Towards a Precise Parton Luminosity Determination at the CERN LHC

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Abstract

A new approach to determine the LHC luminosity is investigated. Instead of employing the proton–proton luminosity measurement, we suggest to measure directly the parton–parton luminosity. It is shown that the electron and muon pseudorapidity distributions, originating from the decay of $W^{+}$, $W^{-}$ and $Z^{0}$ bosons produced at 14 TeV pp collisions (LHC), constrain the $x$ distributions of sea and valence quarks and antiquarks in the range from $\approx 3 \times 10^{-4}$ to $\approx 10^{-1}$ at a $Q^2$ of about $10^4$ GeV$^2$. Furthermore, it is demonstrated that, once the quark and antiquark structure functions are constrained from the $W^{\pm}$ and $Z^{0}$ production dynamics, other $q\bar{q}$ related scattering processes at the LHC like $q\bar{q} \rightarrow W^{+}W^{-}$ can be predicted accurately. Thus, the lepton pseudorapidity distributions provide the key to a precise parton luminosity monitor at the LHC, with accuracies of $\approx \pm 1\%$ compared to the so far considered goal of $\pm 5\%$.

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1 Introduction

Interpretations of essentially all proposed measurements at the LHC, CERN’s 14 TeV proton–proton collider project, require a good knowledge of the parton distribution functions at the relevant $Q^2$ and the collected integrated luminosity. Both omnipurpose experiments, ATLAS [1] and CMS [2], consider a luminosity accuracy of ±5% as their goal [3].

The traditional methods to determine the proton–proton luminosity are size and intensity measurements of the beams at the interaction point, as well as event rates of processes with previously measured or calculable cross sections like elastic proton–proton scattering [4] and QED processes like $pp \rightarrow ppe^+e^-$ [5]. Unfortunately, clean measurements of the above processes, especially at the high luminosity phase of LHC, are very difficult. Consequently it is not obvious that the proton–proton luminosity can be measured with a ±5% accuracy.

Assuming that the proton–proton luminosity is measured, different experimentally observed cross sections are compared with theoretical calculations, using parton distribution functions, $f(x,Q^2)$, where $x$ is the fractional parton momentum ($x = p_{\text{parton}}/E_{\text{beam}}$) of the relevant types of valence and sea quarks (or antiquarks) and gluons at the considered $Q^2$ of the reaction. These parton distributions are determined from experimental observables in lepton–hadron scattering (DIS processes from fixed target and HERA experiments) and Drell–Yan lepton pair production processes at hadron colliders [6]. These results, obtained at different $Q^2$, have then to be extrapolated to the relevant $Q^2$ scale of the studied process. While the $x$ distributions of the valence quarks are now quite well constrained, uncertainties for the $x$ distributions of sea quarks and antiquarks and gluons remain important. As a result of these structure function uncertainties, total cross section predictions of $W^+$, $W^−$ and $Z^0$ boson production at 14 TeV pp collisions (LHC), vary currently between 10–20%. Even though the experimental errors are expected to decrease considerable during the next years, cross section uncertainties related to structure functions will remain important. These uncertainties, combined with the unknown contributions from higher order QCD corrections, are usually considered to limit the use of the reaction $pp \rightarrow W^\pm(Z^0)$ as an absolute proton–proton luminosity monitor at very high center of mass energies [7].

The above problems result in luminosity uncertainties, which are larger than the considered goal of a ±5% proton–proton luminosity accuracy [3]. Consequently, current estimates for the achievable accuracies of some measurements at the LHC appear to be somewhat depressing. This is especially the case when these uncertainties are compared with the possible small statistical errors for many LHC measurements, the current knowledge of quark and lepton couplings to the $W^\pm$ and $Z^0$ boson or the already achieved accuracies in high energy $e^+e^−$ experiments.

