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ABSTRACT

Status report of the JLQCD Collaboration in the lattice QCD calculation of weak matrix elements is made. A precise determination of the kaon B parameter B_K is made with the staggered quark action. B_K is estimated also with the Wilson quark action solving the mixing problem of four-quark operators based on chiral Ward identities. The B meson decay constant f_B and the B_B parameter are calculated with the propagating Wilson quark.

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Recent lattice calculation of weak matrix elements

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Status report of the JLQCD Collaboration in the lattice QCD calculation of weak matrix elements is made. A precise determination of the kaon B parameter B_K is made with the staggered quark action. B_K is estimated also with the Wilson quark action solving the mixing problem of four-quark operators based on chiral Ward identities. The B meson decay constant f_B and the B_B parameter are calculated with the propagating Wilson quark.

1. Introduction

Two years ago, we started the JLQCD Collaboration[1] for large scale lattice QCD simulations using Fujitsu VPP500/80 supercomputer installed at KEK. One of the most important purpose of our collaboration is a systematic and high accuracy calculation of hadronic weak matrix elements which represent the low energy QCD effects in the weak interaction processes involving hadrons. In this talk we briefly report the present status of our calculation of the matrix element B_K appearing in K^0 - $\bar{K}^0$ mixing and f_B and B_B in B^0 - $\bar{B}^0$ mixing. These matrix elements are of much practical importance in that their accurate knowledge is indispensable for exploring the phenomenology of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, especially of CP violation.

2. B_K parameter

Consider the K^0 - $\bar{K}^0$ CP-violation parameter ϵ ,

$$|\epsilon| = B_K \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K} A^2 \lambda^6 \eta \quad (1)$$

$$\times \left[F_1 \left(\frac{m_c^2}{m_W^2}, \frac{m_t^2}{m_W^2} \right) + F_2 \left(\frac{m_t^2}{m_W^2} \right) A^2 \lambda^4 (1 - \rho) \right]$$

where A , λ , ρ and η are the CKM parameters in the Wolfenstein parametrization. F_1 and F_2 contain the short distance QCD effects, which have been computed in the two-loop perturbation theory. The unknown parameter B_K represents the

long distance QCD effects and is defined by

$$\langle \bar{K}^0 | O_{LL} | K^0 \rangle = \frac{8}{3} m_K^2 f_K^2 B_K \quad (2)$$

with O_{LL} the $\Delta S=2$ left-left operator

$$O_{LL} = \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d. \quad (3)$$

We calculate the B_K parameter on the lattice using both the staggered quark action and the Wilson quark action. The staggered action has the advantage that it preserves $U(1)$ subgroup of chiral symmetry. Chiral symmetry plays an essential role in the study of light meson physics. In fact, eq. (2) requires that $\langle \bar{K}^0 | O_{LL} | K^0 \rangle$ vanishes in the chiral limit $m_K^2 \rightarrow 0$, and this property is guaranteed if the underlying theory has chiral symmetry. The staggered action, however, involves four internal flavors and physical interpretation of these flavors is necessary to obtain the final answer. The Wilson action, on the other hand, describes physical number of flavors. This is the advantage of this action. The Wilson action, however, breaks chiral symmetry explicitly. In particular $\langle \bar{K}^0 | O_{LL} | K^0 \rangle$ does not vanish in the chiral limit $m_K^2 \rightarrow 0$. Modification of the lattice O_{LL} operator is necessary to obtain the B_K parameter. Two actions have their own advantage and disadvantage. To demonstrate the reliability of lattice calculation, we should check if both actions give the same answer in the continuum limit.

2.1. B_K parameter with the staggered quark action

A foundation for a lattice QCD calculation of the B_K parameter with the staggered quark ac-

tion was developed several years ago by Gupta, Kilcup, Patel and Sharpe[2–4]. Straightforward calculations are possible since the staggered action preserves chiral symmetry. Theoretically we expect that the B_K parameter calculated with the staggered action has $O(a^2)$ discretization error[4]. A verification of this behavior and a precise determination of the continuum value of B_K require a high statistics measurement of B_K over a wide range of lattice spacing. The statistics of the previous studies were not enough, however, to confirm the $O(a^2)$ behavior. We have therefore decided to carry out a new set of simulations with improved statistics, incorporating advance in methods of analyses [5,6].

