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Possible Effects of Non-Vanishing Particle Sizes in the Early Universe

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Abstract

Possible effects of the non-vanishing sizes of particles (atoms, nuclei, nucleons, quarks, and leptons) in the early universe (the temperature T) are discussed in an extended Friedmann model of the universe (the scale a). Especially pointed out are the following possibilities: 1) if $r_q > (2N_B/\pi)^{-1/3}a$, most of the proposed scenarios for $T > 10^3\text{TeV}$ including the inflationary universe one are unrealistic, 2) $r_q < (2N_B/\pi)^{-1/3}a$ due to the smallness of r_q ($\lesssim 10^{-27}\text{cm}$), 3) $r_q < (2N_B/\pi)^{-1/3}a$ due to the smallness of N_B in which case the baryon number (or quark number) must be generated at $T \lesssim 10^3\text{TeV}$ if $r_q \gtrsim 10^{-17}\text{cm}$ (where r_q and N_B are the effective radius of quarks and the baryon number in the universe), and 4) for $T \gtrsim 10^3\text{TeV}$, the universe was filled not with quark-gluon plasma but with “subquark plasma”.

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Dirac's large number hypothesis (LNH) [1] states that the Eddington large numbers [2], $N_1(\equiv \alpha/Gm_em_p \cong 10^{39})$, $N_2(\equiv m_e/\alpha H \cong 10^{40})$, and $N_3(\equiv 4\pi\rho/3m_p H^3 \cong 10^{80})$, are not independent but related with each other. Recently, we have shown that not only the “dynamical LNH relation” of $N_3 \sim N_1 N_2$ [3] but also the “geometrical LNH relation” of $N_3 \sim (N_2)^2$ [4] holds so that the LNH may not be taken as a hypothesis but become the large number rule (LNR). More recently in Ref. [5], we have proposed the “less large number hypothesis (LLNH)” of $N_G \sim N_S$ where N_G and N_S are the total number of galaxies in the universe and the average total number of stars in a galaxy as both of them are of the order of 10^{11} . We have also suggested the two “geometrical LLNH relations” of $N_G \sim (R_U/R_G)^2$ and $N_S \sim R_G/R_S$ where R_U , R_G , and R_S are the “radius of the universe”, the average radius of a galaxy and that of a star. Furthermore, we have pointed out that the “even less large number hypothesis (ELLNH)” of $n_1 \sim n_2 \sim n_3$ may also work for $n_1 \equiv R_U/D_G$, $n_2 \equiv D_G/R_G$, and $n_3 \equiv R_G/D_S$ where D_G and D_S are the average distance between two neighboring galaxies and that between two neighboring stars since all of them are of the order of 10^3 . In proposing these new large number hypotheses and in deriving these geometrical relations on substructures of the universe, we have emphasized the importance of taking into account the non-vanishing sizes of particles, stars, and galaxies in cosmology. In order to find a theory in which these hypotheses and relations can be accommodated, we must at least extend the Einstein field equations in the standard Friedmann cosmology by taking into account the non-vanishing sizes of particles, stars, and galaxies as van der Waals extended the equation of state for ideal gas (or liquid) by taking into account those of atoms and molecules. In this Letter, we shall discuss possible effects of the non-vanishing sizes of particles (atoms, nuclei, nucleons, quarks, and leptons) in the early universe (the temperature T) in an extended Friedmann model of the universe (the scale a). Especially, we shall point

out the following possibilities: 1) if $r_q > (2N_B/\pi)^{-1/3}a$, most of the proposed scenarios for $T \gtrsim 10^3 \text{TeV}$ including the inflationary universe one are unrealistic, 2) $r_q < (2N_B/\pi)^{-1/3}a$ due to the smallness of r_q ($\lesssim 10^{-27} \text{cm}$), 3) $r_q < (2N_B/\pi)^{-1/3}a$ due to the smallness of N_B in which case the baryon number (or quark number) must be generated at $T \lesssim 10^3 \text{TeV}$ if $r_q \gtrsim 10^{-17} \text{cm}$ (where r_q and N_B are the effective radius of quarks and the baryon number in the universe, respectively), and 4) for $T \gtrsim 10^3 \text{TeV}$, the universe was filled not with quark-gluon plasma but with “subquark plasma”.

