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**PRECISION ELECTROWEAK DATA: PRESENT
STATUS AND FUTURE PROSPECTS**

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Precision electroweak data: present status and future prospects.

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Abstract

The present status of precision electroweak data is reviewed. These data include LEP measurements of the mass and width of the Z, together with various measurements on the Z-fermion couplings. These data are compared to, and combined with, data from the SLC on the left-right polarised asymmetry, A_{LR} , and the left-right forward-backward asymmetries for b and c quarks.

These measurements are combined with hadron collider measurements from the Tevatron and CERN on the mass of the W boson, m_W , as well as other electroweak data, in global electroweak fits in which various Standard Model parameters are determined. A comparison is made between the results of direct measurements of m_W and the top-quark mass, m_t , as determined from the Tevatron, with the indirect results coming from electroweak radiative corrections. Using all precision electroweak data, fits are also made to determine limits on the mass of the Higgs boson, m_H . The influence on these limits of specific measurements, particularly those which are somewhat inconsistent with the Standard Model, is explored.

The data are also analysed in terms of the quasi model independent ϵ variables.

Improvements in the determination of all of these quantities are expected when the Z data at LEP are fully analysed, and further measurements on A_{LR} and related asymmetries performed at the SLC. In addition, substantial improvements in the determination of m_W are expected from measurements at the Tevatron and in the second phase of LEP. An estimate is made of the likely precision of these data, and the implications of the impact of these data on precision electroweak tests are discussed. This discussion is made both in terms of the Standard Model and also in the context of the quasi model independent ϵ variables.

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1 Introduction

Although it is widely accepted that the Standard Model (SM) is incomplete, the many thousands of man-years of experimental effort devoted to testing the SM have so far failed to find any significant and sustainable deviations from its predictions. It is thus of interest to examine the extent to which the SM has been currently tested, using precision electroweak data, and the precision of tests which are likely to be carried out in the next few years. In particular, it is important to compare the direct measurements of both the top-quark mass m_t and the W-boson mass m_W with the values obtained by indirect means using fits to the SM. In addition to a discussion of the data within the context of the SM it is also important to examine the current and expected precision in terms of variables which are quasi model independent.

In this paper the present precision electroweak data are reviewed. A brief review is given in section 2 of the salient points of the SM, including a discussion of the status of the measurements of those quantities which are generally used to formulate the SM predictions. The data used to test the SM are as follows :-

- Measurements of the mass m_W and width Γ_W of the W-boson (section 3).
- Measurements of the mass m_Z and width Γ_Z of the Z-boson, as well as of Z-fermion couplings, including measurement of the left-right polarised asymmetry A_{LR} (section 4).
- Measurements of the Z couplings to heavy-quarks (section 5).

The values of many of the current measurements are taken from [1, 2], and references therein. It should be stressed that many of these measurements are still preliminary, so that some caution must be exercised in drawing conclusions based on these data.

The results of fits to these data are discussed, both in terms of the Standard Model (section 6) and to quasi model independent variables (section 7). In section 8 some estimates are given of the likely future precision on these quantities, and also the improvements expected in both the SM and model independent fits.

2 The Standard Model

The SM is specified by the $SU(3)_{col} \otimes SU(2)_L \otimes U(1)_Y$ coupling constants α_s , g and g' , together with the fermion parameters $\{m_f\}$ and those of the Higgs boson. In practice, the predictions are made in terms of the three best determined

experimental quantities, namely G_μ^1 , m_Z and $\alpha(m_Z)$, the value of the QED coupling at $\sqrt{s} = m_Z$, together with $\alpha_s(m_Z)$ and the parameters m_t and m_H . Of course, genuine tests of the SM itself will become more meaningful when the Higgs mass and all parameters are precisely known.

2.1 The running of α

The fine structure constant is known at $q^2 \simeq 0$ with impressive precision ($4 \cdot 10^{-8}$); however, what is important for the interpretation of heavy gauge boson results is $\alpha(m_Z)$. Here the precision is $\simeq 8 \cdot 10^{-4}$. The running of α is given by

$$\alpha(m_Z) = \frac{\alpha(0)}{1 - \Delta\alpha(m_Z)}, \quad (1)$$

where $\Delta\alpha(m_Z) = \Delta\alpha_{lept} + \Delta\alpha_{had} = -\Pi_{\gamma\gamma}(s)$, with $\Pi_{\gamma\gamma}$ the photon self-energy. The leptonic part $\Delta\alpha_{lept}$ can be calculated analytically, and is well known. The hadronic part $\Delta\alpha_{had}$ cannot be calculated entirely from QCD because of ambiguities in defining the light quark masses m_u and m_d and also the inherent non-perturbative nature of the problem at small energy scales.

Instead, use is made of the data on

$$R_{had}(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (2)$$

from which one can compute

$$\text{Re}\Pi_{\gamma\gamma}(s) = \frac{\alpha s}{3\pi} P \int \frac{R_{had}(s')}{s'(s' - s)} ds' . \quad (3)$$

Most of the sensitivity is to R_{had} at low values of s . In practice, there are difficulties in evaluating the integral :-

- a) from the resonant structure in the data ($\rho, \omega, J/\psi, \Upsilon$)
- b) some data are not very accurate (eg. large systematic errors) and old (not always enough information is given)
- c) somewhat arbitrary choices need to be made in the use of the data (eg. the form of local parameterisation, interpolation or fit, how to deal with inconsistent data and how to cross thresholds etc.)

Fig. 1 shows a summary of the determinations of $\Delta\alpha_{had}$. In reference [3] it was shown that compatible results could be obtained from different methods.

¹ G_μ is determined directly from the muon lifetime τ_μ , which is known to a precision of $0.9 \cdot 10^{-5}$. However, G_μ is determined only to a precision of $1.7 \cdot 10^{-5}$, the dominant error being from second order QED corrections (Particle Data Group, Phys. ReV. D54 (1996)).

Figure 1: Determinations of $\Delta\alpha_{had}(m_Z)$.

The results of references [4–6] rely significantly more on QCD than other estimates. The value from reference [7], $\Delta\alpha_{had} = 0.02752 \pm 0.00046$, which is obtained from fitting the data, is a revised value. The original value was considerably lower than those of the two most recent evaluations [8, 9], which also rely rather directly on the data. The change is due mainly to a correction of a bias in the fit procedure and to the introduction of Crystal Ball data, just below the charm threshold.

In comparing the results in fig. 1 it should be borne in mind that the data used for these evaluations are largely the same. The determinations which rely heavily on QCD need to be treated with some caution as they are not entirely independent of the data, since QCD did not predict all the observed structure! It is proposed to use the value of the two most recent evaluations [8, 9], namely

$$\alpha^{-1}(m_Z) = 128.89 \pm 0.09. \quad (4)$$

Although the largest component of the integral ($\sim 45\%$) is from the “continuum” region $\sqrt{s} > 12$ GeV, by far the largest uncertainty comes from the low s region. More than 75% of the total error is from the region $1 \leq \sqrt{s} \leq 5$ GeV. As discussed in section 6, the error on $\alpha(m_Z)$ is significant in the interpretation of electroweak fits. It is important that this error is reduced by making further precise measurements of R_{had} in this low energy range. Further and improved measurements at DaΦne, Novosibirsk and Beijing would thus help considerably.

2.2 The top quark

The discovery of the top quark at the Fermilab Tevatron, by the CDF [11] and D0 Collaborations [12], is clearly of fundamental importance in the field of electroweak physics. The parameter which is most important in the context of this review is the mass of the top-quark, m_t . Since the top-quark is so massive it decays very quickly on the time-scale of the hadronisation processes. Hence the interpretation of the top-quark mass is more straightforward than that of the lighter quark masses. A combination of the values given in the original publications gives $m_t = 180 \pm 12$ GeV.

Additional, but still preliminary, data have now been analysed and the combined value is now [13–15]

$$m_t = 175 \pm 6 \text{ GeV.} \quad (5)$$

2.3 The strong coupling constant $\alpha_s(m_Z)$

The strong coupling constant $\alpha_s(m_Z)$ has been determined in a variety of processes and at different effective energy scales. Each of the determinations suffers from somewhat different problems; however, the most significant is the extent to which the neglect of higher order processes is justified. The neglected effects can arise from either higher order perturbative corrections or the non-perturbative higher twist contributions. In addition, for those determinations which rely on the properties of the distributions of final state hadrons, there are effects from the hadronisation process, for which the corrections must be modelled. For the most precise determinations of $\alpha_s(m_Z)$, the theoretical uncertainties constitute a significant component of the error.

Recently [16] the results from various determinations have shown some convergence. There has been a reanalysis of the CCFR determination of $\alpha_s(m_Z)$ from a study of scaling violations of structure functions in neutrino-nucleon scattering. At the Warsaw conference a detailed review of the results was made [16] and a new global average determined of

$$\alpha_s(m_Z) = 0.118 \pm 0.003 \text{ .} \quad (6)$$

This value is also compatible with the latest lattice QCD calculations. However, given the significant changes in the results in recent years it would perhaps be preferable to assign a somewhat more conservative error to this global average.

3 Measurement of m_W and Γ_W

Precise measurements of the masses of the heavy gauge W and Z bosons are of fundamental physical importance. So far, the precise measurements of m_W have been from hadron colliders at CERN [10] ($\sqrt{s} = 630$ GeV) and at the Fermilab Tevatron [17–19] ($\sqrt{s} = 1.8$ TeV).

The CDF Collaboration have published final results [18] on both the $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ channels, based on 19 pb^{-1} of data taken at the Tevatron in 1992/3 (Run 1a). The analysis uses variables defined in the plane perpendicular to the incident beams. A high $\mathbf{p}_T^l$ lepton ($E_T^l > 25$ GeV) is required. In addition, the transverse momentum $\mathbf{u}$ of the recoil against the W must be well determined, with $|\mathbf{u}| < 20$ GeV. The calibration of $\mathbf{u}$ is made on $Z \rightarrow e^+e^-$ decays. The missing neutrino vector $\mathbf{p}_T^\nu = -(\mathbf{p}_T^l + \mathbf{u})$ can thus be reconstructed and $E_T^\nu > 25$ GeV is required. The value of m_W is determined from a fit to the W transverse mass M_T^W , where

$$(M_T^W)^2 = 2p_T^l p_T^\nu (1 - \cos\phi_{l\nu}) . \quad (7)$$

In addition, the backgrounds from $W \rightarrow \tau\nu$, $Z \rightarrow \ell\ell$ etc must be minimised and care taken in the event selection that no bias is introduced.

The momentum scale is calibrated from J/ψ decays and cross-checked on Υ and Z decays, which are also used to determine the resolution. The energy scale for electrons is determined from a detailed study of the electron sample, plus use of the momentum scale.

