OPTICAL TRANSDUCTION CHAIN FOR GRAVITATIONAL WAVE BAR DETECTORS

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DFPD 97/GP/18
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Abstract

A new signal extraction chain for gravitational wave bar detectors is proposed. Signal transduction is made by means of a Fabry-Perot cavity. Using a reference cavity for frequency comparison a sensitivity of $h_{\text{min}} = 3 \times 10^{-20}$ is estimated. The first prototype implementation as well as room-temperature measurements on it are reported.

PACS: 0480, 4280

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Introduction

Existing ultracryogenic gravitational wave resonant bar detectors have reached a very low level of brownian noise [1]. To fully exploit the potential sensitivity allowed by the very low temperature and high mechanical quality factor, these detectors must be coupled to a transduction chain able to detect and translate into an usable signal a few quanta increment in the bar energy. Further, this almost single-quantum-limited operation must be reached over an as large as possible detection bandwidth.

At present a promising way to reach high sensitivities and to widen the detector bandwidth appears to be the multimode approach [2]. A multimode optical transduction scheme has been proposed in [3] and partially implemented. This method is based on a He–Ne laser frequency stabilized to an external Fabry–Perot reference cavity. Within this scheme, a three-mode resonant transducer based on a Fabry–Perot cavity was locked to the stabilized laser by changing the length of this cavity by means of a capacitive coupling. Cryogenic operation of such a device has also been successfully demonstrated [4].

A different optical readout is being developed at the I.N.F.N. laboratories in Legnaro, Padova, Italy, in the framework of the A.U.R.I.G.A. experiment [5]. Here, the laser is frequency-locked to the Fabry-Perot cavity housed on the transducer using the Pound–Drever technique [6, 7], thereby eliminating the need for a direct feedback on the transducer itself.

In this paper we present a brief discussion of the principle of operation of this optical transduction chain and we estimate the sensitivity to gravitational waves. The sensitivity is expressed in terms of the minimum gravitational wave amplitude detectable with a resonant bar equipped with the optical transducer. Preliminary room-temperature measurements on a transducer prototype are also reported.
Operation of the detection scheme

The scheme we propose (Fig. 1) to detect small vibrations of a resonant bar is based on a high-finesse Fabry-Perot cavity: one of the cavity mirrors is fixed onto the bar itself and the second is fixed onto a mechanical oscillator coupled to the bar.

This mechanical oscillator, which is called resonant transducer in the following, should have the same resonance frequency (about 1 kHz) but much lighter mass than the resonant bar. For example with a ratio between the masses of the two oscillators equal $10^{-3}$, a frequency difference of 9 Hz will still guarantee an efficiency of 90% in the energy transfer between the two oscillators. The transducer should be so that, once it is coupled to the bar, the normal modes of the system have a mechanical quality factor $Q$ as close as possible to the bar's one. A prototype of such a transducer has been realized: its characteristics are discussed in a following paragraph.

A gravitational wave excitation of the bar will induce a change $\Delta L$ of the Fabry-Perot cavity length $L$ which corresponds to a change $\Delta \nu_l$ of the cavity optical resonance frequency $\nu_l$:

$$\Delta \nu_l = \frac{\Delta L}{L} \nu_l$$

(1)

The optical resonance frequency of this cavity, called transducer-cavity (Fig. 1), is then compared to the optical resonance frequency of a second cavity, called sensing-cavity, whose characteristic frequency is kept as stable as possible.

Frequency comparison between transducer and sensing-cavity is made by frequency-locking a laser to the transducer-cavity; using a beam-splitter a fraction of the laser beam is also diverted onto the sensing-cavity, where a signal proportional to the frequency difference is obtained using the FM sidebands technique [6, 7, 8].

To keep the sensing cavity at resonance one of its mirrors is mounted onto a piezoelectric actuator which corrects the length of the sensing-cavity using the error signal from the FM sidebands technique; the correction is effective only in the low
frequency domain ($\nu \leq 10$ Hz). At frequencies around 1 kHz the same signal is then analysed for gravitational wave detection.

Using the sensing-cavity for frequency comparison has the advantage of not having to apply a feed-back action to the transducer-cavity itself, thereby reducing back-action noise on the detector. This detection scheme could be applied both to room-temperature and to cryogenic systems.