As a solution for the above problem, we propose a new approach to measure the LHC luminosity. This approach is based on:

1. Experiments at the LHC will study the interactions between fundamental constituents of the proton, the quarks and gluons at energies where these partons can be considered
as quasi free. Thus, the important quantity is the parton–parton luminosity at different values of $x_{\text{parton}}$ [8] and not the traditionally considered proton–proton luminosity.

2. Assuming collisions of essentially free partons, the production of weak bosons, $u\bar{d} \rightarrow W^+ \rightarrow \ell^+ \nu$, $d\bar{u} \rightarrow W^- \rightarrow \ell^- \bar{\nu}$ and $u\bar{u}(d\bar{d}) \rightarrow Z^0 \rightarrow \ell^+ \ell^-$ are in lowest order understood to at least a percent level. Cross section uncertainties from higher order QCD corrections are certainly larger, but are obviously included in the measured weak boson event rates. Similar higher order QCD corrections to other $q\bar{q}$ scattering processes at different $Q^2$, like $q\bar{q} \rightarrow W^+W^-$, can be expected. Thus, assuming that the $Q^2$ dependence can in principle be calculated, very accurate theoretical predictions for cross section ratios like $\sigma(pp \rightarrow W^+W^+)/\sigma(pp \rightarrow W^\pm)$ should be possible.

3. It is a well known fact that the $W^\pm$ and $Z^0$ production rates at the LHC, including their leptonic branching ratios into electrons and muons, are huge and provide relatively clean and well measurable events with isolated leptons. With the well known $W^\pm$ and $Z^0$ masses, possible $x$ values of quarks and antiquarks are constrained from $M^2_{W^\pm,Z^0} = sx_qx_{\bar{q}}$ with $s = 4E^2_{\text{beam}}$. The product $x_qx_{\bar{q}}$ at the LHC ($\sqrt{s} = 14$ TeV) is thus fixed to $\approx 3 \times 10^{-5}$. Thus, the rapidity distributions of the weak bosons are directly related to the fractional momenta $x$ of the quarks and antiquarks. Consequently, the observable pseudorapidity distributions of the charged leptons from the decays of $W^\pm$ and $Z^0$ bosons are also related to the $x$ distributions of quarks and antiquarks. The shape and rate of the lepton pseudorapidity distributions provide therefore the key to precisely constrain the quark and antiquark structure functions and their corresponding luminosities.

The aim of this paper is to demonstrate the feasibility of this approach and thus improve the luminosity measurement at the LHC for quark–antiquark related scattering processes. It will be shown that the dynamics of the single weak boson production at the LHC allow to constrain the $q, \bar{q}$ structure functions, the corresponding parton luminosities and therefore also the cross sections of other $q\bar{q}$ related processes. Finally, we suggest that a similar approach to gluon related scattering processes might eventually also lead to similar accuracies for the $x$ distribution of gluons.

2 Event rates and the selection of $pp \rightarrow W^+, W^- \text{ and } Z^0$

The production of $pp \rightarrow W^+, W^- \text{ and } Z^0$ and their identification using the leptonic decays have been discussed extensively in the literature [9]. In particular, these reactions provide clean sources of isolated high $p_t$ electrons or muons, and due to their high rate, are often considered as a clean and excellent calibration tool at the LHC [10]. However, previous studies concluded that their use as a luminosity monitor is limited to relative luminosity measurements only [7]. The reason for these pessimistic conclusions is based on the predicted cross section variations using different sets of structure functions [11]. The size of these cross section variations is as large as 10–20% as can be seen from table 1. The cross section predictions as well as the following simulation results are obtained using the PYTHIA Monte Carlo program [12].
### Weak boson cross sections (LHC 14 TeV)

