_2$ of the strong $SU(2)$ becomes strong ($\tilde{\alpha}_2/\pi \simeq 1$) at the scale $\Lambda = (10^5 - 10^6)$ GeV.