The B_K parameter is obtained from a combination of two- and three-point hadronic correlation functions. The required three-point function is

$$C_3(t_1, t, t_2) = \frac{1}{L^3} \sum_{\mathbf{x}} \langle 0 | \Phi(t_1) O_{LL}(\mathbf{x}, t) \Phi(t_2) | 0 \rangle \quad (4)$$

with $\Phi(t_1)$ the K meson interpolating operator at time t_1 with vanishing spatial momentum $\mathbf{p} = \mathbf{0}$ and L^3 the spatial volume of the lattice. For large time distances $t_1 \ll t \ll t_2$, $C_3(t_1, t, t_2)$ is dominated by the matrix element of the operator O_{LL} sandwiched between K meson states,

$$C_3(t_1, t, t_2) \rightarrow |C|^2 \langle \bar{K}^0 | O_{LL} | K^0 \rangle e^{-m_K(t_2 - t_1)} \quad (5)$$

with C the normalization constant given by

$$C = \frac{\langle K | \Phi | 0 \rangle}{2m_K L^3}. \quad (6)$$

The two-point function we calculate is,

$$C_2(t, t_1) = \frac{1}{L^3} \sum_{\mathbf{x}} \langle 0 | A_0(\mathbf{x}, t) \Phi(t_1) | 0 \rangle. \quad (7)$$

Here A_μ is the axial-vector current. For large time distances $t \ll t_1$, we have

$$C_2(t, t_1) \rightarrow -i C m_K f_K e^{-m_K(t_1 - t)} \quad (8)$$

where we use the relation

$$\langle 0 | A_\mu | K(P) \rangle = -i P_\mu f_K. \quad (9)$$

The B_K parameter, then, is given by the following ratio for large time distances $t_1 \ll t \ll t_2$,

$$R(t_1, t, t_2) = \frac{C_3(t_1, t, t_2)}{\frac{8}{3} C_2^*(t, t_1) C_2(t, t_2)} \rightarrow B_K. \quad (10)$$

We basically use the formalism developed in ref.[2–4] to calculate two- and three- point correlation functions. Following improvements are employed in our simulations, however.

We use an improved interpolating operator Φ to minimize excited-state contaminations to the ground-state signal and reduce statistical errors[7]: our operator creates only the pseudoscalar meson in the Nambu-Goldstone channel. In contrast, the interpolating operator used in the previous studies also creates vector mesons that contaminate signals.

In terms of the staggered quark field, the four-quark operators are extended over a 2^4 hypercube and are gauge non-invariant. The previous calculations employed these operators with fixed gauge. We also use gauge invariant operators[8] constructed by inserting gauge link variables between quark fields.

In order to interpret the lattice results in the continuum theory, renormalization of lattice operators are generally necessary;

$$O_i^{\text{continuum}} = Z_{ij} O_j^{\text{lattice}}. \quad (11)$$

The tadpole-improved[9] one-loop results[10,11] for the lattice renormalization factor Z is used to convert lattice values of B_K to those of the continuum. The naive dimensional regularization(NDR) with the modified minimal subtraction scheme is used in the continuum theory.

Simulations were made in the quenched approximation with degenerate strange and down quark masses[12]. To make a systematic study of the discretization error, we made six independent simulations in a wide range of the lattice spacing. In Table 1 we list the parameters of our simulations. Values of the gauge coupling constant β are chosen so that the lattice spacing a is roughly equally spaced for $\beta \geq 5.85$. The physical scale of lattice spacing is determined from the ρ meson mass. Calculations are done on a $L^3 T$ lattice and #conf. is the number of gauge configurations. The lattice size is increased as we decrease

Table 1

Simulation parameters for B_K calculation with the staggered quark action.