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$\sigma \times BR$</th>
<th>MRS(A)</th>
<th>CTEQ 2L</th>
<th>GRV 94 HO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ud \to W^+ \to \ell^+\nu$</td>
<td>20.18 nb</td>
<td>17.32 nb</td>
<td>21.58 nb</td>
<td></td>
</tr>
<tr>
<td>$d\bar{u} \to W^- \to \ell^-\bar{\nu}$</td>
<td>14.24 nb</td>
<td>12.63 nb</td>
<td>15.40 nb</td>
<td></td>
</tr>
<tr>
<td>$u\bar{u}(dd) \to Z^0 \to \ell^+\ell^+$</td>
<td>3.246 nb</td>
<td>2.854 nb</td>
<td>3.456 nb</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q} \to (Z^<em>,\gamma^</em>) \to \ell^+\ell^+ (M_{\ell}\approx 150 - 200 \text{ GeV})$</td>
<td>9.71 pb</td>
<td>8.98 pb</td>
<td>10.26 pb</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q} \to W^+W^- \to \ell^+\nu\ell^-\bar{\nu}$</td>
<td>3.53 pb</td>
<td>3.30 pb</td>
<td>3.63 pb</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Examples of estimated weak boson production cross sections at the LHC for three different sets of structure functions using PDFLIB and PYTHIA programs [11, 12]. In all cases the leptonic branching ratios into electrons and muons are included.

These cross section variations for single $W^\pm$, $Z^0$ production are strongly correlated with the cross section predictions for other $q\bar{q}$ related processes. As an example, the corresponding cross sections for the reaction $q\bar{q} \to W^+W^-$ are also given in table 1. Thus, even without looking at further details, the uncertainties for multi boson production cross sections at the LHC are reduced to about 5-10% if event rates are estimated relative to the production rates of single $W^\pm$, $Z^0$ events. Furthermore such relative measurements reduce also errors from branching ratios and detection efficiency uncertainties.

For the following studies, the MRS(A) structure function set [11] is used as an example and reference system. Figure 1a shows the expected rapidity distribution of $W^+$ and $W^-$, which directly reflect the difference between the $x$ distributions of the $u$, $d$ valence quarks and the sea quark or antiquarks. For small $W^\pm$ rapidities, corresponding to $x_{1,2}$ values of $\approx 6 \times 10^{-3}$, most $W^\pm$ originate from the annihilation of sea quark–antiquarks and only small differences between $W^+$ and $W^-$ are expected. For larger rapidities the $W^\pm$ originate from the annihilation of quarks and antiquarks with very different $x$ values. For example, to produce a $W^+$ at a rapidity of about 2.5, one finds the corresponding $x_{1,2}$ values of the quark and antiquark to be $x_1 \approx 0.1$ and $x_2 \approx 3 \times 10^{-4}$. As the proton is made of two valence $u$ quarks and one valence $d$ quark the $W^+$ production is much more likely than the $W^-$ production at large rapidities.

Figure 1b shows the pseudorapidity distributions of the charged leptons originating from the $W^\pm$ decays. Because of the V–A interaction, the differences between the pseudorapidity distributions of $\ell^+$ and $\ell^-$ especially at large $\eta$ values are larger than the ones for the $W^+$ and $W^-$. The reason is that the left handed lepton ($\ell^-$) is emitted preferentially in the direction of the incoming quark and the right handed antilepton ($\ell^+$) is emitted opposite to the quark direction. Thus the observable charged lepton pseudorapidities reflect not only the $x$ distributions of quarks and antiquarks but allow also to some extent a distinction between valence and sea quarks at a given $x$.

As discussed in the introduction, we want to demonstrate that the dynamics and event rates of weak boson production at the LHC accurately constrain the quark and antiquark
structure functions and their corresponding luminosity. For this purpose simple event selection criteria are used. These criteria closely follow the design characteristics of the proposed CMS experiment [2]. In detail the following lepton selection criteria are used:

- Electrons and Muons are required to have $p_t > 30$ GeV within a pseudorapidity of $|\eta| < 2.4$.