The results from the 5718 and 3268 events in the selected $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ samples are $m_W(e\nu) = 80.490 \pm 0.145$ (stat) ± 0.175 (syst) GeV and $m_W(\mu\nu) = 80.310 \pm 0.205$ (stat) ± 0.130 (syst) GeV. Combining these, taking into account the common systematic error (0.100 GeV), gives

$$m_W = 80.410 \pm 0.180 \text{ GeV}. \quad (8)$$

The D0 Collaboration have preliminary results using the $W \rightarrow e\nu$ channel, based on $\simeq 13 \text{ pb}^{-1}$ from Run 1a and $\simeq 76 \text{ pb}^{-1}$ from the 1994-6 Run 1b. The results are [19]

$$m_W(\text{Run1a}) = 80.35 \pm 0.14(\text{stat}) \pm 0.17(\text{syst}) \pm 0.16(\text{scale}) \text{ GeV} \quad (9)$$

$$m_W(\text{Run1b}) = 80.38 \pm 0.07(\text{stat}) \pm 0.13(\text{syst}) \pm 0.08(\text{scale}) \text{ GeV}. \quad (10)$$

The energy scale is determined from $Z \rightarrow e^+e^-$ decays and also from π^0 and J/ψ decays. These m_W values have been combined, taking into account common systematic errors, giving

$$m_W = 80.37 \pm 0.15 \text{ GeV}. \quad (11)$$

Figure 2: Direct measurements of m_W with their average. The dashed lines correspond to indirect limits ($\pm 1 \sigma$) from a global fit to the Standard Model.

Fig. 2 shows a summary of the direct measurements of m_W , together with the indirect SM value from a fit to LEP and SLD data (see section 6). In determining the world average value of the direct measurements of m_W the common systematic error of 85 MeV, due to uncertainties in the structure functions, is taken into account, giving

$$m_W = 80.356 \pm 0.125 \text{ GeV.} \quad (12)$$

The traditional, but indirect, way of measuring the total width Γ_W is from the measurement of the ratio of the production of $W \rightarrow \ell \nu$ to $Z \rightarrow \ell \ell$, namely

$$R = \frac{\sigma_W}{\sigma_Z} \cdot \frac{B(W \rightarrow \ell \nu)}{B(Z \rightarrow \ell \ell)} = \frac{\sigma_W}{\sigma_Z} \cdot \frac{\Gamma(W \rightarrow \ell \nu)}{\Gamma_W} \cdot \frac{\Gamma_Z}{\Gamma(Z \rightarrow \ell \ell)}. \quad (13)$$

To extract Γ_W requires input of $B(Z \rightarrow \ell \ell)$ and m_Z from LEP, σ_W/σ_Z from structure functions and the SM and also m_W . The branching ratio $B(W \rightarrow \ell \nu)$ can then be extracted. If, in addition, the SM value of the $W \rightarrow \ell \nu$ partial width is assumed, then a measurement of R gives a value of Γ_W . However, it should be stressed that the SM couplings of the W are invoked both in its production (σ_W/σ_Z depends on $u\bar{d}$ etc) and in its decay.

The recent indirect results are $\Gamma_W = 2.064 \pm 0.085 \text{ GeV}$ from CDF [20] ($W \rightarrow e \nu$ only) and $\Gamma_W = 2.044 \pm 0.093 \text{ GeV}$ from D0 [21] (e and μ). Combining

these results with earlier UA1 [22] and UA2 data [23] gives²

$$\Gamma_W(\text{indirect}) = 2.062 \pm 0.059 \text{ GeV.} \quad (14)$$

Figure 3: Measurements of Γ_W by the indirect and direct methods. The dashed lines give the range expected from the Standard Model.

Recently, a direct measurement of Γ_W , essentially free of SM assumptions, has been performed by the CDF Collaboration [24], from a study of the high energy tail of the M_T^W distribution ($M_T^W > 110$ GeV), giving

$$\Gamma_W(\text{direct}) = 2.11 \pm 0.32 \text{ GeV.} \quad (15)$$

The direct and indirect values are shown in fig. 3 and are compatible with each other and with the SM value of $\Gamma_W^{SM} = 2.077 \pm 0.014$ GeV, leaving room for a possible extra component from new physics of $\delta\Gamma_W^{NEW} < 109$ MeV, at the 95% c.l.

²From [21], where the UA1, UA2 and CDF values are updated, using recent and consistent values for $B(Z \rightarrow \ell^+ \ell^-)$ and $\Gamma(W \rightarrow \ell \nu)$ etc.

4 Measurement of the Z parameters

4.1 Z boson variables

The $Zf\bar{f}$ vertex can be described by effective vector (v_f) and axial-vector (a_f) couplings such that, to a very good approximation, the Born level formulae are retained. It is useful to define the *polarisation parameter*

$$A_f = \frac{2v_f a_f}{(v_f^2 + a_f^2)} . \quad (16)$$

The cross section for $e^+e^- \rightarrow f\bar{f}$, close to the Z pole, may be written in terms of the Z mass m_Z and total width Γ_Z as

$$\sigma_f(s) = \sigma_f^0 \frac{s\Gamma_Z^2}{(s - m_Z^2)^2 + s^2\Gamma_Z^2/m_Z^2} + \gamma + (\gamma/Z)_{interf.} , \quad (17)$$

where the *pole cross section* σ_f^0 is defined as

$$\sigma_f^0 = \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2} . \quad (18)$$

Here, Γ_f is the partial width for $Z \rightarrow f\bar{f}$, which in turn can be written as ³

$$\Gamma_f = \frac{G_F m_Z^3}{6\pi\sqrt{2}} (v_f^2 + a_f^2) f_{QCD} f_{QED} . \quad (19)$$

The final-state QED correction factor is $f_{QED} = 1 + 3\alpha Q_f^2/4\pi$, whereas f_{QCD} is unity for leptons and $f_{QCD} = 3(1 + c_q\alpha_s(m_Z)/\pi + \dots)$ for quarks, with $c_q \simeq 1$.

The formula for the cross-section $\sigma_f(s)$ is an effective Born level formula and must be convoluted with QED radiative corrections before comparing with experiment. The cross sections which are measured are for $e^+e^- \rightarrow \text{hadrons}$ and $e^+e^- \rightarrow \ell^+\ell^-$, for $\ell = e, \mu, \tau$. The hadronic width Γ_{had} is the sum of Γ_q for $q=u, d, s, c, b$. The total Z width, assuming lepton universality, is $\Gamma_Z = \Gamma_{\text{had}} + 3\Gamma_l + \Gamma_{\text{inv}}$, where $\Gamma_{\text{inv}} = 3\Gamma_\nu$ in the SM. The results below are expressed in terms of the ratio of the hadronic to leptonic partial widths R_ℓ ($\ell = e, \mu, \tau$), where

$$R_\ell = \frac{\Gamma_{\text{had}}}{\Gamma_\ell} . \quad (20)$$

The distribution of the angle θ of the outgoing fermion f , with respect to the incident e^- direction, is given at Born level by

$$\frac{d\sigma}{d\cos\theta} = 1 + \cos^2\theta + \frac{8}{3}A_{\text{FB}}\cos\theta . \quad (21)$$

³All the formulae here are for $m_f=0$. In practice, finite mass terms are taken into account.

The *forward-backward asymmetry* is defined as $A_{\text{FB}}^f = (\sigma_F - \sigma_B)/(\sigma_F + \sigma_B)$, where σ_F (σ_B) are the cross sections in the forward (backward) directions. The *pole asymmetry* is defined as

$$A_{\text{FB}}^{0,f} = \frac{3}{4} A_e A_f . \quad (22)$$

Since A_f depends on the ratio v_f/a_f , a measurement of $A_{\text{FB}}^{0,f}$ depends on both v_e/a_e and v_f/a_f . The effective couplings can also be written as

$$a_f = I_f^3 \sqrt{\rho_f} , \quad \frac{v_f}{a_f} = 1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f , \quad (23)$$

where the mixing angle defined for *leptons* ($\sin^2 \theta_{\text{eff}}^{\text{lept}}$) is used for reference. Those defined for quarks have small shifts, due to SM plus any new physics [25].

4.2 LEP lineshape scans

Results, which are still preliminary, are now available from both the 1993 and 1995 LEP lineshape scans. Both these scans consisted of taking data at two off-peak energies, situated about 1.8 GeV below and above the Z peak, and referred to as peak-2 and peak+2 respectively. In both scans the total off-peak luminosity was about 20 pb^{-1} , significantly higher than in the 1991 and earlier scans. Good accuracy on E_{LEP} is crucial in the determination of m_Z and Γ_Z . A precise energy calibration measurement ($E_{\text{LEP}}^{\text{pol}}$, with $\delta E_{\text{cms}} \leq 0.8 \text{ MeV}$) of the average circulating beam energy can be made using the technique of *resonant depolarisation*. Most of the energy calibration measurements were performed after physics data taking at the end of fills. For the 1993 scan, about one-third of the off-peak fills (representing about 40% of the recorded off-peak luminosity) were calibrated. For the 1995 scan about 70% of the recorded off-peak luminosity was calibrated.

Although resonant depolarisation gives a very precise energy value at the time of measurement, the LEP energy varies with time, due to the Earth tides and other effects such as the temperature of the dipole magnets. Since the RF frequency, and thus the orbit length, is fixed, stresses in the local rock structure result in changes to the position of the beams in the quadrupole magnets. This changes the beam energy, since the effective dipole field changes. These energy changes can be tracked accurately using measurements of the horizontal beam orbit positions, x_{orb} . For the 1993 scan a *model* for E_{LEP} was developed, based on x_{orb} , together with correction terms from the magnetic dipole fields and temperatures, the RF cavity voltages as well as other factors [26].

In the 1993 analysis a term was included for an increase in the energy during a fill, based on observations of an NMR device in a reference dipole magnet (outside the LEP ring), which showed an upward drift in the field together with occasional jumps of a few MeV. To understand better these effects in

1995, two NMR devices were inserted in dipole magnets in the LEP ring and, furthermore, in four physics fills energy calibration measurements were made at both the beginning and end of fills to measure the energy change. The NMR devices showed a significant amount of noise, which was largely anti-correlated in the two devices, and which was much smaller between midnight and 5am. The main cause of this noise was traced to local electric trains, the earth currents from which travelled along the LEP beam pipe. When averaged over several fills these NMR devices showed an increase in energy during a fill of typically 5 MeV (much larger than the rise estimated in [26]). The size of the rise was confirmed using the beginning and end energy calibrations, as well as studies of fills in machine development periods in 1993 and 1995 in which energy calibration measurements were made over periods of several hours.

An improved model, including this rise term, has been developed [27], both for the 1995 scan and also for a revised analysis of the 1993 scan and the 1994 peak data. For each energy and year a single normalisation parameter was used and the rms values of $E_{LEP}^{model} - E_{LEP}^{pol}$ were determined. These rms values (typically a few MeV) are of importance in determining the energy error on those fills with no polarisation measurement, since it is assumed that these follow the same distribution. The error for the 1995 scan from this effect is smaller than for 1993 since a much higher fraction of the luminosity was in calibrated fills.

In the 1995 scan there were additional potential systematic effects from the opposite sign vertical dispersion of the beams at the interaction points. This was a feature of the new bunch-train mode of operation. The error from this effect was rendered small by carrying out a series of measurements in which one beam was scanned across the other, and the luminosity recorded, to minimise the vertical offset of the beams.

A preliminary analysis of the energy values and errors for the years 1993 to 1995 has been performed, including estimates of the correlations both between energy points and between years [27]. The correlations between off-peak energy points are important because error components which are highly correlated do not contribute significantly to $\delta\Gamma_Z$. The contribution of the LEP energy uncertainties to the errors on m_Z and Γ_Z are $\delta m_Z(LEP) \simeq \pm 1.5$ MeV and $\delta\Gamma_Z(LEP) \simeq \pm 1.7$ MeV.