β	5.7	5.85	5.93	6.0	6.2	6.4
L^3T	$12^3 24$	$16^3 32$	$20^3 40$	$24^3 64$	$32^3 64$	$40^3 96$
#conf.	150	60	50	50	40	40
$a^{-1}(\text{GeV})$	0.806(14)	1.36(3)	1.59(3)	1.88(4)	2.65(9)	3.47(7)
$La(\text{fm})$	2.9	2.3	2.5	2.5	2.4	2.3
$m_q a$	0.02-0.08	0.01-0.04	0.01-0.04	0.01-0.04	0.005-0.02	0.005-0.02
$m_s a/2$	0.0563	0.0202	0.0160	0.0126	0.0089	0.0070

Figure 1. The Ratio R for the gauge non-invariant operator as a function of t at $\beta=6.4$ and $m_q a = 0.02$ on a $40^3 \times 96$ lattice.

the lattice spacing to keep the physical lattice size approximately constant at $La \approx 2.4$ - 2.5 fm. Four values of quark masses are employed at each β . With degenerate strange and down quark masses, we estimate half the strange quark mass $m_s a/2$ from $m_K/m_\rho = 0.648$.

The ratio $R(t_1, t, t_2)$ in eq.(10) is calculated by changing t in the interval $0 \leq t \leq T-1$ with fixed $t_1 = 0$ and $t_2 = T-1$. As a representative example of our calculation, we show in Fig. 1 the ratio R as a function of t . We clearly see a constant behavior of the ratio for $16 \lesssim t \lesssim 80$ indicating that only K meson contributes to the ratio. The horizontal line is the constant fit for $25 \leq t \leq 69$ with the errors estimated by the jackknife method.

The physical value of B_K is obtained by interpolating the results to the quark mass $m_s a/2$ corresponding to the physical kaon. Chiral per-

Figure 2. $B_K(\text{NDR}, 2\text{GeV})$ as a function of $m_q a$ at $\beta = 6.4$ on a $40^3 \times 96$ lattice. Solid lines are fits to (12) and dashed line corresponds to physical value of m_K/m_ρ .

turbation theory predicts that the B_K parameter has the following quark mass dependence for small $m_q a$ [13]

$$B_K = B[1 - 3c(m_q a) \log(m_q a) + b(m_q a)] \quad (12)$$

where $c = \alpha/(4\pi f_\pi)^2$ with $\alpha = (m_K a)^2/m_q a$ and b is an unknown constant. In Fig. 2, we show the result of our fit taking B , c and b as free parameters both for gauge non-invariant (circles) and invariant (squares) operators. The physical value of B_K is extracted from the fit at $m_q a = m_s a/2$. There is a discrepancy between gauge non-invariant and invariant operators. It represents a systematic uncertainty of $O(g^4)$ arising from the renormalization constants Z which are calculated only to the one-loop order.

In Fig. 3, we plot our results for B_K renor-

Figure 3. Gauge non-invariant(circles) and invariant(squares) $B_K(\text{NDR}, 2\text{GeV})$ as a function of a , together with a quadratic fit to four data points on the left (solid lines) and a linear fit to five points (dashed lines).

malized at $\mu=2$ GeV in the NDR scheme as a function of the lattice spacing a . B_K depends strongly on the lattice spacing. We need to take the continuum limit $a \rightarrow 0$ to get the final answer. Theoretically we expect that B_K has $O(a^2)$ discretization error[4,6] and our data are consistent with this behavior. The solid line in Fig. 3 shows a fit of form $B_K = c_0 + c_1 a^2$ to the four points, excluding the two points on the right at $\beta=5.85$ and 5.7 . The continuum value we found is

$$B_K(\text{NDR}, 2\text{GeV}) = 0.587 \pm 0.007 \pm 0.017 \quad (13)$$

where the first error is statistical and the second is the systematic error of $O(g^4)$ estimated from the difference of gauge non-invariant and invariant operators. Our result has a rather small statistical error. This is achieved both by using several times more gauge configurations compared with the previous work and also by the use of the improved interpolating operator.

We should note, however, that the data by themselves can not establish the $O(a^2)$ behavior. We can make a linear fit of form $B_K = c_0 + c_1 a$ to the five points for $a \leq 0.15\text{fm}$ as shown in Fig. 3 with the dashed lines. To confirm the $O(a^2)$ discretization error numerically, we are now making

a simulation on a $56^3 \times 96$ lattice at $\beta=6.65$ with yet smaller lattice spacing of $a \approx 0.04\text{fm}$.