- In order to select only isolated leptons, the transverse energy deposit from other particles with $p_t > 0.5$ GeV and $|\eta| < 3$, found within a cone R around the lepton ($R \equiv \sqrt{\Delta \phi^2 + \Delta \eta^2} < 0.6$), should be smaller than 5 GeV.

\footnote{Despite the interest in the very forward region, lepton detection up to much larger $|\eta|$ values appears to be very difficult.}
To reduce possible backgrounds from heavy quark decays and to reject high $p_t$ $W^\pm$ and $Z^0$ production due to initial state radiation, events with reconstructed jets with $E_t > 20$ GeV are removed. The jet momentum vector is reconstructed in a cone $R < 0.6$ including all stable particles, with $p_t > 0.5$ GeV and $|\eta| < 3$.

Using these charged lepton selection criteria, $pp \rightarrow W^\pm \rightarrow \ell^\pm \nu$ events are required to have exactly one isolated charged lepton with $30 < p_t < 50$ GeV. The resulting $p_t$ spectra of $\ell^\pm$ and their pseudorapidity distributions are shown in figure 2a and 2b respectively. The used kinematic and geometric event selection criteria, result in an event detection efficiency of about 25% for $W^+ \rightarrow \ell^+ \nu$, and about 28% for $W^- \rightarrow \ell^- \bar{\nu}$.

![LHC q\bar{q} \rightarrow W^\pm \rightarrow \ell^\pm \nu](image1)

![LHC q\bar{q} \rightarrow W^\pm \rightarrow \ell^\pm \nu](image2)

Figure 2: The observable (a) charged lepton $p_t$ and (b) pseudorapidity $\eta$ distributions originating from the reaction $q\bar{q} \rightarrow W^\pm \rightarrow \ell^\pm \nu$ at the LHC ($\sqrt{s} = 14$ TeV and the MRS(A) structure function [11]) including all selection criteria discussed in the text.

To select events of the type $pp \rightarrow Z^0 \rightarrow \ell^+ \ell^-$ the presence of a pair of isolated leptons with opposite charge ($e^+ e^-$ or $\mu^+ \mu^-$), with a $m_{\ell^+ \ell^-} = m_{Z^0} \pm 2$ GeV. In addition, the opening angle between the two leptons in the plane transverse to the beam has to be larger than 135° and
$p_t(Z^0) < 20$ GeV. As these dilepton events are usually considered to be background free, $Z^0$
event with large $p_t$ can be used to constrain the $x$ distribution of gluons, as will be discussed later.

Events of the type $q\bar{q} \rightarrow (Z^*, \gamma^*) \rightarrow \ell^+\ell^-$ with dilepton masses above 100 GeV have a much lower rate. However, these events can be used to study the $Q^2$ evolution, up to masses where the neutral current sector is well understood (e.g. up to masses of about 200 GeV). At least up to these dilepton masses, a measurement of the lepton forward backward charge asymmetry, following the method of reference [13], constrains the ratio of valence and sea $u$ and $d$ quarks at different $x$ values.

Using the above kinematical and geometrical event selection criteria, the efficiency to detect both leptons from $Z^0$ decays is about 16%, and increases to about 23% for dilepton masses in the range between 150–200 GeV.

3 Sensitivity to the $q$ and $\bar{q}$ structure functions

We now study the effects of different structure function parametrisations on the measured $\ell^\pm$ pseudorapidity distributions ($q\bar{q} \rightarrow W^\pm \rightarrow \ell^+\ell^-$), and on the reconstructed $Z^0$ rapidity distribution ($q\bar{q} \rightarrow Z^0 \rightarrow \ell^+\ell^-$).

