Gauge-invariant YFS exponentiation of (un)stable Z-pair production at and beyond CERN LEP 2 energies

S. Jadach

*Institute of Nuclear Physics, ul. Kawory 26a, Kraków, Poland
and CERN, Theory Division, CH-1211 Geneva 23, Switzerland*

W. Płaczek*

Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996-1200

B. F. L. Ward

*Department of Physics and Astronomy, The University of Tennessee, Knoxville, Tennessee 37996-1200
and SLAC, Stanford University, Stanford, California 94309*

(Received 25 April 1997)

We present the theoretical basis and sample Monte Carlo data for the YFS exponentiated calculation of $e^+e^- \rightarrow ZZ \rightarrow f_1 \bar{f}_1 + \bar{f}_2 f_2$ at and beyond CERN LEP 2 energies, where the left-handed part of f_i is a component of an $SU(2)_L$ doublet, $i=1,2$. The calculation is performed for both SM couplings and for anomalous ZZV triple gauge boson couplings in the conventions of Hagiwara *et al.* Our formulas, which are gauge invariant, are illustrated in a prototypical YFS Monte Carlo event generator YFSZZ.
[S0556-2821(97)03823-X]

PACS number(s): 12.15.Lk, 12.60.Cn, 13.10.+q, 13.38.Dg

The problem of the precision calculation of the processes $e^+e^- \rightarrow ZZ + n(\gamma) \rightarrow 4 \text{ fermions} + n(\gamma)$ at the higher energy range of CERN e^+e^- collider LEP 2 and at Next Linear Collider NLC (>300 GeV) type energies is the subject of our paper. We are motivated to investigate this problem for a number of reasons, the most important of which is its role in connection with the verification and tests of the $SU(2)_L \times U(1)$ model of Glashow, Salam, and Weinberg [1] of the electroweak interaction. Indeed, at the NLC these and related processes are expected to play the primary role in such physics studies [2,3] as precision tests of the fundamental non-Abelian triple and quartet gauge field self-interactions in principle, for example. In this paper, we present and illustrate, via sample Monte Carlo data, the rigorous Yennie-Frautschi-Suura (YFS) [4] exponentiated Monte Carlo approach [5] to these processes. The respective Monte Carlo event generator which realizes the calculation is exact in the infrared regime for initial state radiation and is of leading logarithmic accuracy through $O(\alpha^2)$ in the initial state QED hard radiative regime—the importance of initial state radiation for the process under study here has recently been emphasized in Ref. [6] wherein the nonfactorizable infrared corrections have been studied and shown to have no effect in $O(\alpha)$ on the corresponding Z line shape. A higher precision realization of our methods and results is in progress [7].

We recall at this time the current state of the art insofar as Monte Carlo event generators for the processes $e^+e^- \rightarrow ZZ + n(\gamma) \rightarrow 4 \text{ fermions} + n(\gamma)$ are concerned. In Refs. [8,9], both semianalytical and Monte Carlo event generator results

in the literature have been reviewed. What is new in our work in this paper is that we present for the first time the results of a calculation of these processes in which all of the following are simultaneously featured: realistic, finite p_T , simulation of the respective multiple-photon radiative effects on an event-by-event basis in which infrared singularities are canceled to all orders in α ; YFS exponentiated $O(\alpha^2)$ leading logarithm (LL) initial state radiation (ISR); and anomalous triple gauge boson couplings.

Thus, our work represents the first realistic multiphoton radiative effects simulation of the processes under study in which the interplay between the $n(\gamma)$ radiation and possible anomalous coupling effects can be systematically investigated. We illustrate this point in what follows.

Our work is organized as follows. We first build on the recent application of the YFS exponentiated Monte Carlo (MC) algorithm in Ref. [10] to the W -pair production process by extending it to the Z -pair production process of interest to us here. We then present some sample MC data and discuss their implications for Z -pair production studies at high energy e^+e^- colliding beam devices. Finally, we present some summary remarks.

Specifically, in this paper, we extend the result in Ref. [11] to the process $e^+e^- \rightarrow Z + Z + n(\gamma) \rightarrow 4 \text{ fermions} + n(\gamma)$, as is illustrated in Fig. 1. We start with the respective master formula.

Referring to our master formula in Ref. [11], Eq. (9) in this last reference, for the process $e^+e^- \rightarrow W^+ + W^- + n(\gamma) \rightarrow 4 \text{ fermions} + n(\gamma)$ and to the kinematics in Fig. 1, we arrive at the corresponding initial state YFS exponentiated $O(\alpha^2)$ LL master formula for the process $e^+e^- \rightarrow Z + Z + n(\gamma) \rightarrow 4 \text{ fermions} + n(\gamma)$ by substituting the respective Born level differential distribution for the latter process for the former, yielding

*On leave of absence from Institute of Computer Science, Jagiellonian University, Kraków, Poland.