Let us first sketch the familiar picture on the history of our universe in the standard big-bang cosmology [6] in a reverse chronological order. At the present moment and in the Friedmann model of the universe, the space-time is approximately homogeneous and isotropic so that it may be roughly described by the Friedmann-Robertson-Walker metric of

$$ds^2 = -dt^2 + a^2(t)[dr^2/(1 - \kappa r^2) + r^2(d\theta^2 + \sin^2 \theta d\phi^2)] \quad (1)$$

where $\kappa = 0, +1$ or -1 for the spacially flat, closed, or open universe. The time-development of the scale factor $a(t)$ is determined by the Einstein-Friedmann “initial-value equation” of

$$H(\equiv \frac{\dot{a}}{a}) = (\frac{8\pi G\rho}{3} - \frac{\kappa}{a^2})^{1/2} \quad (2)$$

and by the Einstein-Friedmann “acceleration (or dynamic) equation” of

$$q(\equiv -\frac{\ddot{a}}{a}) = \frac{1}{2} + \frac{\kappa}{2a^2H^2} + \frac{4\pi Gp}{H^2}, \quad (3)$$

where the vanishing cosmological constant is assumed, i.e. $\Lambda = 0$. Although the mass-energy density ρ and the pressure p are related with each other by the “law of conservation of energy” as

$$(\rho a^3)_{,t} = -p(a^3)_{,t}, \quad (4)$$

these three Eqs. (2), (3), and (4) are not independent of each other. For the present matter-dominated universe (where $\rho \gg p$) $\rho \propto a^{-3}$ while for an early radiation-dominated universe

(where $p \cong \rho/3$) $\rho \propto a^{-4}$ and the energy density of radiation ρ_r is related with the temperature T as

$$\rho_r = \frac{\pi^2}{30} N(T) (kT)^4 \quad (5)$$

where $N(T)$ is the total number of degrees of freedom of effectively massless particles at the radiation temperature T .

Given an appropriate set of Eqs. (2)-(5) and given the initial (or present) observed values for a , ρ , H , and q (i.e., $a = a_0$, $\rho = \rho_0$, $H = H_0$, and $q = q_0$ at $t = t_0$), the time development of these functions can be traced back from the present moment to an arbitrary past moment. In fact, given $a_0 \cong 1.3 \times 10^{10} \text{yr}$, $\rho_0 = \Omega_0 \rho_c$ with $0.1 < \Omega_0 < 2$ (where the critical density $\rho_c = 3H_0^2/8\pi G = 1.88 \times 10^{-29} h_0^2 \text{gcm}^{-3}$), $H_0 = 100 h_0 \text{kms}^{-1} \text{Mpc}^{-1}$ with $0.5 < h_0 < 0.85$, $q_0 = 1 \pm 0.5$, and $T_0 = 2.726 \pm 0.005 \text{K}$ at $t_0 \cong 1.2 \times 10^{10} \text{yr}$ [7], one can imagine, for example, that $a \sim 1 \text{yr}$ (or $a/a_0 \sim 10^{-10}$), $\rho \sim 10^5 \text{gcm}^{-3}$ (or $\rho/\rho_c \sim 10^{34}$), and $T \sim 10^{10} \text{K}$ (or $T/T_0 \sim 10^{10}$) at $t \sim 1 \text{s}$ (or $t/t_0 \sim 10^{-18}$).

In the above sketched picture on the evolution of the universe, “gravitation dominates the over-all expansion; but otherwise most details of the evolution are governed much less by gravitation than by the laws of thermodynamics, hydrodynamics, atomic physics, nuclear physics, hadron physics, and quark-lepton physics” [6]. The familiar scenario of nucleosynthesis has been extensively developed theoretically and successfully confirmed experimentally for the last more than two decades [8]. Furthermore, various scenarios of baryogenesis [9-11] and those of the inflationary universe [12,13] have been proposed and discussed in detail for the last almost two decades although not confirmed so definitely by observations. In these modern cosmological investigations, however, possible effects of the non-vanishing sizes of particles (atoms, nuclei, nucleons, quarks, and leptons) have never been taken into account seriously. Whether such effects are serious or not depends on the ratio of the total volume