At the LHC, in contrast to proton–antiproton colliders, the antiquarks have to come from the sea. Thus, the pseudorapidity distribution of the positive charged leptons, $u\bar{d} \rightarrow W^+ \rightarrow \ell^+\nu$ contains the information about the sea $\bar{d}$ quarks and the valence or sea $u$ quarks. The negative charged leptons, $d\bar{u} \rightarrow W^- \rightarrow \ell^-\bar{\nu}$ carry consequently the information about the sea $\bar{u}$ quarks and the valence or sea $d$ quarks. The rapidity distribution of charged lepton pairs, from $Z^0, (Z^*, \gamma^*) \rightarrow \ell^+\ell^-$, provide the information about the sum of sea $\bar{u}$ and $\bar{d}$ quarks and the corresponding valence and sea quarks. Consequently, the combination of the different observable lepton pseudorapidity distributions should provide some sensitivity to the $u, d, \bar{u}$ and $\bar{d}$ parton densities over a large $x$ range.

This sensitivity is first investigated by comparing the weak boson production using two quite similar structure function sets, MRS(A) and MRS(H) [11]. The main difference between these two sets lies in the $x$ parametrisation for the light sea quarks. While the older MRS(H) set uses $u, d$ flavor symmetric sea distributions, the MRS(A) set includes a fine tuning of the sea quark parton densities with some isospin symmetry breaking, required to describe the observation of Drell-Yan asymmetries of $A_{DY} = (\sigma_{pp} - \sigma_{pn})/(\sigma_{pp} - \sigma_{pn})$ from the NA 51 experiment [14].

Figure 3a shows the ratio of $\sigma(W^+)/\sigma(W^-)$ as a function of the charged lepton pseudorapidity for the two structure function sets. The different parametrisations thus lead, depending on the lepton pseudorapidity, to a cross section variation of up to about 10%. The double ratio $MRS(H)[\sigma(W^+)/\sigma(W^-)]/ MRS(A)[\sigma(W^+)/\sigma(W^-)]$ is shown in figure 3b. The differences of about 5-10% between the two sets should be compared with the statistical precision, which
is smaller than 1% per bin for an integrated luminosity of only 100 pb$^{-1}$. Furthermore, both sets of structure functions predict almost identical $Z^0$ cross sections. The ratio between the $Z$ cross sections from the two sets, MRS(H)$[\sigma(Z^0)]/\text{MRS(A)}[\sigma(Z^0)]$, is also shown in figure 3b. Combining the obtainable information from $W^+$, $W^-$ and $Z^0$ production, the “fine tuned” isospin splitting of $u$ and $d$ sea quarks between MRS(A) and MRS(H) should be detectable with good accuracy.

![Diagram](image_url)

Figure 3: a) The detected charged lepton cross section ratio, $\sigma(\ell^+\nu)/\sigma(\ell^-\bar{\nu})$, originating from the reaction $q\bar{q} \rightarrow W^\pm \rightarrow \ell^\pm\nu$ as a function of the lepton pseudorapidity for the MRS(H) and MRS(A) structure function parametrisation. b) The relative changes of the charged lepton ratios of 3a) between the MRS(H) and MRS(A) parametrisations and also the cross section ratio of the $Z^0$ production using both parametrisations.

We have shown that the weak boson rapidity distributions are sensitive to small differences between the $x$ distribution of $u$ and $d$ sea quarks and antiquarks. We now go one step further and study how well $q$ and $\bar{q}$ structure function can be constrained from the observable weak boson rapidities. For this purpose the different $\ell^\pm$ cross sections are studied relative to a reference structure function, arbitrarily chosen to be the MRS(A) set.
The fraction of weak bosons which are produced from the annihilation of valence quarks and low $x$ antiquarks increases strongly with increasing rapidity. The valence quark $x$ distribution is already quite well constrained. The main difference between the various structure functions comes from the sea $q$ and $\bar{q}$ parametrisations especially at low $x$. Thus precise measurements of the charged lepton pseudorapidity distributions from $W^\pm$ decays and the rapidity distribution of the $Z^0$ events constrain the low $x$ domain of $\bar{u}$ and $\bar{d}$.

