Comparison of Lattice and Dual QCD Results for Heavy Quark Potentials

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Abstract
Lattice results and Dual QCD results for all heavy quark potentials through order (quark mass)^{-2} are exhibited and compared. The agreement on the whole is quite good, confirming the validity of both of these approaches.
Bali, et al., [1] have recently calculated from lattice theory all of the heavy quark potentials — the central potential, all spin dependent potentials, and all velocity dependent potentials, through order velocity squared, or, equivalently, through order (quark mass$^{-2}$).

We have previously computed all of these same potentials from the Dual Superconducting model of QCD, (i.e.) Dual QCD [2, 3]. Our purpose in this note is to compare the results of these two methods.

The definitions of the potentials by Bali, et al., [1] are the same as in Dual QCD, except for those proportional to velocity squared. (Bali, et al., include in their calculation some numbers called $c_2, c_3, c_4$ etc. which represent ratios of the running coupling $\alpha_s$ at various energies. We have set all these ratios equal to one because in dual QCD the coupling constant, in the classical approximation used to derive the potentials, does not run.) The comparison of the potentials is given in Table 1.

<table>
<thead>
<tr>
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<th>Bali, et al.</th>
<th>Dual QCD</th>
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<tbody>
<tr>
<td>$V_0$</td>
<td>$\frac{1}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( \nabla^2 V_0 + \nabla^2 V_a^E - \nabla^2 V_a^B \right)$</td>
<td>$\frac{1}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( \nabla^2 V_0 + \nabla^2 V_a \right)$</td>
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<td>$V_1'$</td>
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<td>$V_2'$</td>
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<tr>
<td>$V_3$</td>
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<tr>
<td>$V_4$</td>
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<tr>
<td>$V_b$</td>
<td>$\frac{i}{3} (-V_+ + V_- + \frac{1}{2}V_{\parallel} - \frac{1}{2}V_L)$</td>
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<tr>
<td>$V_c$</td>
<td>$\frac{i}{2} (-V_+ + V_- - V_{\parallel} + V_L)$</td>
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<tr>
<td>$V_d$</td>
<td>$\frac{i}{6} (V_+ + V_- + \frac{1}{2}V_{\parallel} + \frac{1}{2}V_L)$</td>
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<tr>
<td>$V_e$</td>
<td>$\frac{i}{3} (V_+ + V_- - V_{\parallel} - V_L)$</td>
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In dual QCD, the potential $V_a$ can also be broken up into an electric and a magnetic part:

$$V_a = V_a^E - V_a^B$$

where [3]

$$\nabla^2 V_a^E = -\nabla^2 V_0^{NP}$$

and

$$\nabla^2 V_a^B = -\frac{4}{3} e^2 \nabla \cdot \nabla' G^{NP}(\vec{x}, \vec{x}') \bigg|_{\vec{x} = \vec{x}' = \vec{z}_j}.$$
The dual QCD result for $V_a^B$ is weakly singular and requires a cutoff [4]. The lattice calculation of course cannot show this. The spin-spin potential $V_4$ has a delta function term and the term $\nabla^2(V_0 + V_a^E)$ in dual QCD is simply proportional to a delta function at the origin [3], though these naturally do not show up cleanly in the lattice calculation. All of the remaining potentials are finite and well behaved in both approaches.

The comparison of the two sets of results are shown in Figures 1 through 10. Fig. 1 shows the lattice and the dual QCD calculations of the central potential $V_0(R)$. The units are GeV and Fermis. The dual QCD parameters given in Reference (2) have been changed to produce a best fit to the lattice $V_0$ for $\beta = 6.2$. The new parameters are $\alpha_s = 0.2048$ and the string tension $\sigma = 0.2384 GeV^2$. These changes significantly worsen the fits for the $c\bar{c}$ and $b\bar{b}$ spectra given in Reference (2). The resulting effective $\chi^2$ is 11.4, about 6 times that of our earlier fit. The average error increases from 13 MeV to 29 MeV. While our method of calculation differs considerably from that used by Bali, et. al. the quality of our fit described here is comparable to theirs.

The next figure shows the comparison of the quantity $\nabla^2 V_a^E$. The agreement, evidently, is not bad. We recall, however, as mentioned before, that in dual QCD $\nabla^2(V_0 + V_a^E)$ is simply a delta function. This result does not hold on the lattice, so some discrepancy in $\nabla^2 V_a^E$, especially at small $R$, is not surprising.

There is no figure for $\nabla^2 V_a^B$, because, also as mentioned above, in dual QCD this quantity is weakly divergent and is not very sensitive to the required cutoff. A detailed analysis and comparison of $\nabla^2 V_a$ in dual QCD and on the lattice is given in Reference (4).

The remaining Figures (3 through 10) show the Dual QCD and lattice predictions for the rest of the potentials, namely $V_1', V_2', V_3, V_4, V_b, V_c, V_d$ and $V_e$. All of these agree remarkably well (within the lattice calculation uncertainties), with a few relatively minor exceptions. For example, at small $R$, Figures 5 and 6 show the dual QCD potential spin-spin potentials to be well above the lattice points. To understand the possible origin of these differences, consider the interaction of a point magnetic dipole with a sphere of constant magnetization in which dipole and magnetization directions are determined by the two spin directions. For the dipole outside of the sphere the interaction potential is of the form of $V_3$ and produces the usual perturbative QCD result. For the dipole inside the sphere the interaction is a constant and has the spin dependence of $V_4$. If one takes the radius of the sphere to zero holding its
magnetic moment constant this potential becomes a delta function at the origin. Because of the fact that the finite lattice size represents a granularity in space, one might expect a modification of the small R behavior of both of these potential in a lattice calculation.

Figures 9 and 10 for $V_d$ and $V_e$ show that the lattice results are consistent with zero for these two potentials. The dual QCD results are also nearly flat and very small, so we agree with what one gets on the lattice.

Overall, the agreement between two such different methods of calculations is remarkably good, and gives us hope that both are correct, and give reasonably reliable results.

We are indebted to Rajan Gupta for calling our attention to Reference (1). We would also like to thank Gunnar Bali for making their lattice results available to us in the numerical form necessary for the detailed fits and comparison.
Figure Captions

Fig. 1 Comparison of the dual QCD central potential (solid line) with the lattice central potential (points) for $\beta = 6.2$. The dual QCD parameters are $\alpha_s = .2048$, and $\sigma = -.2384(\text{GeV})^2$. The lattice string tension, in contrast, is $\sigma = .2190(\text{GeV}^2)$.

Fig. 2 A similar comparison (using the same parameters) for the quantity $\nabla^2 V_a^E$.

Fig. 3 The same for $-V_1'$. (Note the minus sign.)

Fig. 4 The same for $V_2'$.

Fig. 5 The same for $V_3$.

Fig. 6 The same for $V_4$. Note that aside from the first two points, the lattice results are all consistent with zero.

Fig. 7 And for $V_b$.

Fig. 8 And for $V_c$.

Fig. 9 And for $V_d$.

Fig. 10 And finally the comparison for $V_e$.

References