2.2. B_K parameter with the Wilson quark action

The Wilson quark action explicitly breaks chiral symmetry due to the presence of the Wilson term which is necessary to have physical number of flavors. As a result, O_{LL} operator does not have the correct chiral property, i.e., $\langle \bar{K}^0 | O_{LL} | K^0 \rangle$ does not vanish in the chiral limit $m_K^2 \rightarrow 0$. The O_{LL} operator mixes with different operators under renormalization. We have to solve the mixing problem so that the continuum operator has the desired chiral property. It is well known that for this problem the currently accessible values of gauge coupling is not weak enough for perturbation theory to be reliable[14]. Recently the Roma group has applied the non-perturbative method[15] to solve this problem and yielded a continuum operator with a good chiral behavior[16]. However, the underlying mechanism of improvement in this approach is not quite clear. Here we propose a non-perturbative method to solve the operator mixing problem based on chiral Ward identities[17], thus incorporating explicitly the chiral properties of the Wilson quark action.

For the study of operator mixing, it is convenient to decompose the O_{LL} operator into $O_{VV+AA} = (O_{VV} + O_{AA})/2$ and O_{VA} with $O_{\Gamma\Gamma'} = \bar{s}\Gamma d \bar{s}\Gamma' d$ where Γ and Γ' represent the Dirac matrices. While O_{VA} is multiplicatively renormalizable, O_{VV+AA} mixes with other operators having different chiral properties

$$\begin{aligned} \hat{O}_{VV+AA} &= Z_{VV+AA}(O_0 + z_1 O_1 + \cdots + z_4 O_4) \\ \hat{O}_{VA} &= Z_{VV+AA} \cdot z_5 O_5. \end{aligned} \quad (14)$$

Here $\hat{O}_{VV+AA}$ and $\hat{O}_{VA}$ are the continuum operators and O_i are lattice operators in the Fierz eigenbasis

$$\begin{aligned} O_0 &= (O_{VV} + O_{AA})/2 \\ O_1 &= (O_{SS} + O_{TT} + O_{PP})/2 \\ O_2 &= (O_{SS} - O_{TT}/3 + O_{PP})/2 \\ O_3 &= (O_{VV} - O_{AA})/2 + (O_{SS} - O_{PP}) \\ O_4 &= (O_{VV} - O_{AA})/2 - (O_{SS} - O_{PP}) \end{aligned}$$

Table 2
Simulation parameters for B_K calculation with the Wilson quark action.

β	5.9	6.1	6.3
L^3T	24^364	32^364	40^396
#conf.	300	100	50
$a^{-1}(GeV)$	1.95(5)	2.65(11)	3.41(20)
$m_s a/2$	0.0294(14)	0.0198(16)	0.0144(17)

$$O_5 = O_{VA}. \quad (15)$$

The continuum operators $\hat{O}_{VV+AA}$ and $\hat{O}_{VA}$ close under the chiral transformation induced by the third component of the axial current $A_\mu^3 = \bar{\psi} \frac{1}{2} \lambda^3 \gamma_\mu \gamma_5 \psi$; denoting $\hat{O}_1 = \hat{O}_{VV+AA}$ and $\hat{O}_2 = \hat{O}_{VA}$, we have $\delta \hat{O}_1 = -i \hat{O}_2$ and $\delta \hat{O}_2 = -i \hat{O}_1$. We choose the mixing coefficients z_i in eq. (14) such that the quark matrix elements of $\{\hat{O}_i\}$ satisfy the relevant chiral Ward identities to $O(a)$. The identities can be derived in a standard manner[18] and take the form given by

$$\begin{aligned} & 2m \langle \sum_x P^a(x) \hat{O}_i(0) \prod_{k=1}^4 \bar{\psi}(p_k) \rangle \\ & + i \langle \delta \hat{O}_j(0) \prod_{k=1}^4 \bar{\psi}(p_k) \rangle \\ & + i \sum_l \langle \hat{O}_l(0) \prod_{k \neq l} \bar{\psi}(p_k) \delta \bar{\psi}(p_l) \rangle + O(a) = 0 \end{aligned} \quad (16)$$

with P^a the pseudoscalar operator $P^a = \bar{\psi} \frac{1}{2} \lambda^a \gamma_5 \psi$ and p_k the momentum of external quark. We calculate the quark matrix elements appearing in eq. (16) at equal external quark momentum $p_k^2 = p^2$ in the Landau gauge. The remaining overall factor Z_{VV+AA} is determined by the non-perturbative renormalization method of ref. [15].