The sensitivity of the measurable lepton rapidity distribution to the different sets of structure functions is shown in figure 4 and figure 5. Figure 4 shows the observable ratio of the $\ell^+$ to $\ell^-$ event rates for three different sets of structure functions and an integrated luminosity of $100 \text{ pb}^{-1}$. The difference between the various low $x$ sea quark parametrisations are thus reflected in the observable lepton pseudorapidities. Consequently, the shape of the $\ell^\pm$ pseudorapidity distributions provide a strong constraint on the underlying $x$ distribution of quarks and antiquarks with $x$ between $\approx 3 \times 10^{-4}$ and $\approx 10^{-1}$.

![Figure 4](image-url)  

Figure 4: The ratio of the accepted cross sections $\sigma(u\bar{d} \rightarrow W^+ \rightarrow \ell^+\nu)$ and $\sigma(d\bar{d} \rightarrow W^- \rightarrow \ell^-\bar{\nu})$ as a function of the lepton pseudorapidity for four different structure functions [11].

Figures 5a-c show the ratio of the predicted $\ell^\pm$ cross sections from different structure functions relative to the reference MRS(A) set. The statistical fluctuations shown in figures 5a and 5b correspond to the errors from roughly one day of data taking at the initial LHC luminosity of $10^{33} \text{ sec}^{-1} \text{ cm}^{-2}$. The expected $Z^0$ event rates are roughly a factor of 10 smaller and the errors shown in figure 5c correspond to about 10 days of data taking.
Figure 5: Rapidity dependence of the $\ell^\pm$ cross section predictions from different sets of structure functions relative to the one obtained from the MRS(A) parametrisation; a) for $\ell^+$, b) for $\ell^-$ and c) for the reconstructed $Z^0$. 
Having demonstrated that the $\ell^\pm$ pseudorapidity distributions, originating from weak boson decays, are very sensitive to details of the quark and antiquark $x$ distributions one can now relate the rate of $\ell^\pm$ events in a selected pseudorapidity interval to the quark and antiquark luminosity at the given $x$. Obviously, once the shape of the pseudorapidity distribution is accurately known, the $\ell^\pm$ event rates need to be measured only for a small pseudorapidity interval. For example, counting of $\ell^\pm$ events from the process $pp \rightarrow W^\pm \rightarrow \ell^\pm + + \nu$ could be restricted to the pseudorapidity range of $|\eta| < 0.5$. Including all selection criteria one would observe roughly 150 000 “clean” luminosity events, corresponding to a statistical error of 0.3%, per day at the initial LHC luminosity ($\approx 100$ pb$^{-1}$/day).

Once the quark and antiquark luminosity at $Q^2 \approx 10^4$ GeV$^2$ and in the $x$ range between $\approx 5 \times 10^{-4}$ and $10^{-1}$ are determined, accurate cross section predictions of other $q\bar{q}$ related processes should be possible. This is studied for the reaction $q\bar{q} \rightarrow W^+W^-$. The correlation between cross section predictions for single and pair production of weak bosons has been pointed out already in section 2 (see table 1). For example, the total cross section predictions for the process $pp \rightarrow W^\pm$ between the CTEQ 2L and the MRS(A) parametrisations differ by about 15%. However, as we suggest to use the process $\sigma(q\bar{q} \rightarrow W^\pm)$ as a reference process, one has to relate the cross section of for example $\sigma(q\bar{q} \rightarrow W^+W^-)$ to the reference reaction $\sigma(q\bar{q} \rightarrow W^\pm)$. Comparing now the prediction for the relative cross sections between CTEQ 2L and the MRS(A) one finds that the difference is reduced to $\approx 7.5%$.