Frequency stabilization of the sensing-cavity is obtained by placing it inside the cryostat at the middle point of the bar thus exploiting the high mechanical isolation and very low temperature. Optical access to the cavities is accomplished by means of optical fibres.

**Noise sources and sensitivity**

Noise contributions in the detection system are essentially due to laser power noise, electronic noise and thermal noise.

Laser power noise origins from fluctuations in the radiation pressure on the transducer-cavity mirrors. The spectral density of the equivalent noise force acting on the mirror fixed to the resonant transducer is given by:

$$S_{BA} = \left(\frac{2A}{cT_i}\right)^2 S_P e^{-\left[\frac{N^2}{Hz}\right]}$$

where $A$ is the fraction of light transmitted by the cavity, $T_i$ is the transducer-cavity mirror transmittivity, $c$ is the speed of light and $S_P$ is the laser power noise spectral density.

The bar and the transducer are kept at temperature $T = 0.1$ K thus determining a stochastic noise force whose spectral density is given by:

$$S_{ri} = 2K_b T_i \frac{m_i}{\tau_i} e^{-\left[\frac{N^2}{Hz}\right]}$$

4
where $K_B$ is the Boltzmann's constant, $m_i$ is the mass and $\tau_i$ the oscillation amplitude decay time of the bar and transducer oscillators for $i=1, 2$ respectively.

These noises drive the system of the two harmonic oscillators and therefore become narrow band noises at the point where frequency comparison between sensing and transducer cavities is made.

The laser is frequency-locked to the transducer-cavity by means of the Pound-Drever technique; the locking signal is obtained from the analysis of the light reflected by the cavity and collected by photodiode Pd1 of Fig.1. It is thus possible to reduce the laser frequency noise to the level given by the detection noise on the photodiode. Its shot-noise level is given by:

$$S_{vt} = 2.46 \frac{ec^2}{\eta P_0 F_t L_s^2} \left[ \frac{Hz^2}{Hz} \right]$$

(4)

where $e$ is the electronic charge, $\eta$ the photodiode sensitivity, $P_0$ the laser power, $F_t$ the transducer-cavity finesse, $L_s$ the transducer-cavity length. We assumed an index of modulation of 1.08 for the FM sidebands technique [8], cavity mirror losses equal to mirror transmittivity and 50% beam-splitters.

Light readout for locking of the sensing-cavity is also affected by detection noise $S_{vt}$ on photodiode Pd2 of Fig.1, that can be computed from an equation analogous to (4).

The sensing cavity is also affected by noises arising from the use of the piezoelectric actuator, namely thermal noise of the actuator and electrical noise of its driver. The total spectral density, expressed as frequency noise, is given by:

$$S_{vp} = (2K_B T \Re\{Z\} + S_v) d_{pe} \frac{V_L^2}{L_s^2} \left[ \frac{Hz^2}{Hz} \right]$$

(5)

where $\Re\{Z\}$ represents the real part of the piezo impedance, $S_v$ the voltage noise of the driver, $L_s$ the sensing-cavity length, $V_L$ the light frequency and $d_{pe}$ the piezoelectric constant.
Since the detection is made by performing a frequency comparison, the total noise is given as a frequency noise spectral density, according to:

\[
S_{\text{vis}}(\omega) = \left\{ F^2(\omega)(S_{\text{dA}} + S_{\text{T1}}) + G^2(\omega)S_{\text{T1}\text{r}} \right\} \frac{v^2}{L^2} + S_{\nu} + S_{\nu\text{r}} + S_{v\nu} \left[ \frac{H^2}{Hz} \right]
\]  

(6)

where \( F(\omega) \) and \( G(\omega) \) are the transfer function of the coupled oscillator system for forces acting on the transducer and on the bar, respectively. This noise can be converted into an equivalent noise at the input of the detector, as follows:

\[
S_{a}(\omega) = \frac{S_{\text{vis}}(\omega)}{\left( \frac{v_{L}m_{L}L_{\delta}\omega^2}{L_{\nu}n^2} \right)^2 G^2(\omega)} \left[ \frac{1}{Hz} \right]
\]  

(7)

where \( L_{\delta} \) is the bar length. The plot of the square-root of \( S_{a} \) is shown in Fig.2 with the numerical values of the system parameters given in Tab.I.