In Table 2 we list the parameters of our simulations. Three simulations were made in the range of the lattice spacing $a \sim 0.06 - 0.1 \text{ fm}$ to study continuum extrapolation. The physical scale of lattice spacing is determined from the ρ meson mass. Simulations were made in the quenched approximation with degenerate strange and down quark masses as in the case of the staggered ac-

Figure 4. Mixing coefficients $z_1, \dots, z_4$ for $K = 0.15034$ at $\beta = 6.3$. Vertical line corresponds to $p \approx 2 \text{ GeV}$.

tion. At each β , four values of the hopping parameter K (which is related to the quark mass) are employed ranging approximately from $3/8m_s a$ to $9/8m_s a$ so that the physical point $m_s a/2$ for the K meson with degenerate quarks can be interpolated from data.

In Fig. 4, a representative result for the mixing coefficients $z_1, \dots, z_4$ is plotted as a function of the external quark momenta. Mixing coefficients are almost scale independent in the range $0.2 \lesssim p^2 a^2 \lesssim 1.0$. In the following analysis, we use values of coefficients at $p \approx 2 \text{ GeV}$, which corresponds to $0.3 \lesssim p^2 a^2 \lesssim 1.0$ for our runs at $\beta = 5.9 - 6.3$.

To check if the continuum operator $\hat{O}_{VV+AA}$ has the desired chiral property, we study the ratio

$$R_0 = \frac{\langle \bar{K}^0 | \hat{O}_{VV+AA} | K^0 \rangle}{|\langle 0 | \hat{P} | K^0 \rangle|^2} \Big|_{m_K^2=0} \quad (17)$$

extrapolated to $m_K^2 = 0$. Since the pseudoscalar matrix element $\langle 0 | \hat{P} | K^0 \rangle$ in the denominator does not vanish in the chiral limit, this quantity measures the contributions of chiral symmetry breaking terms. We calculate $\hat{O}_{VV+AA}$ using the mixing coefficients obtained both from the Ward identity (WI) method and from the one-loop perturbation theory (PT). In Fig. 5, we show the a dependence of the ratio R_0 . We clearly see the

Figure 5. Effect of chiral symmetry breaking for Ward identity (WI) and perturbative (PT) methods.

Figure 6. B_K as a function of a .

improvement made by the WI method. While the PT values stay negative for finite a , R_0 from the WI method becomes consistent with zero even for the finite lattice spacing $a \sim 0.06 - 0.08 \text{ fm}$. We expect that the PT values have both the $O(a)$ discretization error and the $O(g^4)$ one-loop truncation error. If extrapolated linearly in a , the continuum PT value is also consistent with zero within errors suggesting that remaining $O(g^4)$ truncation error is actually small.

Our results for $B_K(\text{NDR}, 2\text{GeV})$ are summarized in Fig. 6. Filled squares denote the re-

sult from the WI method. We obtain reasonable values even at finite lattice spacing $a \sim 0.06 - 0.08 \text{ fm}$. The large statistical errors, however, make it difficult to take a reliable continuum extrapolation.

The large statistical error stems from that of the mixing coefficients. To reduce them at each β , we use an alternative definition of the B_K parameter $\langle \bar{K}^0 | \hat{O}_{VV+AA} | K^0 \rangle / \langle \bar{K}^0 | \hat{O}_{VV+AA} | K^0 \rangle_{VS}$ in which the denominator is replaced by the vacuum saturation value of the $\hat{O}_{VV+AA}$ operators, thus canceling the large statistical fluctuation between the numerator and the denominator (WI[VS] method). While this method introduces an additional $O(a)$ discretization error, the difference is expected to vanish in the continuum limit. Filled circles in Fig. 6 show our results. Errors are significantly reduced. Now the linear extrapolation in a is possible and the continuum value of B_K is given by

$$B_K(\text{NDR}, 2\text{GeV}) = 0.59(8). \quad (18)$$

This value is consistent with the result for the staggered action $B_K(\text{NDR}, 2\text{GeV}) = 0.587(7)(17)$ in spite of the rather different approach to the continuum limit.

We can also linearly extrapolate the PT value yielding $B_K(\text{NDR}, 2\text{GeV}) = 0.59(10)$ in agreement with the results from WI or WI[VS] methods. We have to resolve various systematic uncertainties associated with such a large extrapolation, however, before we can trust this approach.