The uncertainty of the LEP cms energy spread ($\simeq 55 \pm 1$ MeV) also gives rise to an error on Γ_Z , amounting to ± 0.2 MeV; significantly improved with respect to [26].

4.3 LEP data on cross sections and lepton asymmetries

Data at, or close to, the Z peak have been collected since the start-up of LEP in 1989 until 1995. The total number of Z decays analysed so far by the four

experiments is 14.4 million hadronic and 1.6 million leptonic events. The results from data up to 1992 have been published, whereas those from the 1993 and 1995 scans, and from the $\simeq 60 \text{ pb}^{-1}$ on-peak data in 1994, are preliminary.

To match these impressive statistics the systematic errors need to be well understood; this is indeed the case. The experimental error on the *luminosity*, which is determined from the $e^+e^- \rightarrow e^+e^-$ cross section at small angles, is now determined to better than 0.1% by the four experiments. This requires knowledge of both the absolute and relative positions of the detectors at the 10-20 μm level; an impressive achievement.

The event selection efficiency for $\sigma(e^+e^- \rightarrow \text{hadrons})$ is known at a precision of $\sim 0.1\%$. The lepton cross section efficiencies are less well determined, and the systematic errors on these measurements are roughly comparable to the statistical errors. The errors on A_{FB}^ℓ are mainly statistical in nature.

The theoretical error on the luminosity has improved significantly since the 1994 ICHEP in Glasgow, when the error was $\delta\mathcal{L}/\mathcal{L}(\text{theory}) = 0.25\%$. New calculations, using BHLUMI V4 [28], now include $\mathcal{O}(\alpha^2 L^2)$ terms (where L denotes the leading log term), as well as improved treatment of the γ -Z interference contributions. The error is now $\delta\mathcal{L}/\mathcal{L}(\text{theory}) = 0.11\%$. Note, however, that this error is common between the LEP experiments, and is much larger than that from the combined experimental component, which is about 0.05%. It is anticipated that the theory error will be further reduced in the future to 0.06%.

quantity	value	error	m_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{\text{FB}}^{0,e}$	$A_{\text{FB}}^{0,\mu}$	$A_{\text{FB}}^{0,\tau}$
$m_Z(\text{GeV})$	91.1863	0.0020	1.00	0.09	-0.01	0.01	-0.02	-0.01	0.02	0.06	0.04
$\Gamma_Z(\text{GeV})$	2.4946	0.0027		1.00	-0.14	0.00	-0.01	0.00	0.00	0.00	0.00
$\sigma_h^0(\text{nb})$	41.508	0.056			1.00	0.07	0.12	0.08	0.01	0.00	0.00
R_e	20.754	0.057				1.00	0.05	0.04	-0.01	0.00	0.00
R_μ	20.796	0.040					1.00	0.05	-0.01	0.01	0.00
R_τ	20.814	0.055						1.00	0.00	0.00	0.01
$A_{\text{FB}}^{0,e}$	0.0160	0.0024							1.00	0.01	0.01
$A_{\text{FB}}^{0,\mu}$	0.0162	0.0013								1.00	0.01
$A_{\text{FB}}^{0,\tau}$	0.0201	0.0018									1.00

Table 1: Results and correlation matrix of the 9 parameter fit to the LEP data. The χ^2/df of the average is 22/27, a probability of 74%.

For the purposes of combining the LEP data each experiment provides the results of a fit to their cross section and asymmetry data in terms of 9 variables. These are chosen to have small experimental correlations and are m_Z , Γ_Z , σ_h^0 , R_ℓ and $A_{\text{FB}}^{0,\ell}$ ($\ell = e, \mu, \tau$). The common errors from the LEP energy scale and energy spread, as well as that from the theoretical error on $\delta\mathcal{L}/\mathcal{L}$, are taken into account. The results of a 9 parameter fit to the combined LEP data are given in table 1. If *lepton universality* is imposed (evidence for this is discussed below), then there are 5 variables. The results of the fit to the combined LEP

quantity	value	error	m_Z	Γ_Z	σ_h^0	R_ℓ	$A_{\text{FB}}^{0,\ell}$
$m_Z(\text{GeV})$	91.1863	0.0020	1.00	0.09	-0.01	-0.01	0.08
$\Gamma_Z(\text{GeV})$	2.4946	0.0027		1.00	-0.14	-0.01	0.00
$\sigma_h^0(\text{nb})$	41.508	0.056			1.00	0.15	0.01
R_ℓ	20.778	0.029				1.00	0.01
$A_{\text{FB}}^{0,\ell}$	0.0174	0.0010					1.00

Table 2: Results and correlation matrix of the 5 parameter fit to the LEP data. The χ^2/df of the average is 38/31, a probability of 18%.

Without Lepton Universality		With Lepton Universality	
$\Gamma_e(\text{MeV})$	83.96 ± 0.15	$\Gamma_l(\text{MeV})$	83.91 ± 0.11
$\Gamma_\mu(\text{MeV})$	83.79 ± 0.22	$\Gamma_{\text{had}}(\text{MeV})$	1743.6 ± 2.5
$\Gamma_\tau(\text{MeV})$	83.72 ± 0.26	$\Gamma_{\text{inv}}(\text{MeV})$	499.5 ± 2.0

Table 3: Quantities derived from the 9 and 5 parameter fits.

data are given in table 2. It can be seen from tables 1 and 2 that the Z mass and width are determined to $\delta m_Z = 2.0 \text{ MeV}$ and $\delta \Gamma_Z = 2.7 \text{ MeV}$ respectively. These are impressive accuracies, and it is anticipated that the final errors will be further reduced.

Other quantities can be derived from these 9 or 5 parameter fits. Some of these are given in table 3. In addition, $\Gamma_{\text{inv}}/\Gamma_l = 5.952 \pm 0.023$ can be extracted, and when combined with the SM ratio $\Gamma_\nu/\Gamma_l = 1.991 \pm 0.001$, this gives the number of light neutrinos $N_\nu = 2.989 \pm 0.012$.

4.4 τ polarisation

The τ polarisation, $\mathcal{P}_\tau$, is determined from studies of the decay distributions of the τ leptons produced in Z decays. It is defined as:

$$\mathcal{P}_\tau \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \quad (24)$$

where σ_R and σ_L are the τ -pair cross sections for the production of a right-handed and left-handed τ^- , respectively.

The angular distribution of $\mathcal{P}_\tau$ as a function of the angle θ between the e^- and the τ^- , for $\sqrt{s} = m_Z$, is given by:

$$\mathcal{P}_\tau(\cos \theta) = -\frac{\mathcal{A}_\tau(1 + \cos^2 \theta) + 2\mathcal{A}_e \cos \theta}{1 + \cos^2 \theta + 2\mathcal{A}_\tau \mathcal{A}_e \cos \theta}, \quad (25)$$

with $\mathcal{A}_e$ and $\mathcal{A}_\tau$ defined in equation (16). In equation (25) the small corrections for the effects of photon exchange and the γZ interference and electromagnetic

radiative corrections for initial and final state radiation are neglected. All of these effects are taken into account in the experimental analyses.

When averaged over all production angles $\mathcal{P}_\tau$ gives a measurement of $\mathcal{A}_\tau$. Measurements of $\mathcal{P}_\tau(\cos\theta)$ provide nearly independent determinations of both $\mathcal{A}_\tau$ and $\mathcal{A}_e$, thus allowing a test of the universality of the couplings of the Z to ee and $\tau\tau$.

ALEPH	('90 - '92), final	$0.136 \pm 0.012 \pm 0.009$
DELPHI	('90 - '94), prel.	$0.138 \pm 0.009 \pm 0.008$
L3	('90 - '94), prel.	$0.152 \pm 0.010 \pm 0.009$
OPAL	('90 - '94), final	$0.134 \pm 0.009 \pm 0.010$
LEP Average		0.1401 ± 0.0067

Table 4: LEP results for $\mathcal{A}_\tau$. The χ^2/df for the average is 1.1/3. The systematic component of the error, obtained by combining the individual systematic errors, is ± 0.0045 .

ALEPH	('90 - '92), final	$0.129 \pm 0.016 \pm 0.005$
DELPHI	('90 - '94), prel.	$0.140 \pm 0.013 \pm 0.003$
L3	('90 - '94), prel.	$0.156 \pm 0.016 \pm 0.005$
OPAL	('90 - '94), final	$0.129 \pm 0.014 \pm 0.005$
LEP Average		0.1382 ± 0.0076

Table 5: LEP results for $\mathcal{A}_e$. The χ^2/df for the average is 1.8/3. The systematic component of the error, obtained by combining the individual systematic errors, is ± 0.0021 .

Each of the LEP experiments has made separate $\mathcal{P}_\tau$ measurements using the five τ decay modes $e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$, $\pi\nu$, $\rho\nu$ and $a_1\nu$. The $\rho\nu$ and $\pi\nu$ are the most sensitive channels, contributing weights of about 40% each in the average. In addition, DELPHI have used an inclusive hadronic analysis. The LEP combination is made on the results from each experiment already averaged over the τ decay modes.

Tables 4 and 5 show the results for $\mathcal{A}_\tau$ and $\mathcal{A}_e$ obtained by the four experiments and their combination. No common systematics are included in these averages. For further discussion, see [29]. The statistical correlation between the extracted values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$ is small ($\leq 5\%$).

The current LEP average values are

$$\mathcal{A}_e = 0.1382 \pm 0.0076 \quad \mathcal{A}_\tau = 0.1401 \pm 0.0067 \quad . \quad (26)$$

These values are compatible and, assuming lepton universality, can be combined to give

$$\mathcal{A}_\ell = 0.1393 \pm 0.0050 \quad . \quad (27)$$

4.5 Measurement of A_{LR}

The high values of longitudinal polarisation ($P_e \simeq 80\%$) achieved at the SLC have allowed the SLD experiment to make an extremely precise, but still preliminary, measurement of

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e, \quad (28)$$

where $\sigma_L(\sigma_R)$ is the total cross-section for a left (right) handed polarised incident electron beam. Combining all the data from 1992-5 gives [30]

$$A_e = 0.1542 \pm 0.0037, \quad \sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23061 \pm 0.00047. \quad (29)$$

This value is compatible with the less precise value of A_e from τ -polarisation only at the 1.9σ level, and at the 2.4σ level compared to the value of A_ℓ from τ -polarisation if lepton-universality is assumed. However, the A_{LR} result is compatible with the value $A_e = 0.1523 \pm 0.0044$ from $A_{\text{FB}}^{0,\ell}$ (assuming lepton universality) at the 0.3σ level.

4.6 Lepton universality

The data from the leptonic partial decay widths, forward-backward asymmetries, τ -polarisation (A_τ and A_e) and A_{LR} have been used to fit to v_ℓ and a_ℓ ($\ell=e, \mu, \tau$) and thus to test *lepton universality*. The results are shown in table 6 and fig. 4, where it can be seen that the data are consistent with the universality hypothesis. The signs are plotted taking $a_e < 0$. Using this convention (this is justified from ν -electron scattering results [31]), the signs of all couplings are uniquely determined from LEP data alone.