The sensitivity of the detector can be expressed in terms of the amplitude \( h_{\text{min}} \) of the gravitational pulse which leads to a signal-to-noise ratio of unity. Assuming as gravitational signal the so-called "standard pulse" [5], with the values listed in Tab.I we obtain a sensitivity of \( h_{\text{min}} = 3 \times 10^{-20} \). The corresponding squared signal-to-noise ratio per unit bandwidth is shown in Fig.3. From this plot a detection bandwidth of about 50 Hz can be estimated.

The sensitivity is often expressed in terms of the effective temperature \( T_{\text{eff}} \) of the system, defined as the minimum observable energy fluctuation: the effective temperature for the proposed detection chain is 3 \( \mu \)K.

**Transducer prototype**

**Prototype description**

The prototype in implementation has been designed for a bar identical with the bar of the A.U.R.I.G.A. detector: it is currently kept at room-temperature and consists of a 3 m long, \( 2.3 \times 10^3 \) kg, Al5056 cylinder whose first longitudinal mode resonance
frequency is about 875 Hz. The bar is suspended from a cable around the middle section and is isolated from vertical mechanical noise with an attenuation of about -130 dB at resonance.

The transducer prototype consists of a resonant part and supports for the optics as schematically shown in Fig.4. The resonant part is a circular plate (2.7 mm thick, 90.0 mm radius) supporting a concentric 1.15 kg load. This load is uniformly distributed over the inner portion of the plate within a 60.0 mm radius circle. The whole construction is made out of Al5056 which allows one to reach a high mechanical quality factor when working at low temperatures. The transducer can be viewed as a simple harmonic oscillator with the plate acting as a spring and the load as a neutral oscillating mass. The dimensions of this object are chosen so that the resonance frequency of the first symmetric mode of the loaded plate equals the bar frequency. Fig.5 shows the shape of the plate surface of the first symmetric mode of the plate. The load holds at its center one of the Fabry–Perot transducer-cavity mirrors (see Fig.4).

The entrance mirror of the transducer cavity and the optical fiber carrying the laser light to the cavity are fixed to the bar (see Fig.4) by means of two non-resonant supports, i.e. not showing resonance frequencies close to the bar and transducer ones. These supports are designed so that both mirror and optical fibre can be tilted to adjust the beam entrance angle in the cavity; the fibre can also be translated orthogonally to the beam propagation direction. Mirror and fibre supports are made out of Ergal.

The transducer prototype was assembled with a (24 ± 2) mm long, 1400 ± 200 finesse Fabry-Perot cavity aligned on it: this showed the satisfying operation of mirror support.

**Mechanical Q measurements**

Measurements have been performed to determine the total mechanical quality factor at room-temperature of the transducer-bar assembly. The transducer itself was rigidly coupled to one side of the bar and a mirror was glued in its proper place on the oscillating mass. The quality factor Q was determined by measuring the decay time of
the amplitude of a mechanical excitation fed into the system. Results from these measurements are:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Quality factor Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;-&quot;</td>
<td>857.028 ± 0.004</td>
<td>19500 ± 400</td>
</tr>
<tr>
<td>&quot;+&quot;</td>
<td>893.85 ± 0.01</td>
<td>9200 ± 300</td>
</tr>
</tbody>
</table>

From these measurements we observed that the transducer and the bar are not perfectly tuned since they exhibit a difference in resonance frequency of about 13 Hz, with the transducer having the higher frequency. This difference could be further reduced by decreasing the thickness of the circular plate supporting the oscillating mass. This frequency difference can also explain the fact that the quality factor of the "+" mode is less than the quality factor of the "-" mode: the "-" mode, which is dominated by the bar contribution, exhibits the higher Q due to the bar itself. The quality factors of this prototype transducer at room-temperature are comparable to those obtained with standard capacitive transducers in similar experimental conditions [9].

Conclusions

In this paper a new type of detection scheme for gravitational-wave resonant bar-detectors has been proposed. The expected sensitivity to gravitational bursts is \( h_{\text{min}} = 3 \times 10^{-20} \) with a detection bandwidth of about 50 Hz. This sensitivity is of utmost interest as it is in the range of sensitivities predicted for electromechanical transducers read by SQUID amplifiers with energy resolution of about 100fA. It should be noted that presently available SQUID’s show sensitivities at least a factor of ten worse than this. It is then apparent that the proposed optical transducer presents a very interesting viable alternative.