As a next step, the parametrisations of the $q, \bar{q}$ structure functions, especially at low $x$, should be adjusted such that the observed $\ell^\pm$ pseudorapidity distributions are described. As the final experimental accuracy for the lepton pseudorapidity distributions will be limited by systematics, the limitations of the structure function “fine tuning” are difficult to estimate. It is nevertheless worth pointing out that neither the $\ell^\pm$ momentum and charge determination nor differences between $\ell^+$ and $\ell^-$ detection are expected to be problematic. Furthermore, backgrounds from different sources and efficiency uncertainties can be controlled by the simultaneous analysis of the $W^\pm$ and $Z^0$ events with isolated electrons and muons. We therefore do not expect any principle problem of measuring the shape and the rate of the charged lepton pseudorapidity distribution with a $\pm 1%$ accuracy. Thus even small differences for the sea quark parametrisation, like those between MRS(A) and MRS(H), as shown in figure 3a and b, should be detectable. One could thus use the difference in cross section for the two sets as a pessimistic limitation of the proposed method. Differences between relative cross section predictions for different $q\bar{q}$ scattering processes and the two parametrisations indicate therefore the size of the remaining uncertainties. For example the cross section ratios $\sigma(q\bar{q} \rightarrow W^+W^-)/\sigma(q\bar{q} \rightarrow W^\pm)$ are $4.74 \times 10^{-4}$ for MRS(A) and $4.76 \times 10^{-4}$ for MRS(H). Other $q\bar{q}$ scattering processes like $\sigma(q\bar{q} \rightarrow W^\pm Z^0)/\sigma(q\bar{q} \rightarrow W^\pm)$ show similar stability with predicted ratios of $1.78 \times 10^{-4}$ for MRS(A) and $1.79 \times 10^{-4}$ for MRS(H).

Following the above procedure, i.e. constraining the $q, \bar{q}$ structure functions and the corresponding parton luminosities, the event rate of weak boson pair production appears to be predictable with an accuracy of at least $\pm 1%$. 

4 Outlook and conclusions

A new approach to the LHC luminosity measurement demonstrates that the $x$ distributions of valence and sea quarks and their corresponding parton luminosities can be constrained very accurately, using the $\ell^\pm$ pseudorapidity distributions from the decay of weak bosons. It is also shown that this method leads to very accurate rate predictions of other $q\bar{q}$ scattering processes. For example, the strong correlation between the weak boson pair production and the single boson production leads to an estimated experimental luminosity accuracy at the $\pm 1\%$ level. This should be compared to the often considered optimistic goal of $\pm 5\%$ accuracy.

We have not investigated the achievable theoretical accuracies, but believe that many theoretical uncertainties, like the $\alpha_s(Q^2)$ uncertainties or still unknown higher order QCD corrections, contribute in very similar ways to the single and pair production of weak bosons. Furthermore, the experimental possibility to measure the $x$ distributions of sea and valence quarks and the corresponding luminosities to within $\pm 1\%$ should encourage our theoretical colleagues to match this experimental accuracy.

Finally, we argue that the gluon $x$ distribution and the corresponding gluon luminosity can also be constrained in a similar way from accurate measurements of the rapidity distribution of gluon dominated scattering processes. In fact, as the $q, \bar{q}$ luminosity can accurately be measured from the weak boson rapidity distribution, the rapidity distribution of gluon dominated scattering processes has only to be measured relative to the weak boson rapidity distributions. Once the gluon $x$ distribution is known relative to the $x$ distribution of quarks, the weak boson rate also provides the luminosity monitor for gluon related signal and background processes.

The possible experimental accuracy thus depends mainly on how accurate the rapidity and $Q^2$ distributions of gluon related scattering processes can be measured. A very clean signature with a well measurable $Q^2$ is much more important than a huge cross section.