4.7 γ -Z interference term

In the fits described above the γ -Z interference term for the hadronic cross-section is fixed to its SM value ($j_{\text{had}}^{\text{tot}} = 0.22 \pm 0.03$)⁴. The S-matrix formalism allows a convenient way to introduce such terms in a model independent way. The sensitivity to $j_{\text{had}}^{\text{tot}}$ increases significantly if data above or below the Z pole are included. Data from the LEP runs between $\sqrt{s} = 130$ and 140 GeV are now included in these fits. If the term $j_{\text{had}}^{\text{tot}}$ is left free in the fits, then the error on m_Z increases significantly above 2.0 MeV, and is very strongly correlated to $j_{\text{had}}^{\text{tot}}$ (correlation coefficient $\simeq -85\%$). The error on Γ_Z is essentially unchanged. A fit to the combined LEP data gives

$$\delta m_Z = \pm 4.0 \text{ MeV}, \quad j_{\text{had}}^{\text{tot}} = -0.21 \pm 0.20. \quad (30)$$

⁴The notation here is that of the S-matrix formalism [32].

	Without Lepton Universality:	
	LEP	LEP+SLD
v_e	-0.0368 ± 0.0015	-0.03828 ± 0.00079
v_μ	-0.0372 ± 0.0034	-0.0358 ± 0.0030
v_τ	-0.0369 ± 0.0016	-0.0367 ± 0.0016
a_e	-0.50130 ± 0.00046	-0.50119 ± 0.00045
a_μ	-0.50076 ± 0.00069	-0.50086 ± 0.00068
a_τ	-0.50116 ± 0.00079	-0.50117 ± 0.00079
	With Lepton Universality:	
	LEP	LEP+SLD
v_ℓ	-0.03688 ± 0.00085	-0.03776 ± 0.00062
a_ℓ	-0.50115 ± 0.00034	-0.50108 ± 0.00034
$v_\nu = a_\nu$	$+0.5009 \pm 0.0010$	$+0.5009 \pm 0.0010$

Table 6: Values of the lepton vector and axial-vector couplings from LEP data alone and with the addition of the SLD measurement of A_{LR} , without and with the assumption of lepton universality.

The value of $j_{\text{had}}^{\text{tot}}$ is about 2.2σ from the SM prediction. The errors on both these terms can be reduced by the inclusion of TOPAZ data from TRISTAN [33] at $\sqrt{s} = 57.8$ GeV, giving

$$\delta m_Z = \pm 3.5 \text{ MeV}, \quad j_{\text{had}}^{\text{tot}} = -0.07 \pm 0.16 . \quad (31)$$

The additional physics which is perhaps most likely to influence these parameters is from a possible Z' boson. However, taking into account the present experimental limits on the usual Z' models (η, ψ , L-R etc), the expected contributions to $j_{\text{had}}^{\text{tot}}$ are rather small compared to the SM value. Furthermore, if such a Z' boson were to be discovered in the future, the corrections to m_Z and Γ_Z could be computed. It thus appears reasonable to use the errors on m_Z and Γ_Z obtained with $j_{\text{had}}^{\text{tot}}$ fixed to its SM value (table 2).

5 Heavy flavour measurements at LEP and SLC

It is of intrinsic interest to extract the Z couplings to individual quark flavours, in contrast to the results described in section 4 which are summed over 5 flavours. In addition, the quantity

$$R_b^0 = \frac{\Gamma_b}{\Gamma_{\text{had}}} \quad (32)$$

is of particular interest⁵. The propagator effects for the t -quark and Higgs, as well as QCD effects, largely cancel in this ratio. However, there are significant

⁵The symbols R_b^0 and R_c^0 denote specifically the ratios of Z partial widths.

Figure 4: The 68% probability contours in the vector axial-vector plane from LEP measurements. The solid contour results from a fit assuming lepton universality. Also shown is the one standard deviation band resulting from the A_{LR} measurement of SLD. The lines with arrows correspond to the SM prediction for $m_t = 175 \pm 6$ GeV and $60 \leq m_H [\text{GeV}] \leq 1000$ and point in the direction of increasing m_t and m_H .

SM vertex corrections from tWb couplings. These are essentially independent of m_H and lead to a decrease of Γ_b with increasing m_t , rather than an increase as for the other quark partial-widths. Furthermore, R_b^0 is sensitive to physics beyond the SM (eg. from light $\tilde{t}, \tilde{\chi}$).

Extracting relatively pure samples of events corresponding to individual quark flavours is far from easy. Measurements exist for both c and b quarks, which can be separated from light (u,d,s) quarks, and from each other, using their characteristic properties (see table 7).

The measured quantities are R_b and R_c , as well as the forward-backward asymmetries A_{FB}^b and A_{FB}^c . The main selection criteria (*tags*) are as follows :-

- **c-quarks:** D,D* mesons plus lifetime and lepton tags. The harder mo-

quantity	B	D ⁺	D ⁰
lifetime (ps)	1.6	1.0	0.4
$\langle x_E = E_{had}/E_{beam} \rangle$	0.7	0.5	0.5
decay charged multiplicity	5.5	2.2	2.2

Table 7: Some properties of B hadrons and D mesons.

momentum fraction in direct c compared to $b \rightarrow c$ is also used. For A_{FB}^c , the D/D^{*} charges and the lepton charges, in semileptonic decays, are used to distinguish c from $\bar{c}$.

- **b-quarks:** lifetime, mass and lepton tags. The mass tag exploits the fact that the b-quark decay products have relatively large invariant masses. For A_{FB}^b the lepton charge is used (evaluating the contributions from $b \rightarrow \ell$, $b \rightarrow c \rightarrow \ell$, $b \rightarrow \bar{c} \rightarrow \ell$), as is also the jet-charge for a specific hemisphere with respect to the thrust-axis $Q_{hemi} = \sum |p_{||}^i|^\kappa Q_i / \sum |p_{||}^i|^\kappa$, where $p_{||}^i$ is the momentum component of a hadron, with charge Q_i , parallel to the thrust axis. The power κ is optimised for sensitivity. For R_b the most accurate results are from double-tag methods, as discussed below.

The main background in the tagged c-quark sample is from b-quarks and is roughly 20 ± 2 % per hemisphere. In the b-quark sample, the c-quark background dominates and is roughly 5 ± 0.5 % per hemisphere. In addition, there are smaller backgrounds from u,d and s-quarks. For the lifetime tag the OPAL Collaboration, and the most recent measurement from the DELPHI Collaboration, use the distribution of the decay length significance per hemisphere, for which there is a clear excess for positive (forward) decay lengths due to b-quarks. The backward direction is used to control systematic effects. The ALEPH, DELPHI (for older measurements) and SLD Collaborations use a somewhat different approach, which involves constructing a probability that a group of tracks contains some significant lifetime information.

The main systematic uncertainties arise from:-

- the fraction of D^{*}, D⁺, D_s, Λ_c etc in $c\bar{c}$ events (particularly important for R_b)
- b and c hadron lifetimes
- charm decay modes
- fraction of gluon-splitting $g \rightarrow c\bar{c}, b\bar{b}$ in hadronic Z events
- semi-leptonic decay models and branching ratios
- light quark fragmentation models
- correlations between hemispheres for double-tags

Figure 5: Results on A_{FB}^c . The values plotted are corrected to correspond to $A_{FB}^{0,c}$. The average value is from a fit to all LEP heavy flavour data. The OPAL lepton result is updated with respect to Summer 1995. The χ^2/df of the average is 4.1/6, a probability of 66%. The SM prediction for $m_t = 175$ GeV, $m_H = 100$ GeV is shown as a dashed line.

The results for A_{FB}^c and A_{FB}^b are given in figs. 5 and 6 respectively. The quoted values are corrected, if necessary, for QCD effects⁶ and to correspond to $\sqrt{s} = 91.26$ GeV. In order to obtain $A_{FB}^{0,c}$ and $A_{FB}^{0,b}$ from the experimentally measured results, corrections are applied to get to $\sqrt{s} = m_Z$, for QED effects and for the contributions of γ exchange and γ -Z interference. These amount in total to additive corrections of 0.0062 for A_{FB}^c and 0.0025 for A_{FB}^b . The results for A_{FB}^b have also been corrected for the effects of $b\bar{b}$ mixing. For the A_{FB}^c measurements, the lepton results have roughly equal statistical and systematic errors; whereas for the D* results the limit is still statistics. For A_{FB}^b , the statistical errors are larger than the systematic errors for both the lepton and jet-charge measurements. The asymmetries $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$ are rather weakly correlated, and both the pole asymmetries, and their energy dependence (see fig. 7), are compatible with the SM.

Measurements of the heavy-quark forward-backward asymmetries using a longitudinally polarised beam by the SLD Collaboration give directly values of A_b and A_c . Using lepton, kaon, D-meson and jet-charge plus lifetime tags, the values $A_b = 0.863 \pm 0.049$ and $A_c = 0.625 \pm 0.084$ are obtained [34].

The results for R_b^0 are given in fig. 8 and those for R_c^0 in fig. 9. The largest

⁶The treatment of QCD effects has been improved with respect to the results discussed in [1].

Figure 6: Results on A_{FB}^b . The values plotted are corrected to correspond to $A_{FB}^{0,b}$. The average value is from a fit to all LEP heavy flavour data. The ALEPH lepton and jet charge and the OPAL lepton results are updated with respect to Summer 1995. The D^* results from DELPHI and OPAL are included in the average, but are not plotted as they are less precise than the other measurements. The χ^2/df of the average is 6.3/8, a probability of 61%. The SM prediction for $m_t = 175$ GeV, $m_H = 100$ GeV is shown as a dashed line.

background for R_b is from c-quark events, so the results for R_b and R_c are correlated. In fig. 8 the results are given for the SM value, $R_c^0 = 0.172$. These results are discussed further below. It should be stressed that many of the heavy quark results are still preliminary.

5.1 Combining the heavy flavour results

The combination has been carried out by a LEP/SLD working group [35]. Details of the procedure used for the LEP experiments can be found in [36]. Each experiment provides, for each measurement, a complete breakdown of the systematic errors, adjusted if necessary to agreed meanings of these errors. Direct measurements of A_b and A_c by SLD, obtained by measuring A_{FB}^b and A_{FB}^c with a polarised beam, are also included. A multi-parameter fit is then performed to get the best overall values of $R_b^0, R_c^0, A_{FB}^{0,b}, A_{FB}^{0,c}, A_b$ and A_c , plus their covariance matrix. The results of a fit to both the LEP and SLD data are given in table 8. The effective mixing parameter $\bar{\chi}$ and the leptonic branching ratios $b \rightarrow \ell$ and

Figure 7: Energy dependence of the heavy quark forward-backward asymmetries.

$b \rightarrow c \rightarrow \ell$ are also included in the fit.