In conclusion, our calculation demonstrates the validity of the chiral Ward identity method to construct the continuum operator having the correct chiral property. The staggered and Wilson quark actions give consistent continuum value for B_K parameter, which shows explicitly the reliability of our lattice QCD calculation.

3. Matrix elements of heavy-light systems

In the determination of the CKM matrix elements, $B^0 - \bar{B}^0$ mixing also plays an important role. The $B^0 - \bar{B}^0$ mass difference is given by

$$\begin{aligned} \Delta m_{B^0} &= f_B^2 B_B \frac{G_F^2}{6\pi^2} m_B m_t^2 F\left(\frac{m_t^2}{m_W^2}\right) \eta_{QCD} \\ &\times A^2 \lambda^6 [(1 - \rho)^2 + \eta^2] \end{aligned} \quad (19)$$

where F and η_{QCD} are perturbatively calculable quantities and long distance QCD effects are represented by the unknown f_B and B_B parameters.

As we have seen in the previous section, the continuum extrapolation is an indispensable procedure to obtain physical result. The potential problem of the b quark system is that the b quark mass $m_b \sim 4.5\text{GeV}$ is larger than currently accessible lattice spacing $a^{-1} \sim 2.0\text{--}3.5\text{GeV}$, so that the $O(m_b a)$ effect may become uncontrollably large. This expectation needs not to be true, however, for the heavy-light system such as B meson since the heavy quark plays only a role of a slowly moving color source in the bound state and the energy scale relevant for the dynamics is Λ_{QCD} and not m_b . Heavy quark behaves non-relativistically and we might not have large $O(m_b a)$ discretization errors. It has been argued[19] that we can use the Wilson quark action to describe the non-relativistic heavy quark dynamics provided that suitable definition of the heavy quark mass and the wave function normalization is employed in the analysis. We have therefore calculated the B meson decay constant f_B using the Wilson quark action both for the heavy and light quarks and studied systematically the continuum extrapolation of f_B parameter [20,21].

Let us define the pole mass m_{pole} and the kinetic mass m_{kin} of the Wilson quark action by the expansion of the energy of the one particle state as

$$E = m_{\text{pole}} + \frac{\vec{p}^2}{2m_{\text{kin}}} + \dots \quad (20)$$

where $\vec{p}^2 = \sum_{i=1}^3 (2 \sin p_i/2)^2$. Lorentz invariance requires $m_{\text{pole}} = m_{\text{kin}}$. For heavy quarks, however, they differ by the large discretization effect. It has been argued[19] that for the heavy-light system the relevant dynamical scale is controlled by the kinetic mass, rather than the pole mass which we usually use for light quarks. We adopt this view and compare the mass dependence of the decay constant using both the pole mass and the kinetic mass.

The wave function normalization is also modified for heavy quark by the Kronfeld-Lepage-Mackenzie(KLM) factor[19,22] $Z_{KLM} = \exp m_{\text{pole}} a/2$ which reduces to 1 in the massless

Table 3

Simulation parameters for f_B calculation.

β	5.9	6.1	6.3
$L^3 T$	$16^3 40$	$24^3 64$	$32^3 80$
#conf.	150	100	100
$a^{-1}(\text{GeV})$	1.84(3)	2.54(7)	3.36(11)

limit.

We have calculated the decay constant by taking a number of alternative prescriptions for the heavy mass parameter and the wave function normalization to examine the validity of our treatment of heavy-light system.

(IA) We use the pole mass with the naive normalization as in the case of the relativistic quark.

(IB) Alternatively we take the KLM normalization with the pole mass.

(IIA) We use the kinetic mass measured from the simulations together with the KLM normalization. The kinetic mass is extracted from four values of momenta up to $p_i = (1, 1, 1)$ with the three parameter fit of the form $E = m_{\text{pole}} + \vec{p}^2/2m_{\text{kin}} - c\vec{p}^4/8m_{\text{kin}}^3$.

(IIB) The measured kinetic mass has rather large statistical error and is not well determined. To circumvent this problem, we take another definition of the kinetic mass[23] estimated from the tree level heavy quark mass difference $(m_{\text{kin}}^{(\text{quark})} - m_{\text{pole}}^{(\text{quark})})$ by

$$m_{\text{kin}}^{(\text{meson})} = m_{\text{pole}}^{(\text{meson})} + (m_{\text{kin}}^{(\text{quark})} - m_{\text{pole}}^{(\text{quark})}) \quad (21)$$

with $m_{\text{pole}}^{(\text{meson})}$ the measured meson pole mass. We keep the KLM normalization.