of particles in the universe v ($\equiv 4\pi r^3 N/3$ where r is the effective radius of a particle and N is the total number of particles in the universe) to the volume of the universe V ($\equiv 2\pi^2 a^3$). The ratio can be easily estimated to be, for example, $v/V \sim 10^{-21}$ at $t \sim 10^5 \text{ yr}$ (when $a/a_0 \sim 10^{-3}$ and $T \sim 10^4 \text{ K}$ or 1 eV) for $N = N_B \cong 10^{80}$ and $r = a_\infty$ (the Bohr radius) $\cong 0.53 \times 10^{-8} \text{ cm}$, and $v/V \sim 10^{-15}$ at $t \sim 1 \text{ s}$ (when $a/a_0 \sim 10^{-10}$ and $T \sim 10^{10} \text{ K}$ or 1 MeV) for $N = N_B$ and $r = r_p$ (the proton charge radius) $\cong 0.81 \times 10^{-13} \text{ cm}$. Therefore, such effects were absolutely negligible at and after the period of nucleosynthesis (when $t \gtrsim 1 \text{ s}$, $a/a_0 \gtrsim 10^{-10}$, and $T \lesssim 10^{10} \text{ K}$ or 1 MeV). However, if quarks (and leptons) have non-vanishing sizes as expected in composite models of quarks and leptons [14–16], at much earlier time the universe in the supposed quark-gluon plasma state would become most packed with quarks (and leptons). In fact, suppose that the effective radius of quarks r_q is of the order of 10^{-17} cm (or $1/2 \text{ TeV}$) as suggested by the recent CDF experimental observation [17] and as anticipated rather theoretically [18] or by precise comparison between currently available experimental data and calculations in composite models of quarks (and leptons) [19]. Then, we can imagine that at $t \sim 10^{-18} \text{ s}$ (when $a/a_0 \sim 3 \times 10^{-19}$, and $T \sim 10^{19} \text{ K}$ or 10^3 TeV) v ($\equiv 4\pi r_q^3 N_B$) $\sim V$ ($\equiv 2\pi^2 a^3$) $\sim 10^{30} \text{ cm}^3$ and, therefore, $v/V \sim 1$.

Now, we can imagine the following four possibilities: 1) if $r_q > (2N_B/\pi)^{-1/3} a$, most of the proposed scenarios on the early universe for $T \gtrsim 10^3 \text{ TeV}$ including the inflationary universe one at the grand unification or even higher energy (or temperature) scale ($T > 10^{15} \text{ GeV}$) [12, 13] are unrealistic since the universe would never have experienced such high energy density (or temperature) in the phase of quark-leptons, gauge bosons and Higgs scalars. 2) $r_q < (2N_B/\pi)^{1/3} a$ always in the history of the universe due to the smallness of r_q . If this is the case, the upper bound of r_q must be of the order of 10^{-27} cm (or $1/2 \times 10^{13} \text{ GeV}$). 3) $r_q < (2N_B/\pi)^{-1/3} a$ due to the smallness of N_B . If this is the case, the baryon number

(or quark number) must be generated not at the grand unification or even higher energy (or temperature) scale ($T > 10^{15} \text{ GeV}$) [10] but at $T \lesssim 10^3 \text{ TeV}$ [11] if $r_q \gtrsim 10^{-17} \text{ cm}$. 4) For $T > 10^3 \text{ TeV}$, the universe was filled not with quark-gluon plasma but with “subquark plasma” in which quarks were dissociated into subquarks or, in other words, quarks had not yet been formed from subquarks. The last possibility of the early universe in composite models of quarks and leptons was first discussed by Chikashige and Ito in some detail already more than a decade ago [20].

In order to discuss the early universe by taking into account the non-vanishing sizes of particles in more detail, we must modify or extend every one of Eqs. (2)–(4). It is easy to extend Eq. (4), the “law of conservation of energy”, into the following form [5]:

$$[\rho_r(a^3 - b) + \rho_m a^3]_{,t} = -p(a^3)_{,t} \quad (6)$$

where ρ_m is the energy density of matter ($\rho_m = \rho - \rho_r$) and b is a constant representing the non-vanishing size of particles ($b \sim 3N_B r_q^3$ for quarks). Also, it is easy to solve this equation: for example, for a radiation-dominated universe where $\rho_m \ll \rho_r$ and $p = \rho_r/3$, $\rho_r \propto (a^3 - b)^{-4/3}$ so that the energy density of radiation may naturally blow up when $a^3 \rightarrow b$ (or $V \rightarrow v$). However, it seems difficult to extend either one of Eqs. (2) and (3), the Einstein-Friedmann field equations, so that the non-vanishing sizes of matter particles may be accommodated. The reason for this is simple: Neither Einstein’s picture of gravitation in general relativity nor Friedmann’s picture of the universe is consistent with particle physicist’s picture of particles with non-vanishing sizes. Probably, either completely particle-theoretical description of the universe in the continuous space-time or drastic modification of the space-time metric into a discontinuous one seems to be necessary. Which way to proceed is a subject for future investigations.