The LEP values of $A_{FB}^{0,b}$, $A_{FB}^{0,c}$ and $A_\ell = 0.1466 \pm 0.0033$ (from $A_{FB}^{0,\ell}$ and P_τ) can be used to give $A_b = 0.890 \pm 0.029$ and $A_c = 0.667 \pm 0.047$. Combining the LEP ($A_{FB}^{0,b}$, $A_{FB}^{0,c}$ and A_ℓ) and SLD (A_b , A_c and A_e) results gives $A_b = 0.867 \pm 0.022$ and $A_c = 0.646 \pm 0.040$, to be compared to the SM values of 0.935 and 0.668 respectively (for $m_t = 175$ GeV and $m_H = 100$ GeV). The LEP plus SLD value for A_b is about 3.1 standard deviations below the SM value; whereas that for A_c is compatible with the SM. However, it should be noted that this procedure involves the use of A_ℓ values which are not fully compatible. If the SM value from the global fit to all data ($A_\ell = 0.1458$) is used, then $A_b(\text{LEP}) = 0.895 \pm 0.021$, and the LEP plus SLD combined value is then $A_b = 0.890 \pm 0.019$. This value is only 2.4 σ from the SM value.

Since the results for R_b and R_c have changed rather significantly with respect to [1], it is worth examining these measurements in a little more detail.

5.2 Measurement of R_b

The most accurate measurements of R_b all employ a double-tag method. This involves determining the jet axis of the event (*thrust-axis*) and then employing lifetime, leptonic or other tags to each hemisphere to determine the number of hemispheres N_t with a tag and the number of events N_{tt} with two tags. For a sample of N_{had} hadronic Z decays one has

quantity	value	error	R_b^0	R_c^0	$A_{FB}^{0,b}$	$A_{FB}^{0,c}$	$\mathcal{A}_b$	$\mathcal{A}_c$
R_b^0	0.2178	0.0011	1.000	-0.23	0.00	0.00	-0.03	0.01
R_c^0	0.1715	0.0056		1.000	0.04	-0.06	0.05	-0.07
$A_{FB}^{0,b}$	0.0979	0.0023			1.000	0.10	0.04	-0.02
$A_{FB}^{0,c}$	0.0735	0.0048				1.000	0.01	0.10
$\mathcal{A}_b$	0.863	0.049					1.000	0.12
$\mathcal{A}_c$	0.625	0.084						1.000

Table 8: Results of fits to the LEP and SLD heavy flavour data, plus the correlation matrix. The χ^2/df of the average is 51/(87-13), a probability of 98%.

Figure 8: Results on R_b , together with the average value for R_b^0 from a fit to all the LEP and SLC heavy flavour data. The ALEPH, DELPHI, L3 lifetime, OPAL and SLD results are new or updated with respect to Summer 1995. The χ^2/df of the average is 5.1/5, a probability of 40%. The SM prediction for $m_t = 175$ GeV, $m_H = 100$ GeV is shown as a dashed line.

$$\frac{N_t}{2N_{\text{had}}} = \epsilon_b R_b + \epsilon_c R_c + \epsilon_{uds} (1 - R_b - R_c), \quad (33)$$

$$\frac{N_{tt}}{N_{\text{had}}} = C_b (\epsilon_b)^2 R_b + (\epsilon_c)^2 R_c + (\epsilon_{uds})^2 (1 - R_b - R_c), \quad (34)$$

where ϵ_b , ϵ_c and ϵ_{uds} are the tagging efficiencies per hemisphere for b, c and light-quark events, and $C_b \neq 1$ accounts for the fact that the tagging efficiencies

between the hemispheres may be correlated. In practice, $\epsilon_b \gg \epsilon_c \gg \epsilon_{uds}$, $\mathcal{C}_b \approx 1$, and the correlations for the other flavours are neglected. These equations can be solved to give R_b and ϵ_b which, neglecting the c and uds backgrounds and the correlations, are approximately given by:

$$\epsilon_b \approx 2N_{tt}/N_t, \quad (35)$$

$$R_b \approx N_t^2/(4N_{tt}N_{\text{had}}). \quad (36)$$

The double-tagging method has the advantage that the tagging efficiency is derived directly from the data, reducing the systematic error of the measurement. The residual background of other flavours in the sample, and the evaluation of the correlation between the tagging efficiencies in the two hemispheres of the event are the main sources of systematic uncertainty in such an analysis.

In practice there are significant complications from:-

- *backgrounds*, typically $\sim 0.5\%$. The main background is from $Z \rightarrow c\bar{c}$, which means that R_b is correlated with R_c .
- *hemisphere correlations*. These arise from QCD effects (eg the b and $\bar{b}$ end up in the same hemisphere), primary vertex (eg errors wrongly estimated), back-to-back ‘holes’ in the detector acceptance etc. These lead to an error $\sim 0.5\%$.

These (plus other) corrections are found using both the data and Monte Carlo. It is worth stressing that the Monte Carlo programs have been the subject of much effort and that much experimental input has been used to make them as reliable as possible.

The combined LEP/SLD value of $R_b^0 = 0.2178 \pm 0.0011$ ($R_c^0 = 0.172$) thus has a relative precision of about 0.5%. The excellent precision on R_b comes primarily from the double-tag results. The lepton results are much less precise as can be seen from fig. 8. The combined statistical error of all the measurements is about 0.0007 and that from the internal experimental systematics (track resolution, detection efficiencies of leptons etc) is about 0.0005. The error due to common systematics is about 0.0007. The largest common systematic errors are from uncertainties on the inclusive $D \rightarrow K^0 X$ (0.0002), the D^+ decay multiplicity (0.0003), QCD effects in hemisphere correlations (0.0003) and gluon splitting into b and c quark pairs (0.0004). In total more than 20 sources of error to R_b are considered.

At the time of the Summer Conferences in 1995, the average value was $R_b^0 = 0.2205 \pm 0.0016$ (for $R_c^0 = 0.172$) [1], more than three standard deviations above the SM value. The measured value was in the direction expected from light SUSY particles ($\tilde{t}, \tilde{\chi}$). However, the size of the difference was difficult to explain entirely by such particles, if the limits from direct searches, particularly those from the Autumn 1995 LEP run at $\sqrt{s} \simeq 135$ GeV, were to be respected.

The change from 1995 to 1996 is $\delta R_b^0 = 0.0027$; representing a shift of 1.6σ of the 1995 error. However, it should be recalled that the 1995 error of 0.0016 was dominated by the common systematic error of 0.0013, which was needed when combining the results.

The change in the R_b^0 average is due to several factors :

- a) *A change in the central value of R_c^0 (section 5.3).* The correlation between R_b and R_c is large. It was $\rho = -0.35$ for the Summer 1995 average and is $\rho = -0.23$ for the present average.
- b) *New data.* There are new results from ALEPH, DELPHI, L3 and SLD. The ALEPH Collaboration have developed a very pure tag, built from five different tags; two being sensitive mainly to uds and c quarks respectively. Charm rejection is improved by using invariant mass information. The SLD Collaboration also use invariant mass information to improve their b-tag. The DELPHI result has been updated by including the 1994 data and L3 have a measurement using the 1994 data with a lifetime tag.
- c) *Changes in some of the common input parameters.* For example, changes in the gluon-splitting rate and other parameters have lowered R_b somewhat. However, this change is smaller than the above two changes.

However, R_b^0 is still largely systematics limited. Each experiment chooses cuts to optimise its overall error. Although considerable effort [35] has been made to ensure common input parameters, and of the definition of systematic errors between experiments, it should be stressed that systematic errors do not, in general, have the same precise meaning in terms of confidence levels as statistical errors. However, the total systematic error on R_b^0 is made up from many components, so it is to be expected that some cancellations in the signs of the contributions will occur (the components are added in quadrature). It is thus unlikely that they could all be “wrong” in the sense that they shift R_b^0 in the same direction.

The background of c in b samples can, to some extent, be reduced by making harder cuts on the lifetime or lepton properties used. However, such cuts might well increase the error from hemisphere correlations. Thus increased statistics alone do not guarantee a more accurate result.

5.3 Measurement of R_c

Tagging charm quarks with high efficiency and purity is unfortunately difficult. The cleanest tag is to use $c \rightarrow D^{*+} \rightarrow D^0 \pi^+ (D^0 \rightarrow K^- \pi^+)$, but this tags only about 0.5% of c-decays, so is statistically limited. Other D^0 modes, which are somewhat less clean, can also be used, as can other ground-state charmed

Figure 9: Results on R_c , together with the average value for R_c^0 from a fit to all the LEP heavy flavour data. The ALEPH (92-94) lepton and D^* , the DELPHI charm-count plus D^* and the OPAL D^* results are new or updated with respect to Summer 1995. The other results have been updated as discussed in the text. The χ^2/df of the average is 0.9/6, a probability of 99%. The SM prediction for $m_t = 175$ GeV, $m_H = 100$ GeV is shown as a dashed line.

hadrons and c-quark leptonic decays. However, the overall charm tag efficiency is low compared to that for b-quarks.

There are also analyses which use a “slow” π tag (the π in the D^* decay has a small p_T) in a double-tag. However, this tag is rather loose because there is a considerable background at low p_T from fragmentation processes.

Several methods have been used in the determination of R_c . These are :

- i) *Single charm-counting rate.* This requires measuring the production rates of the ground-state charmed hadrons (D^0, D^+, D_s , as well as charmed baryons). The strange-charmed baryons are not well measured and a correction of 1.15 ± 0.05 is applied to the Λ_c rate to account for these. The total rate gives $R_c \times \text{Prob}(c \rightarrow \text{hadrons})$, so if all the ground-state charmed hadrons are detected the measurement gives R_c . DELPHI and OPAL have used this method.
- ii) *Exclusive D^{*+} measurement.* This determines the product $R_c \times P_{c \rightarrow D^{*+}}$, which is an important input to R_c double-tag measurements. Both DEL-

Figure 10: The 68% and 95% confidence level contours in the R_b^0 , R_c^0 plane. The SM predictions for m_t values from 140 to 220 GeV are also shown; as is the Summer 1995 result.

PHI and OPAL determine the rate of $D^{*+} \rightarrow D^0 \pi^+ \rightarrow K^- \pi^+ \pi^+$. The contribution from b decays must be accounted for in these measurements.

- iii) *Double-tag measurements.* Here three methods have been used. These are exclusive D^* / inclusive π_{slow} (ALEPH, DELPHI and OPAL), exclusive D-meson double-tag (ALEPH) and inclusive π_{slow} double-tag (DELPHI).

The LEP average value for R_c^0 , made in Summer 1995, was $R_c^0 = 0.1540 \pm 0.0074$. This was some 2.4 standard deviations below the SM value of 0.172. The Summer 1996 value of $R_c^0 = 0.1715 \pm 0.0056$, and is very close to the SM value. For the 1995 average, roughly half of the error weight comes from common systematic errors between the measurements. In particular the results relied on the measurement of $Y_c = P(c \rightarrow D^{*+}) \times BR(D^{*+} \rightarrow \pi^+) = 0.178 \pm 0.013$, made at low ($\sqrt{s} \sim 10$ GeV) energy and on the assumption that this quantity was energy independent. The LEP data now determine this quantity directly, so that the 1996 average does not depend on the use of low energy data. The LEP value is $Y_c = 0.163 \pm 0.007$. Although this value is only about 1σ different to the low energy measurement, because of the strong correlation (-60%) between R_c^0 and Y_c , this causes a sizeable shift in R_c^0 . If Y_c is fixed to 0.178 then R_c^0 is shifted to 0.1637.

The relative precision of the average value of R_c^0 ($=0.1715 \pm 0.0056$) is 3.3%. The statistical component of the error is 0.0037, and that from internal experimental systematics 0.0029. The largest common systematic errors are 0.0022 from $BR(c \rightarrow \ell)$ and 0.0011 from $BR(D_s \rightarrow \phi \pi)$.