Gluon related scattering processes are $gg \rightarrow X$ and $gq(\bar{q}) \rightarrow X$. As these processes involve jets, measurement problems should be minimized by using processes with small backgrounds and well measurable $p_t$. Candidates for such processes are high $p_t$ events with one or more jets and an isolated $\gamma$ or a $Z^0(\rightarrow \ell^+\ell^-)$. As the energy and momentum of isolated photons and leptons can be measured very accurately, the $p_t$ of the jets, assuming transverse momentum conservation, can also be determined. Thus, the observables are well measured and should provide accurate $Q^2$ measurements.

The production of events with isolated high $p_t$ photons or $Z^0$ are dominated by $gq \rightarrow \gamma(Z^0)q$, and $q\bar{q} \rightarrow \gamma(Z^0)g$. As shown in table 2, the expected cross sections for these reactions, including the branching ratios $Z^0 \rightarrow \ell^+\ell^-$, and relatively high $p_t$ of $\gamma$ and $Z^0$ are still quite large. Furthermore, the calculable background corrections from the process $q\bar{q}$ are expected to be small as the cross sections are dominated by the $gg$ scattering process.

Previous studies of $\gamma$-jet final states have shown that jet events with isolated $\pi^0$'s provide a considerable background [15]. This large background will therefore limit the achievable accuracy of such a final state. However, the leptonic $Z^0$ decays provide an excellent signature
Table 2: PYTHIA cross section estimates for high $p_t$ final states of the type $Z_0^0$($\rightarrow \ell^+\ell^-$)q(g) and $\gamma q(g)$.

<table>
<thead>
<tr>
<th>reaction</th>
<th>$\sigma$ [pb]</th>
<th>$\sigma$ [pb]</th>
<th>$\sigma$ [pb]</th>
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</thead>
<tbody>
<tr>
<td>$q\bar{q} \rightarrow Z_0^0$($\rightarrow \ell^+\ell^-$)g</td>
<td>36.4</td>
<td>6.01</td>
<td>0.71</td>
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<td>$gg \rightarrow Z_0^0$($\rightarrow \ell^+\ell^-$)g</td>
<td>150</td>
<td>34.8</td>
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<td>$qq \rightarrow \gamma g$</td>
<td>717</td>
<td>74.5</td>
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<tr>
<td>$gg \rightarrow \gamma q$</td>
<td>6590</td>
<td>615</td>
<td>49.3</td>
</tr>
</tbody>
</table>

and should allow the selection of essentially background free $Z_0^0$–jet events. We are not aware of any detailed LHC study which demonstrates that this process can indeed be measured with accuracies of a few %, but see no obvious reason why the rapidity distribution of the clean $Z_0^0$–jet events can not be measured with an accuracy close to ±1%.

Unfortunately, the accurate $Q^2$ determination due to inherent uncertainties of jet energy measurements especially at large rapidities will probably limit the interpretation of the observable rapidity distribution with respect to the gluon $x$ distribution. Nevertheless, such direct measurements of the gluon structure function will provide the highest possible accuracy for the $x$ distribution of gluons and might eventually lead to cross section predictions with % accuracies for other gluon related scattering processes.

To summarise, we have shown that the rapidity distributions of $W^\pm$ and $Z_0^0$ events at the LHC provide directly and accurately the $x$ distributions of quarks and antiquarks. Their rates are thus a measure of the corresponding parton luminosities. We have shown that such an approach might eventually lead to perhaps 1% accurate cross section predictions of $q\bar{q}$ related scattering processes, like $q\bar{q} \rightarrow W^+W^-$ at the LHC. We suggest that the detailed measurement of the rapidity distribution of the process $qq \rightarrow Zq$ might provide similar accuracies for the gluon structure function and the corresponding gluon luminosity.

**Acknowledgements**

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References


[11] For the simulations we have used the PYTHIA 5.7 frame [12] and the structure functions MRS(A), MRS(H), CTEQ2L and the GRV 94 HO as implemented within the PDFLIB. For details and further references see H. Plothow–Besch, Comput. Phys. Commun. 75, 396 (1993); PDFLIB (version 6.06) W5051 CERN Computer library.