To study the continuum extrapolation we made three independent quenched simulations. The parameters of our simulations are listed in Table 3. The physical scale of lattice spacing is determined from the charmonium spin-averaged 1S-1P mass splitting. The physical lattice size is kept approximately constant at $La \approx 1.9\text{ fm}$.

We calculate the decay constant f_P for the pseudoscalar meson P made from the heavy quark Q and the light quark q . Figure 7 shows the scale invariant quantity $\Phi_P(m_P) = [\alpha_s(m_P)/\alpha_s(m_B)]^{2/\beta_0} f_P \sqrt{m_P}$ as a function of

Figure 7. $\Phi_P(m_P)$ versus $1/m_P$ with the prescription IA (open symbols) and with IIB (filled symbols). Results with the static approximation are also shown at $1/m_P=0$.

$1/m_P$ for cases (IA) and (IIB). Significantly smaller values of Φ_P for heavy quarks for case (IA) compared with (IIB) is mainly due to the use of the naive normalization; as is well known[24] Φ_P even decreases towards a large quark mass. In contrast, it increases for case (IIB) with the KLM normalization and is smoothly extrapolated to the value estimated in the static limit[25].

In Fig. 8, we plot the D meson decay constant f_D as a function of the lattice spacing a . Since c quark mass is smaller than the presently used lattice spacing $a^{-1} \sim 2.0\text{--}3.5\text{GeV}$, we expect that continuum extrapolation can be taken reliably assuming linear $O(m_Q a)$ behavior and the above four prescriptions give consistent continuum answer up to $O((m_Q a)^2)$ effects. Although case (IIA) suffers from large statistical error coming from that of the measured kinetic mass, our expectation is reasonably realized in our data.

The results of the B meson decay constant f_B is shown in Fig. 9. As expected we observe a rather large a dependence in the simplest prescription (IA). It is not clear whether we can trust linear extrapolation in this case. The a dependence, however, is significantly reduced in case (IIB) and the size of the a dependence is comparable with that of f_D in the prescription (IIB). We think that large $O(m_Q a)$ effects are removed by

the non-relativistic prescription and the remaining errors are reasonably small for (IIB).

After continuum extrapolation we obtain

$$\begin{aligned} f_D &= 202(8)^{+24}_{-11} \text{ MeV} \\ f_{D_*} &= 216(6)^{+22}_{-15} \text{ MeV} \\ f_B &= 179(11)^{+2}_{-31} \text{ MeV} \\ f_{B_*} &= 197(7)^{+0}_{-35} \text{ MeV} \end{aligned} \quad (22)$$

where we take the value with prescription (IIB) for our central values. The first error is statistical and the second represents a spread over four prescriptions.

To confirm the above numbers, it is desirable to calculate the decay constant using a different quark action and compare the continuum values as we have done for B_K parameter. At present we are calculating the heavy-light decay constant using the clover quark action[26]. This action contains one extra term compared to the Wilson action. By tuning the coefficient of this term we can remove leading $O(a)$ discretization errors from physical quantities. Although more computer time is required for this action, we expect to finish the calculation soon.

Finally we briefly discuss the B meson B_B parameter. At present we have calculated it using the Wilson quark action with the mixing coefficient obtained by the one-loop perturbation theory at $\beta = 6.1$ and 6.3 [20]. Our results renormalized at $\mu=5\text{GeV}$ are

$$\begin{array}{ccc} \beta & 6.1 & 6.3 \\ B_B & 0.895(47) & 0.840(60) \\ B_{B_*} & 0.889(24) & 0.878(32) \end{array} \quad (23)$$

These numbers are consistent with the previous results[27,28]. We plan to study the continuum extrapolation of B_B parameter with the clover quark action by extending the chiral Ward identity method to calculate the mixing coefficient nonperturbatively.

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Figure 8. Continuum extrapolation of f_D with four methods: IA (open circles), IB (open diamonds), IIA (crosses) and IIB (filled circles).

Figure 9. Same as Figure 8 for f_B .

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