As discussed above, the determinations of R_b^0 and R_c^0 are correlated. Fig. 10 shows the 68% and 95% confidence level contours in the R_b^0, R_c^0 plane, as well as the SM prediction for various values of m_t .

5.4 Inclusive Hadron Charge Asymmetry $\langle Q_{FB} \rangle$

This involves measurements of the hadronic charge asymmetry based on the mean difference in jet charges measured in the forward and backward event hemispheres, $\langle Q_{FB} \rangle$. Measurements of $\langle Q_{FB} \rangle$ have been made by the ALEPH, DELPHI and OPAL experiments. However, the values cannot be directly compared as some of them include detector dependent effects such as acceptances and efficiencies. The results are best compared using the values of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ extracted in each analysis, as given in Table 9. It can be seen that the systematic errors are larger than the statistical errors and are dominated by fragmentation and decay modelling uncertainties.

Experiment		$\sin^2 \theta_{\text{eff}}^{\text{lept}}$
ALEPH	90-94, final	$0.2322 \pm 0.0008 \pm 0.0011$
DELPHI	91-94, prel.	$0.2311 \pm 0.0010 \pm 0.0014$
OPAL	91-94, prel.	$0.2326 \pm 0.0012 \pm 0.0013$
Average		0.2320 ± 0.0010

Table 9: Summary of the determination of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ from inclusive hadronic charge asymmetries at LEP. For each experiment, the first error is statistical and the second systematic.

5.5 $Z \rightarrow q\bar{q}\gamma$ radiative events

Since the coupling of a γ to a quark is $\propto Q_q^2$, the number of Z events which result in a $q\bar{q}\gamma$ final state is $N_{q\bar{q}\gamma} \simeq 4R_u + 4R_c + R_d + R_s + R_b$. The number of hadronic Z events $N_{q\bar{q}} \simeq R_u + R_c + R_d + R_s + R_b$. Measurements of isolated photon production have been made by all the LEP experiments and the observed yield compared to matrix element calculations of $\mathcal{O}(\alpha\alpha_s)$. The fraction of radiative events can be used to extract [37] $(R_u + R_c)/2 = 0.147 \pm 0.019$ (SM=0.172). Alternatively, this result can be expressed as $(R_d + R_s + R_b)/3 = 0.235 \pm 0.013$ (SM=0.218). This result suffers from different systematic errors to those of the direct measurements of R_b and R_c .

5.6 Extraction of heavy quark couplings

An alternative approach in trying to understand the possible implications of the heavy flavour results is to extract the individual quark couplings. The

Figure 11: Results of a fit to the b-quark vector and axial-vector couplings. The contours are for the 70, 95 and 99% confidence limits.

measurements used are $R_b^0 = \Gamma_b/\Gamma_{\text{had}}$ (which, using Γ_{had} from the lineshape, gives $v_b^2 + a_b^2$), $R_c^0 (v_c^2 + a_c^2)$, A_e from LEP/SLD (v_e/a_e), $A_{\text{FB}}^{0,b} (v_b/a_b, v_e/a_e)$, $A_b (v_b/a_b)$, $A_{\text{FB}}^{0,c} (v_c/a_c, v_e/a_e)$ and $A_c (v_c/a_c)$. The constraint $\alpha_s(m_Z) = 0.123 \pm 0.006$ [42] is imposed. The results for v_b and a_b are shown in table 10 and fig. 11. Also shown are the SM predictions corresponding to $m_t = 175 \pm 6$ GeV and $60 \leq m_H [\text{GeV}] \leq 1000$. The corresponding results for v_c and a_c are shown in table 10 and fig. 12. Note that there is a strong anti-correlation between v_b and a_b .

parameter	fitted value	v_b	a_b	v_c	a_c
v_b	-0.306 ± 0.012	1.00	-0.97	-0.19	0.07
a_b	-0.526 ± 0.007		1.00	0.20	-0.02
v_c	0.185 ± 0.013			1.00	-0.36
a_c	0.503 ± 0.010				1.00

Table 10: Results, plus correlation matrix, of a fit to the vector and axial-vector couplings of b and c quarks.

The b-quark couplings can also be expressed in terms of the left-handed $\ell_b = (v_b + a_b)/2$ and right-handed $r_b = (v_b - a_b)/2$ couplings. The results are shown in fig. 13. The corresponding results for the c-quark are shown in fig. 14.

Figure 12: Results of a fit to the c-quark vector and axial-vector couplings. The contours are for the 70, 95 and 99% confidence limits.

6 Global electroweak fits

6.1 Determinations of $\sin^2\theta_{\text{eff}}^{\text{lept}}$

The values of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ determined by the various methods are shown in fig. 15. All the results are statistically compatible, and have an impressive precision of $\delta\sin^2\theta_{\text{eff}}^{\text{lept}} = \pm 0.00024$. It is an interesting question as to whether one expects agreement amongst these measurements. The various methods have different sensitivities to different couplings, so differences could be sensitive to different physics beyond the SM. The sensitivity of the heavy quark asymmetries A_{FB}^b and A_{FB}^c to $\sin^2\theta_{\text{eff}}^{\text{lept}}$ is mainly through A_e , rather than A_b and A_c (see eqns. 16, 22 and 23), which are rather insensitive to variations of the SM parameters. As discussed in section 4.1, the value of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ extracted for heavy quarks has a small SM dependence. The range of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ extracted from either A_{FB}^b or A_{FB}^c is $\sim 2 \cdot 10^{-5}$, for variations within the SM of $100 < m_t < 200$ GeV and $60 < m_H < 1000$ GeV.

6.2 Standard Model Constraints

The precise electroweak measurements performed at LEP can be used to

Figure 13: Results of a fit to the b-quark left and right-handed couplings. The contours are for the 70, 95 and 99% confidence limits.

check the validity of the Standard Model and, within its framework, to infer valuable information about its basic parameters. Their accuracy makes them sensitive to the top quark mass, m_t , and to the mass of the Higgs boson, m_H , through the loop corrections. The leading top quark mass dependence is quadratic and allows a determination of m_t . The main dependence on m_H is logarithmic and therefore, with the present data accuracy, the constraints on m_H are much weaker.

The various measurements used in the SM fits are summarised in Table 11. In addition to the data discussed so far, the average value of $\sin^2\theta_W$ determined from ν -nucleon scattering [2, 38–40] is also used. Also given is an estimate of the systematic component of the error. The ‘pulls’ of the 19 data points with respect to the SM fit B of table 13, and discussed below, are also given. The distribution of the pulls is shown in fig. 16, together with a fit to a gaussian distribution. The fit has a good χ^2 probability ($> 90\%$) and the gaussian rms is 0.86 ± 0.35 ; compatible with unity. Hence all the results are compatible with the expectations of the Standard Model.

Table 12 shows the constraints obtained on m_t and $\alpha_s(m_Z)$ when fitting the measurements to the most recent SM calculations [41]⁷. The central values given in table 12, and the first errors quoted, refer to $m_H = 300$ GeV. The second

⁷In the fits presented here the ZFITTER package [41] is used.

Figure 14: Results of a fit to the c-quark left and right-handed couplings. The contours are for the 70, 95 and 99% confidence limits.

Figure 15: Measurements of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$. The $\chi^2/\text{df} = 12.8/6$, corresponding to a probability of 5%. The SM prediction of 0.23146, for $m_t = 175$ GeV, $m_H = 100$ GeV, is shown as a dashed line.

errors correspond to the variation $60 \leq m_H [\text{GeV}] \leq 1000$. The constraint from the direct measurement of m_t is not used in these fits. The value of $1/\alpha(m_Z)$

Figure 16: Distribution of the pulls shown in Table 11 with respect to the SM fit B of Table 13. Also shown is a fit to a gaussian distribution.

used is that given in equation 4, constrained by the quoted error. The value $G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ is also used in the fits. The estimated theoretical uncertainties are $\Delta m_t \lesssim 1 \text{ GeV}$, $\Delta \alpha_s(m_Z) \lesssim 0.001$ and $\Delta \log(m_H) \lesssim 0.1$. An uncertainty in $1/\alpha(m_Z)$ of 0.09 (section 2) causes an error of 0.00023 on the SM prediction of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ and an error of 4 GeV on m_t , for fixed m_H . The effect on the prediction for Γ_t is negligible.

The fitted values of m_t are compatible with the direct Tevatron measurement of $m_t = 175 \pm 6 \text{ GeV}$. The value of $\alpha_s(m_Z)$ depends mainly on R_ℓ , Γ_Z and σ_h^0 , and is in good agreement with event shape analyses at LEP; $\alpha_s(m_Z) = 0.123 \pm 0.006$ [42]. The strong coupling constant can also be determined from R_ℓ alone. Imposing $m_t = 175 \pm 6 \text{ GeV}$ as constraint, $\alpha_s(m_Z) = 0.124 \pm 0.004 \pm 0.002$ is obtained, where the second error corresponds to $60 \leq m_H [\text{GeV}] \leq 1000$.

The value of $1/\alpha(m_Z)$ is used as a constraint in the above fits. If $1/\alpha(m_Z)$ is left free in Fit C in table 12, then the fitted value found is

$$\alpha^{-1}(m_Z) = 129.1 \pm 0.1 \pm 0.3, \quad (37)$$

where the central value is for $m_H = 300 \text{ GeV}$ and the error corresponds to $60 \leq m_H [\text{GeV}] \leq 1000$.

	Measurement	'syst' error	SM fit	Pull
a) <u>LEP</u>				
m_Z [GeV]	91.1863 ± 0.0020	0.0015	91.1861	0.1
Γ_Z [GeV]	2.4946 ± 0.0027	0.0017	2.4960	-0.5
σ_h^0 [nb]	41.508 ± 0.056	0.055	41.465	0.8
R_ℓ	20.778 ± 0.029	0.024	20.757	0.7
$A_{\text{FB}}^{0,\ell}$	0.0174 ± 0.0010	0.0007	0.0159	1.4
A_τ	0.1401 ± 0.0067	0.0045	0.1458	-0.9
A_e	0.1382 ± 0.0076	0.0021	0.1458	-1.0
R_b^0	0.2179 ± 0.0012	0.0009	0.2158	1.8
R_c^0	0.1715 ± 0.0056	0.0042	0.1723	-0.1
$A_{\text{FB}}^{0,b}$	0.0979 ± 0.0023	0.0010	0.1022	-1.8
$A_{\text{FB}}^{0,c}$	0.0733 ± 0.0049	0.0026	0.0730	0.1
$\sin^2 \theta_{\text{eff}}^{\text{lept}} ((Q_{\text{FB}}))$	0.2320 ± 0.0010	0.0008	0.23167	0.3
b) <u>SLC</u>				
$\sin^2 \theta_{\text{eff}}^{\text{lept}} (A_{\text{LR}})$	0.23061 ± 0.00047	0.00014	0.23167	-2.2
R_b^0	0.2149 ± 0.0038	0.0021	0.2158	-0.2
A_b	0.863 ± 0.049	0.032	0.935	-1.4
A_c	0.625 ± 0.084	0.041	0.667	-0.5
c) <u>$p\bar{p}$ and νN</u>				
m_W [GeV]	80.356 ± 0.125	0.110	80.353	0.3
$1 - m_W^2/m_Z^2$	0.2244 ± 0.0042	0.0036	0.2235	0.2
m_t [GeV]	175 ± 6	4.5	172	0.5

Table 11: Measurements used in SM fits, together with an estimate of the systematic component of the error. Also given is the corresponding SM fit value for Fit B of table 13 and the pull (the difference between the measurement and the fit, divided by the measurement error).

