On the Second Quantization of M(atrix) Theory

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Abstract

The second quantization of M(atrix) theory is considered. It provides a possible framework to the recent Susskind proposal that \( U(N) \) supersymmetric Yang Mills theories for all \( N \) might be embedded in a single dynamical system. The second quantization of M(atrix) theory can also be useful for the study of the Lorentz symmetry of the theory and for the consideration of processes with creation and annihilation of D-branes.

Recently Susskind [1] has suggested that there is the connection between M-theory and \( U(N) \) supersymmetric Yang-Mills theory not only in the large \( N \) limit but even at finite \( N \). He also conjectured that there is an embedding the super Yang-Mills theories for all \( N \) in a single dynamical system. In this note we attempt to pursue this idea by using the second quantization of M(atrix) theory. The second quantization would also be useful for the consideration of processes with creation and annihilation of D-branes [3]. Another problem which requires the consideration of processes which change the size of matrices is the problem of the Lorentz symmetry because the moment exchange in the eleventh dimension means the exchange of zero-brane charge [4].

The bosonic part of M(atrix) theory [2] is described by the following Lagrangian

\[
L = \frac{1}{2} tr[\dot{Z}^i \dot{Z}^i] + \frac{1}{2} [Z^i, Z^j]^2
\]  

(1)

where \( Z^i \) are Hermitian \( N \times N \) matrices and \( i, j = 1, \ldots, 9 \). Let us set

\[
Z^j_{ab} = X^j_{ab} + iY^j_{ab}, \ j = 1, \ldots, 9; \ a, b = 1, \ldots, N
\]  

(2)

where \( X^j_{ab} \) and \( Y^j_{ab} \) are real numbers. One has \( X^j_{ab} = X^j_{ba} \) and \( Y^j_{ab} = -Y^j_{ba} \). The free Hamiltonian corresponding to the Lagrangian (1) is

\[
H_0 = -\frac{1}{2} \sum_{i=1}^{9} (\sum_{a=1}^{N} \frac{\partial^2}{\partial X^j_{aa}^2}) + \frac{1}{2} \sum_{1 \leq a < b \leq N} (\frac{\partial^2}{\partial X^j_{ab}^2} + \frac{\partial^2}{\partial Y^j_{ab}^2}))
\]  

(3)

Let us consider the creation and annihilation operators satisfying the following relations

\[
[\psi_\alpha(x), \psi^*_\beta(y)] = \delta_{\alpha\beta}\delta^{(0)}(x - y)
\]  

(4)
where $\alpha, \beta = 1, 2, 3; \ x, y \in R^9$. The second quantized Hamiltonian corresponding to (3) reads

$$H_0 = -\frac{1}{2} \sum_{\alpha=1}^{3} \epsilon_\alpha \int d^9 x \psi_\alpha^*(x) \Delta \psi_\alpha(x)$$

(5)

where $\Delta = \sum_{i=1}^{9} \frac{\partial^2}{\partial x_i^2}$ and $\epsilon_1 = 1, \epsilon_2 = \epsilon_3 = 1/2$. One gets the Hamiltonian in the form (3) by acting (5) to the multiparticle vector

$$\Phi_N = \prod_{a=1}^{N} \psi_1^*(X_{aa}) \prod_{1 \leq a < b \leq N} \psi_2^*(X_{ab}) \psi_3^*(Y_{ab}) |0>$$

(6)

The second quantized Hamiltonian corresponding to the Lagrangian (1) is

$$H = H_0 + H_{\text{int}}$$

(7)

where $H_0$ is given by (5) and $H_{\text{int}}$ has the form

$$H_{\text{int}} = \sum_{\alpha, \beta, \gamma, \delta=1}^{3} \int d^9 x d^9 y \psi_\alpha^*(x) \psi_\beta^*(y) V_{\alpha\beta\gamma\delta}(x, y) \psi_\gamma(y) \psi_\delta(x)$$

(8)

Here $V_{\alpha\beta\gamma\delta}$ corresponds to the commutator term in (1).

The Hamiltonian (7) doesn’t depend on $N$ and it describes the $U(N)$ matrix quantum mechanics for any $N$. We can fix $N$ if we take a vector of the form (6). In this sense we get the embedding of $U(N)$ theory to the single theory for all $N$. Applications of the Hamiltonian (7) will be considered in another work.

The analogous second quantization of the matrix string theory [5, 6, 7] leads to the third quantization of theory of strings.

A string field theory approach to D-branes based on the connection between Witten’s string field theory and matrix models [8] has been considered in [9].

References