References

- [1] P.A.M. Dirac, *Nature* **139**, 323 (1937); *Proc. R. Soc. Lond.* **A165**, 199 (1938); **A333**, 403 (1973); **A338**, 439 (1974).
- [2] A.S. Eddington, in *Mathematical Theory of Relativity* (Cambridge Univ. Press, New York, 1924).
- [3] H. Terazawa, *Phys. Lett.* **101B**, 43 (1981).
- [4] H. Terazawa, in *Proc. 3rd Alexander Friedmann International Seminar on Gravitation and Cosmology*, St. Petersburg, 1995, edited by Yu. N. Gnedin *et al.* (Friedmann Laboratory Publishing, St. Petersburg, 1995), p. 116.
- [5] H. Terazawa, *Mod. Phys. Lett.* **A11**, 2971 (1996).
- [6] For a review, see, for example, C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (W.H. Freeman and Co., San Francisco, 1973).
- [7] For a review, see, for example, Particle Data Group, R.M. Barnett *et al.*, *Phys. Rev. D* **54**, 1 (1996).
- [8] For a review, see, for example, S. Weinberg, *Gravitation and Cosmology* (John Wiley and Sons, New York, 1972).
- [9] A.D. Sakharov, *ZhETF Pis'ma* **5**, 32 (1967) [*Sov. Phys. JETP Lett.* **5**, 24 (1967)] and *ZhETF* **76**, 1172 (1979) [*Sov. Phys. JETP* **49**, 594 (1979)]; V.A. Kuzmin, *ZhETF Pis'ma* **13**, 335 (1970).
- [10] A. Yu Ignatiev, N.V. Krasnikov, V.A. Kuzmin, and A.N. Tavkhelidze, *Phys. Lett.* **76B**, 436 (1978); M. Yoshimura, *Phys. Rev. Lett.* **41**, 281 (1978), **42**, 746(E) (1979).
- [11] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985).
- [12] A.A. Starobinsky, *JETP Lett.* **30**, 682 (1980); *Phys. Lett.* **B91**, 99 (1980); A.H. Guth, *Phys. Rev. D* **23**, 347 (1981); K. Sato, *Mon. Not. Roy. Astron. Soc.* **195**, 467 (1981); A.D. Linde, *Phys. Lett.* **108B**, 289 (1982); A. Albrecht and P.J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [13] For reviews, see, for example, A.D. Linde, *Particle Physics and Inflationary Cosmology* (Hardwood Academic, Chur, Switzerland, 1990); in *Proc. 1st International Symposium on Particles, Strings and Cosmology*, Boston, 1990, edited by P. Nath and S. Reucroft (World Scientific, Singapore, 1991), p. 635.
- [14] J.C. Pati and A. Salam, *Phys. Rev. D* **10**, 275 (1974); H. Terazawa, Y. Chikashige, and K. Akama, *Phys. Rev. D* **15**, 480 (1977); H. Terazawa, *Phys. Rev. D* **22**, 184 (1980).
- [15] For a review, see, for example, H. Terazawa, in *Proc. XXII International Conference on High Energy Physics*, Leipzig, 1984, edited by A. Meyer and E. Wieczorek (Akademie der Wissenschaften der DDR, Zeuthen, 1984), Vol. I, p. 63.
- [16] For recent reviews, see, for example, H. Terazawa, in *Proc. 4th International Conference on Squeezed States and Uncertainty Relations*, Taiyuan, Shanxi, China, 1995, edited by D. Han *et al.*, NASA Conference Publication **3322** (NASA Goddard Space Flight Center, Greenbelt, Maryland, 1996), p. 179; in *Proc. XII-th International Workshop on High Energy Physics (HADRON-96)*, Novy Svet (Crimea), 1996, edited by G. Bugrij *et al.* (Bogoliubov Institute for Theoretical Physics, Kiev, 1996), p. 109.
- [17] CDF Collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* **77**, 438 (1996); **77**, 5336 (1996).

- [18] See, for example, H. Terazawa, Prog. Theor. Phys. 79, 734 (1988); Phys. Rev. Lett. 65, 823 (1990); Mod. Phys. Lett. A6, 1825 (1991); K. Akama, H. Terazawa, and M. Yasuè, Phys. Rev. Lett. 68, 1826 (1992).
- [19] See, for example, K. Akama and H. Terazawa, Phys. Lett. B321, 145 (1994); Mod. Phys. Lett. A9, 3423 (1994); H. Terazawa, Mod. Phys. Lett. A11, 2463 (1996); K. Akama and H. Terazawa, to be published in Rapid Communications, Phys. Rev. D55, No. 5, 1 March (1997).
- [20] Y. Chikashige and I. Ito, a paper contributed to XXII International Conference on High Energy Physics, Leipzig, 1984; I. Ito, in Proc. 1985 INS International Symposium on Composite Models of Quarks and Leptons, Tokyo, 1985, edited by H. Terazawa and M. Yasuè (INS, Univ. of Tokyo, 1985), p. 436.