	A) LEP	B) LEP + SLD	C) LEP + SLD + $p\bar{p}$ and νN
m_t (GeV)	$171 \pm 8 \substack{+17 \\ -19}$	$177 \substack{+7 \\ -8} \substack{+17 \\ -19}$	$177 \pm 7 \substack{+16 \\ -19}$
$\alpha_s(m_Z^2)$	$0.122 \pm 0.003 \pm 0.002$	$0.121 \pm 0.003 \pm 0.002$	$0.121 \pm 0.003 \pm 0.002$
χ^2/df (prob)	10/9 (35%)	20/12 (7%)	20/14 (13%)

Table 12: Results of SM fits for m_t and $\alpha_s(m_Z)$. No external constraint on $\alpha_s(m_Z)$ is imposed. The central values and the first errors quoted refer to $m_H = 300$ GeV. The second errors correspond to the variation $60 \leq m_H [\text{GeV}] \leq 1000$. The constraint from the direct measurement of m_t is not used in these fits.

6.3 Fits to m_H

In the fits discussed so far m_H has been held fixed. In table 13 the results are shown of SM fits in which m_H is left as a free parameter. Fig. 17 shows the 70 and 95% c.l. contours of m_t and m_H for a fit to the data used for Fit B

Figure 17: Results of a fit to all data showing the 70 and 95% c.l. contour plots for m_t vs m_H .

	A) LEP	B) LEP + SLD + $p\bar{p}$ + νN + m_t
m_t [GeV]	155^{+18}_{-13}	172 ± 6
m_H [GeV]	86^{+202}_{-51}	149^{+148}_{-82}
$\log(m_H)$	$1.93^{+0.52}_{-0.39}$	$2.17^{+0.30}_{-0.35}$
$\alpha_s(m_Z^2)$	0.121 ± 0.003	0.120 ± 0.003
χ^2/df (prob)	9/8(34%)	19/14(16%)
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	0.23198 ± 0.00026	0.23167 ± 0.00023
$1 - m_W^2/m_Z^2$	0.2249 ± 0.0009	0.2235 ± 0.0006
m_W (GeV)	80.278 ± 0.049	80.352 ± 0.033

Table 13: Results of the fits to A) LEP data alone and to B) all data, including the direct determination of m_t . As the sensitivity to m_H is logarithmic, both m_H and $\log(m_H)$ are given. In the lower part of the table derived results for $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, $1 - m_W^2/m_Z^2$ and m_W are given. The errors quoted do not include the SM theoretical uncertainties discussed in the text.

of table 13. The sensitivity to m_H comes mainly from the various asymmetry measurements.

The value of $1/\alpha(m_Z)$ used is that given in equation 4, with the error used as a constraint in the fit. The uncertainty on $1/\alpha(m_Z)$ gives rise to an error of about 1 GeV on m_t and of 0.2 on $\log(m_H)$; which is relatively large. The effect of the uncertainty on $1/\alpha(m_Z)$ is included in the errors given in table 13. If $\alpha(m_Z)$ were known more precisely then the limits on m_H could be much improved.

Fig. 18 shows the distribution of $\Delta\chi^2 \equiv \chi^2 - \chi_{min}^2$ as a function of m_H , for fit B in table 13. The band shown in fig. 18 is an estimate of the theoretical error due to missing higher order corrections. Taking these corrections into account, the 95% upper limit on m_H is 550 GeV.

The fit results given in table 13 thus favour a relatively light Higgs mass. However, the fitted value of m_H does depend on which measurements are used in the fit. For example, if fit B is repeated without the SLD A_{LR} measurement then m_t and $\alpha_s(m_Z^2)$ are almost unchanged, but the fitted value for m_H becomes 259^{+234}_{-131} GeV. If fit B is repeated without the inclusion of any heavy flavour data, then again m_t and $\alpha_s(m_Z^2)$ are almost unchanged, but m_H becomes 71^{+121}_{-52} GeV.

The value of $\alpha_s(m_Z^2)$ obtained from the LEP electroweak data (which comes primarily from the hadronic line shape data) is somewhat larger than values obtained from some other data. To see the influence of these hadronic line shape data, fit B in table 13 was repeated without the inclusion of the lineshape data sensitive to $\alpha_s(m_Z^2)$. The 5 parameters of table 2 were replaced in the fit by m_Z , Γ_l and $A_{FB}^{0,\ell}$. The fitted value of m_t was again 172 ± 6 GeV and $m_H = 147^{+149}_{-81}$ GeV. The remaining variables still have, however, some sensitivity to $\alpha_s(m_Z^2)$. The values given are for $\alpha_s(m_Z^2) = 0.123$. If $\alpha_s(m_Z^2) = 0.110$ is used, then the fitted m_H value is increased by 18 GeV, but m_t is unchanged.

The expected precision from the current level of accuracy of these quantities can be gauged by performing SM fits to the data in which the “measured” values of all quantities are set to their SM values corresponding to $m_t = 175$ GeV, $m_H = 100$ GeV and $\alpha_s(m_Z) = 0.123$, but with their errors and correlations unchanged. The resulting fit (corresponding to fit B of table 13) gives $m_t = 175 \pm 6$ GeV, $m_H = 100^{+102}_{-54}$ GeV. The precision depends on the actual SM values assumed. Taking the same values of m_t and $\alpha_s(m_Z)$, but imposing $m_H = 300$ GeV, the values $m_t = 175 \pm 6$ GeV, $m_H = 300^{+258}_{-145}$ GeV and $\alpha_s(m_Z) = 0.123 \pm 0.003$ are obtained.

Although, as discussed above, the distribution of the pulls around the SM fits are compatible with that expected if the SM is valid, it is of interest to quantify how large the possible excess in the measured value of R_b^0 could be compared to the SM. If R_b^0 has an additional component then this will also affect Γ_{had} and related quantities. In particular, one would expect an influence on the extracted value of $\alpha_s(m_Z)$. A method to quantify this is to define $\Gamma_b = \Gamma_b(SM) + \Delta\Gamma_b$, and then make a fit to all the data, propagating this definition

Figure 18: $\Delta\chi^2 \equiv \chi^2 - \chi_{min}^2$ vs m_H for fit B in Table 12. The shaded band is an estimate of the theory error due to missing higher order corrections.

to all variables depending on Γ_{had} . A repeat of Fit B in table 13 yields

$$\Delta\Gamma_b = 4.5 \pm 2.5 \text{ MeV}, \alpha_s(m_Z) = 0.112 \pm 0.005, m_t = 174 \pm 6 \text{ GeV}, m_H = 185_{-101}^{+181} \text{ GeV}, \quad (38)$$

with a χ^2 of 16/13 df, compared to 19/14 for Fit B. Note that there is a strong correlation (-0.8) between $\Delta\Gamma_b$ and $\alpha_s(m_Z)$, so that the extra piece in Γ_b is accommodated by having a somewhat lower value of $\alpha_s(m_Z)$.

Until rather recently [16] the ‘low energy’ determinations of $\alpha_s(m_Z)$, and also those from QCD lattice calculations, were smaller than those extracted from LEP from either the electroweak data or studies of hadronic final states. However, as discussed in section 2.3, new determinations of $\alpha_s(m_Z)$, particularly from the CCFR neutrino experiment and from lattice QCD, are consistent with a new global average of $\alpha_s(m_Z) = 0.118 \pm 0.003$ [16]. The value of $\alpha_s(m_Z)$ from electroweak measurements is consistent with this new average, whether or not an extra piece in Γ_b is allowed. Thus, achieving consistency for $\alpha_s(m_Z)$ determinations is no longer an argument in favour of an extra piece in Γ_b .

In conclusion, the values and limits on m_H show some sensitivity to the inclusion or exclusion of various electroweak data. The data in question are also somewhat incompatible with the SM fits to all data. In particular, the fitted value of m_H is sensitive to the inclusion of the A_{LR} data. Hence some caution must still be exercised in concluding that the existing data show that

Figure 19: A comparison of direct versus indirect determinations of m_t and m_W . The contour corresponds to the 70% confidence level. The filled part of the contour is that compatible with $m_H \geq 65$ GeV from direct searches.

the Higgs is necessarily relatively light.

6.4 Direct versus indirect determinations of m_t and m_W

Standard Model fits to m_t and m_H can also be expressed in terms of m_t and m_W , since m_W is fixed in the SM for a given value of m_H and the other SM parameters. Fig. 19 shows a comparison of the direct versus indirect (from an SM fit to all the LEP and SLD data) determinations of m_W and m_t . It can be seen that there is good agreement between the direct and indirect determinations of both m_t and m_W .

7 Quasi model-independent variables

The information content of all of the precision electroweak quantities can, to a very good approximation, be described by four quasi model independent quantities $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_b [43], plus $\alpha_s(m_Z^2)$.

The ϵ 's are defined as follows :

$$\epsilon_1 = \Delta\rho \quad (39)$$

$$\epsilon_2 = c_0^2 \Delta\rho + \frac{s_0^2 \Delta r_W}{(c_0^2 - s_0^2)} - 2s_0^2 \Delta\kappa \quad (40)$$

$$\epsilon_3 = c_0^2 \Delta\rho + (c_0^2 - s_0^2) \Delta\kappa \quad (41)$$

where the axial-vector and vector Z-lepton couplings are (see eqn. 23)

$$a_\ell = -\frac{1}{2}(1 + \Delta\rho/2) \quad (42)$$

and

$$v_\ell/a_\ell = x_\ell = 1 - 4 \sin^2 \theta_{\text{eff}}^{\text{lept}} = 1 - 4(1 + \Delta\kappa)s_0^2, \quad (43)$$

with

$$s_0^2 c_0^2 = \frac{\pi \alpha(m_Z)}{\sqrt{2} G_F m_Z^2}. \quad (44)$$

Numerically, $s_0^2 = 0.23118$. The quantity Δr_W is given by m_W

$$(1 - \frac{M_W^2}{M_Z^2}) \frac{M_W^2}{M_Z^2} = \frac{\pi \alpha(m_Z)}{\sqrt{2} G_F M_Z^2 (1 - \Delta r_W)}. \quad (45)$$

Inserting the value of s_0^2 , the expression for ϵ_3 becomes

$$\epsilon_3 = 0.77 \Delta\rho + 0.54 \Delta\kappa. \quad (46)$$

In this approach it is assumed that new physics, beyond the SM, enters through gauge-boson propagator (vacuum polarisation) functions and/or vertex corrections to the $Z\ell^+\ell^-$ vertex, assuming lepton universality. The parameters ϵ_1 and ϵ_3 are determined from the LEP and SLD Z leptonic data on $\Gamma_l, A_{\text{FB}}^{0,\ell}, A_\tau, A_e$ and A_{LR} . The quantity ϵ_2 is then primarily determined by the direct measurements of the W mass.