FIG. 1. The process $e^+e^- \rightarrow ZZ + n(\gamma) \rightarrow f_1 + \bar{f}_1 + f_2 + \bar{f}_2 + n(\gamma)$.

$$\begin{aligned}
 d\sigma = & e^{2\alpha \text{Re } B + 2\alpha \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^n \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} \\
 & \times \exp \left[i y \left(p_1 + q_1 - p_2 - q_2 - \sum_j k_j \right) + D \right] \\
 & \times \bar{\beta}_n(k_1, \dots, k_n) \frac{d^3 p_{f_1} d^3 p_{\bar{f}_1} d^3 p_{f_2} d^3 p_{\bar{f}_2}}{p_{f_1}^0 p_{\bar{f}_1}^0 p_{f_2}^0 p_{\bar{f}_2}^0}, \quad (1)
 \end{aligned}$$

where we have introduced the same notation as that used in Ref. [11] for the YFS infrared functions B , $\tilde{B}$, and D and for the YFS hard photon residuals $\bar{\beta}_n$. Thus, at the $\bar{\beta}_0$ level to which we work (i.e., $\bar{\beta}_1$ and $\bar{\beta}_2$ are included here in the LL approximation only), we may identify $\frac{1}{2}\bar{\beta}_0$ with $d\sigma_{\text{Born}}/d\Omega_1 d\Omega_2$ where $d\Omega_{1(2)}$ is the differential decay solid angle of $f_1(f_2)$ in the respective Z 's rest frame. Our respective Born cross section $d\sigma_{\text{Born}}$ is taken from Ref. [12], where we allow for anomalous couplings gauge boson couplings f_4 and f_5 in the notation of this latter reference. We have realized the result (1) using MC methods of two of us (S.J. and B.F.L.W. [5,10,13]), in complete analogy with our work in Ref. [11]. The resulting program is called YFSZZ 1.0, and it is available from the authors [14]. We will now illustrate some of its applications. Such multiple photon radiative effects in the process under study here have not appeared elsewhere.

Specifically, in Fig. 2, we show the results for 10^6 simulated (weighted) events for the case that $f_1 = e$ and $f_2 = \mu$ in Fig. 1. We show this for the NLC type c.m. system (c.m.s.) energy $E_{\text{c.m.}} = 500$ GeV. To illustrate the interplay of the $n(\gamma)$ radiation and the anomalous gauge boson couplings f_4 and f_5 , we calculate five different scenarios, $f_i = 0$, $i = 4, 5$ (SM gauge boson couplings) at the Born level; $f_i = 0$, $i = 4, 5$ (SM gauge boson couplings) at the $O(\alpha^2)$ LL YFS exponentiated $\bar{\beta}_0$ level; $f_4 = 0.01$, $f_5 = 0$; $f_4 = 0$, $f_5 = 0.01$; and, finally, $f_4 = f_5 = 0.01$, where the last three scenarios are all done at the $O(\alpha^2)$ LL YFS exponentiated $\bar{\beta}_0$ level. The

FIG. 2. Total cross sections and angular distributions for the process in Fig. 1 for $f_1 = e$ and $f_2 = \mu$ at $E_{\text{c.m.}} = 500$ GeV. σ_{tot}^0 corresponds to the respective Born total cross section. The light solid curve, dark solid curve, solid dot, open diamond, and solid star correspond, respectively, to the differential distributions the polar angle (θ) of one of the Z 's relative to the incoming e^- direction, for example, for the standard model (SM) Born cross section, for the SM $O(\alpha^2)$ LL YFS exponentiated cross section, for the $f_4 = 0.0$, $f_5 = 0.01$ $O(\alpha^2)$ LL YFS exponentiated cross section, for the $f_4 = 0.01$, $f_5 = 0.0$ $O(\alpha^2)$ LL YFS exponentiated cross section, and for the $f_4 = 0.01$, $f_5 = 0.01$ $O(\alpha^2)$ LL YFS exponentiated cross section, respectively.