Further, it would be very useful to be able to simultaneously detect bar vibrations at harmonics other than the fundamental. Since cross-talk problems might arise in such an application, an optomechanical device would be an ideal candidate to operate in the
same apparatus along with an entirely different transducer system, such as an
electromechanical SQUID–based device.

Experimental work is currently under way to fully test the predictions on
sensitivity and bandwidth of the system. In particular a transducer prototype has been
assembled on a room–temperature bar detector facility and testing is in progress. The
mechanical quality factor of this transducer at room–temperature has been measured,
and found to be comparable to the quality factors of standard capacitive transducers.

Acknowledgements

We gratefully acknowledge the essential suggestions, criticisms, and
encouragement of Prof. J.–P. Richard and Dr. Yi-Pang, who shared with us their
experience and knowledge on transducer systems. We also thank Prof. M. Inguscio,
Prof. E. Polacco, and Dr. M. Prevedelli for useful discussions.

References

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detectors, in: Gravitational Wave Experiments, eds. E. Coccia, G. Pizzella and F.


Figure captions

Fig.1
Schematic drawing of a gravitational wave bar detector instrumented with the optical transduction chain.

Fig.2
Plot of the square root of $S_n(\omega)$ as a function of frequency (see eq. (7)).

Fig.3
Plot of the calculated spectral density of the signal–to–noise ratio of the optical transducer system as a function of frequency.

Fig.4
Schematic cross-sectional exploded view of the mechanical resonant transducer. The figure is not drawn to scale.

Fig.5
Shape of the first symmetric mode of the plate of the resonant transducer, in arbitrary units.
Tab. I

Numerical values at 0.1K of the parameters used for estimating the sensitivity of the optical transduction chain.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar dynamical mass</td>
<td>$m_i$</td>
<td>1150 kg</td>
</tr>
<tr>
<td>Bar length</td>
<td>$L_p$</td>
<td>3 m</td>
</tr>
<tr>
<td>Bar decay time</td>
<td>$\tau_1$</td>
<td>1600 s</td>
</tr>
<tr>
<td>Bar and transducer resonance frequencies</td>
<td>$\omega / 2\pi$</td>
<td>1 kHz</td>
</tr>
<tr>
<td>Transducer dynamical mass</td>
<td>$m_{\tau}$</td>
<td>1.2 kg</td>
</tr>
<tr>
<td>Transducer decay time</td>
<td>$\tau_2$</td>
<td>1600 s</td>
</tr>
<tr>
<td>Transducer-cavity length</td>
<td>$L_\tau$</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Transducer-cavity finesse</td>
<td>$F_\tau$</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td>Sensing-cavity length</td>
<td>$L_s$</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Sensing-cavity finesse</td>
<td>$F_s$</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td>Photodiode sensitivity</td>
<td>$\eta$</td>
<td>0.6 A/W</td>
</tr>
<tr>
<td>Laser frequency</td>
<td>$n_\lambda$</td>
<td>2.82 \times 10^{14} Hz</td>
</tr>
<tr>
<td>Laser power</td>
<td>$P_\lambda$</td>
<td>0.01 W</td>
</tr>
<tr>
<td>Laser noise power spectral density</td>
<td>$S_{n_\lambda}$</td>
<td>$10^{-18}$ W^2/Hz</td>
</tr>
<tr>
<td>Piezoelectric constant</td>
<td>$d_{\pi z}$</td>
<td>$8 \times 10^{-11}$ m/V</td>
</tr>
<tr>
<td>Piezo driver voltage noise</td>
<td>$S_v$</td>
<td>$4 \times 10^{-17}$ V^2/Hz</td>
</tr>
<tr>
<td>Piezo impedance real part @ 1kHz</td>
<td>Re{$Z$}</td>
<td>20 $\Omega$</td>
</tr>
</tbody>
</table>
Bs = Beam-splitter
Pd1, Pd2 = Photodiode
\[ \sqrt{\frac{S_h(\omega)}{[1/\sqrt{\text{Hz}]}} \]