respective total cross sections, along with the Born cross section denoted by σ_{tot}^0 , are given in the top of the figure, whereas the differential cross section in the polar production angle of one of the Z 's is plotted in the figure for each case as indicated. The figure illustrates that these anomalous couplings give angular distributions with general shapes similar to that of the SM couplings, but that they make a significant increase in the cross section normalization. In this last regard, the coupling f_4 produces a stronger effect than does the same strength of the coupling f_5 , and when both are present with the same strength, the effects essentially add linearly insofar as the total cross sections are concerned. This approximate additive linearity is consistent with the fact that f_4 is CP violating and f_5 is not. At the level of the differential cross sections, there is a more detailed interplay between the radiative corrections and the anomalous couplings so that there are regions near the forward and backward directions where the effect of f_5 exceeds that of f_4 whereas in the complementary region the effect of f_4 exceeds that of f_5 , for the same strength of the respective couplings. Evidently, the multiple-photon character of the radiative effects, affecting as it does the normalization and the detailed angular profile of the cross sections in the figure for example, must be taken into account in any precision analysis of such anomalous gauge boson effects. YFSZZ 1.0 allows one to do this on an event-by-event basis.

In Ref. [15], semianalytical results are presented for the process in Fig. 1 which feature the exact $O(\alpha)$ ISR correc-

tion with soft photon exponentiation. Indeed, the authors in Ref. [15] have isolated both the complete $O(\alpha)$ correction and the so-called universal part of this correction associated with structure function evolution of the incoming $e^\pm$ beams. For the process which we present in Fig. 2, we may therefore compare directly the ratio the $\sigma_{\text{tot}}/\sigma_{\text{tot}}^0$ for the case of SM couplings, 1.081 ± 0.002 , with the analogous ratio in Ref. [15] for the respective universal correction, which is 1.0798. The excellent agreement between these two numbers gives us an important cross check on our work.

In summary, we have developed and illustrated the first ever multiple-photon, finite p_T , amplitude based Monte Carlo event generator YFSZZ 1.0 for the process $e^+e^- \rightarrow ZZ$

$+n(\gamma) \rightarrow f_1 \bar{f}_1 + \bar{f}_2 f_2 + n(\gamma)$ in which infrared singularities are canceled to all orders in α . We find that there is an interplay between the $n(\gamma)$ radiation and the presence of possible anomalous couplings in the gauge boson sector. We look forward with excitement to further applications of our calculation in high energy e^+e^- colliding beam devices.

Two of us (S.J. and B.F.L.W.) acknowledge the kind hospitality of Professor G. Veneziano and the CERN Theory Division while this work was completed. This work was partly supported by Polish Government Grant No. KBN 2P30225206 and by U.S. Department of Energy Contracts Nos. DE-FG05-91ER40627 and DE-AC03-76ER00515.

-
- [1] S. L. Glashow, Nucl. Phys. **22**, 579 (1961); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
 - [2] See, for example, S. Kuhlman *et al.*, SLAC-R-0485, 1996, and references therein.
 - [3] T. Barklow *et al.*, in Proceedings of the 1996 Snowmass Workshop.
 - [4] D. R. Yennie, S. Frautschi, and H. Suura, Ann. Phys. (N.Y.) **13**, 379 (1961).
 - [5] S. Jadach and B. F. L. Ward, Phys. Rev. D **38**, 2897 (1988).
 - [6] W. Beenakker, A. P. Chapovsky, and F. A. Berends, hep-ph/9706339, CERN-TH/97-114, 1997.
 - [7] S. Jadach, W. Płaczek, and B. F. L. Ward (in preparation).
 - [8] D. Bardin *et al.*, in *Physics at LEP2*, edited by G. Altarelli *et al.* (CERN, Geneva, 1996), Vol. 2, p. 3.
 - [9] T. Ohl, in *Proceedings of the 1996 Cracow International Symposium on Radiative Corrections*, edited by S. Jadach [Acta Phys. Pol. B **28**, 847 (1997)].
 - [10] S. Jadach and B. F. L. Ward, Comput. Phys. Commun. **56**, 351 (1990).
 - [11] S. Jadach, W. Płaczek, M. Skrzypek, and B. F. L. Ward, Phys. Rev. D **54**, 5434 (1996).
 - [12] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. **B282**, 253 (1987); D. Zeppenfeld (private communication).
 - [13] S. Jadach and B. F. L. Ward, Phys. Rev. D **40**, 3582 (1989).
 - [14] S. Jadach, W. Płaczek, and B. F. L. Ward, “The MC Event Generator YFSZZ 1.0,” available at the WWW URL <http://enigma.phys.utk.edu/pub/YFSZZ/>.
 - [15] D. Bardin *et al.*, Nucl. Phys. **B477**, 27 (1996).