Data on the $Zq\bar{q}$ vertices can also be introduced if it is further assumed that all deviations from the SM are contained in the vacuum polarisation functions and/or the $Zb\bar{b}$ vertex. Further, $\alpha_s(m_Z)$ must also be introduced to describe the QCD corrections. A new parameter ϵ_b is needed to describe the $Zb\bar{b}$ vertex and is defined such that

$$a_b = a_\ell(1 + \epsilon_b) \quad (47)$$

and

$$\frac{v_b}{a_b} = \frac{1 - \frac{4}{3} \sin^2 \theta_{\text{eff}}^{\text{lept}} + \epsilon_b}{1 + \epsilon_b}. \quad (48)$$

The determination of ϵ_b comes essentially from the measurement of R_b . As noted previously, the forward-backward asymmetries for heavy-quarks depend mainly on A_e . For the other quarks, small corrections are made in order to obtain $\Delta\rho$ and $\Delta\kappa$ from the extracted values of the quark vector and axial-vector couplings. These corrections are computed in the SM. The results are also

corrected for QCD effects where necessary, using the ZFITTER package [41]. With these additions, all the LEP and SLD measurements can be included in the fits. Note that the measurements of the total or partial Z widths, which constrain $\Delta\rho$, contribute to both ϵ_1, ϵ_2 and ϵ_3 , whereas measurements of $\sin^2\theta_{eff}$, from forward-backward or other asymmetry measurements, contribute only to ϵ_2 and ϵ_3 .

In the SM, the main dependence on m_t is either quadratic or logarithmic, whereas the dependence on m_H is only logarithmic. The parameter ϵ_1 depends on m_t^2 and $\ln(m_H)$, ϵ_2 depends on $\ln(m_t)$, ϵ_3 depends on $\ln(m_H)$ and $\ln(m_t)$ and ϵ_b depends on m_t^2 . The SM contributions to the $Zb\bar{b}$ vertex come from t-W-b loops.

Of course, the usefulness of these variables, or the closely related S, T and U variables [44]⁸, is in studying physics beyond the SM. The aim is thus to reduce the errors on the ϵ 's to be as small as possible, so as to be as sensitive as possible to such physics.

The results of a fit to *leptonic electroweak data* ($\Gamma_l, A_{FB}^{0,\ell}, A_\tau, A_e$ and A_{LR}), plus the direct measurement of m_W and $1 - m_W^2/m_Z^2$ from ν -N scattering, are given in table 14. In this, and subsequent fits, the value of $\alpha(m_Z)$ is constrained to the value and error given in section 2.1. The results of a fit to *all electroweak data* (all the data in table 11, with the exception of the direct top-quark mass measurement) are given in table 15. The contour plots for the 70% confidence levels for ϵ_1 , ϵ_2 and ϵ_3 are shown in fig. 20. The results are, as expected, compatible with the SM expectations at the values of m_t and m_H discussed in previous sections. It can be seen that the value of ϵ_b is strongly correlated to that of $\alpha_s(m_Z)$. In the SM, $\epsilon_b = -5.8 \cdot 10^{-3}$ for $m_t = 175$ GeV, and is essentially independent of m_H .

parameter	fitted value	ϵ_1	ϵ_2	ϵ_3
ϵ_1	$(4.2 \pm 1.3)10^{-3}$	1.00	0.30	0.84
ϵ_2	$(-6.8 \pm 3.1)10^{-3}$		1.00	0.14
ϵ_3	$(3.2 \pm 1.4)10^{-3}$			1.00

Table 14: Results of the fit to the ϵ variables to leptonic electroweak data, plus m_W . The correlation matrix is also given. The χ^2 for the fit is 6/4 df, a probability of 20%.

8 Future precision on electroweak quantities

Assuming that the SM Higgs boson will eventually be discovered, then it will be of interest to compare the directly measured value of m_H with the indirect

⁸These are related approximately as follows: $\epsilon_1 = \alpha T$, $\epsilon_2 = -\alpha U/4x$ and $\epsilon_3 = \alpha S/4x$, where $x = \sin^2\theta_{eff}$. Other roughly equivalent formalisms exist in the literature.

parameter	fitted value	ϵ_1	ϵ_2	ϵ_3	ϵ_b	$\alpha_s(m_Z)$
ϵ_1	$(4.4 \pm 1.3)10^{-3}$	1.00	0.31	0.89	0.00	-0.37
ϵ_2	$(-7.6 \pm 3.1)10^{-3}$		1.00	0.21	0.00	-0.14
ϵ_3	$(4.4 \pm 1.3)10^{-3}$			1.00	0.00	-0.27
ϵ_b	$(-5.0 \pm 2.8)10^{-3}$				1.00	-0.75
$\alpha_s(m_Z)$	0.113 ± 0.006					1.00

Table 15: Results of the fit to the ϵ variables to all electroweak data, with the exception of the direct measurement of m_t . The correlation matrix is also given. The χ^2 for the fit is 16/11 df, a probability of 14%.

Figure 20: Results of a fit to ϵ_1 , ϵ_2 and ϵ_3 , showing the 70% confidence level contours and the expectations in the SM. For ϵ_1 versus ϵ_2 there is little sensitivity to m_H .

determination from precision electroweak quantities. This will then be a genuine test of the Standard Model. In order to estimate the sensitivity of the indirect

measurement of m_H some assumptions must be made on the expected precision of the various electroweak quantities which contribute. In this section some estimates are made, and fits are performed to compute the consequent expected accuracy on m_H .

8.1 Future precision on m_W

The Tevatron data so far analysed, and shown in fig. 2, come from the 1992/3 data taking (Run 1a) for CDF and are based on about 19pb^{-1} , whereas those from D0 are based on about 13pb^{-1} from Run 1a and about 76pb^{-1} from the 1994/6 data taking (Run 1b). The D0 result is still preliminary. When all the available data are analysed it is envisaged that the total combined error on m_W will be reduced to between about 80 to 90 MeV [45].

The Tevatron programme is foreseen to restart, with a much higher luminosity (due to the main injector and other improvements), at the turn of the century. Predictions of the error on m_W which will then be obtainable are clearly more difficult. However, assuming that the increased data samples allow on-going reduction of the systematic errors one might envisage that the combined value from the Tevatron will be in the range $\delta m_W = 30$ to 40 MeV, with a large part of this error coming from systematics common between the two experiments [45].

In the even further future more precise measurements may well be possible at both the Tevatron and the LHC. In addition, the indirect determination of m_W would be improved by higher precision deep-inelastic neutrino-nucleon scattering experiments and from precise measurements of the neutral and charged current cross-sections at HERA. These possible improvements are not considered in the fits below.

Estimates of the final expected precision from LEP 2 are also still somewhat uncertain. In 1996 about 10pb^{-1} of data was taken at centre of mass values of both 161 and 172 GeV. The $e^+e^- \rightarrow W^+W^-$ cross-section at 161 GeV, which is just above the W^+W^- threshold, is very sensitive to m_W [46], [47]. Preliminary results give $m_W = 80.42 \pm 0.22$ (expt) ± 0.03 (LEP) GeV, where the first error is the combined experimental error from the 4 LEP experiments and the second from the uncertainty in the beam energy. The experimental error is mainly statistical so cannot be decreased without more data. The measurement of m_W by this method is strictly only valid within the SM. For the LEP data at 172 GeV, and higher energies, the W^+W^- cross-section is no longer sensitive to m_W . The determination of m_W is by direct reconstruction of the W decay products in W^+W^- events in the semi-leptonic $WW \rightarrow jj\ell\nu$ and purely hadronic $WW \rightarrow jjjj$ decay modes. In each case kinematic fitting is used to improve the precision. A detailed discussion of the methods and expected precision can be found in [47].

The expected statistical precision for the semi-leptonic and purely hadronic decay modes is roughly the same. However, there are potentially large systematic errors for the purely hadronic final states. These arise from the effects of colour recombination and Bose-Einstein statistics effects, which can lead to “cross-talk” between the two W-bosons, and thus bias the extracted value of m_W . These effects [47] are non-perturbative and estimates of this bias thus rely on hadronisation models. At present the range of such model predictions is $\delta m_W \simeq 50$ to 100 MeV, so that it would appear that this channel cannot be used in a final precision measurement, without further theoretical and experimental understanding of these effects.

Thus the most precise determination is expected to be from the semi-leptonic final state $e^+e^- \rightarrow W^+W^-$ ($W \rightarrow jj$, $W \rightarrow \ell\nu$, $\ell=e,\mu$). The expected accuracy at the end of LEP 2 running from this channel is about 45 MeV.

Combining all potential data one might then expect a final world average value of the W mass error in the range δm_W between 25 and 35 MeV. In the following the value taken is $\delta m_W = 30$ MeV.

8.2 The Z lineshape parameters

The current precision on the mass and width of the Z, together with that of the pole cross-section σ_h^0 for hadronic final states, the ratio of the hadronic to leptonic partial widths R_ℓ and the pole lepton forward backward asymmetry $A_{FB}^{0,\ell}$ are given in table 16. Although most of the LEP 1 data have been analysed, the remaining 1995 data will somewhat reduce the experimental component of the errors. Further, some reduction in the LEP energy error can be expected.

The error on σ_h^0 is dominated by the theoretical uncertainty on the low angle Bhabha cross section, which is currently known to 0.11% [28], and is expected in the future to be known to about 0.06%. The experimental component of the luminosity error is typically $\sim 0.1\%$, or better, with little common error between the experiments. Hence, when the experiments are combined, the main component of the luminosity error will still be the “theory” error. Almost all of the data on $A_{FB}^{0,\ell}$ has been analysed, so little improvement in the accuracy is expected.

Futher determinations of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ from measurements of the decays $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$ at hadron colliders will also be made. The errors on these could well be comparable to those obtained at LEP and SLD. These possible improvements are not considered in the fits below.

8.3 τ polarisation

Additional information of the leptonic couplings of the Z come from studies of the τ -polarisation. The present and expected future accuracies on the quantities A_τ and A_e are given in table 17. The quantity A_e comes essentially from the forward-backward asymmetry of the τ -polarisation, and its error is mainly statistical in nature. The quantity A_τ is essentially the mean value of the τ -polarisation, averaged over polar angles, and has a sizeable systematic component. The estimated improvement in the statistical error takes into account the fraction of the total data analysed so far. It is assumed that the systematic errors can be reduced with increasing statistics; but not as fast as the statistical error.

quantity	current error	estimated error at end of LEP 1
δm_Z (MeV)	2.0	1.8
$\delta \Gamma_Z$ (MeV)	2.7	2.4
$\delta \sigma_h^0$ (nb)	0.056	0.035
δR_ℓ	0.029	0.027
$\delta A_{FB}^{0,\ell}$	0.0010	0.0010

Table 16: Current and estimated future precision of the Z parameters derived from fits to the cross section lineshapes for hadrons and leptons and from the lepton forward-backward asymmetries.

quantity	current error	estimated error at end of LEP 1
δA_e	0.0076	0.0055
δA_τ	0.0067	0.0053

Table 17: Current and estimated future precision of the τ -polarisation parameters.

8.4 Heavy quark couplings of the Z

Our knowledge of the heavy-quark couplings to the Z is also important for electroweak tests. The quantities analysed by the LEP experiments are the ratios of the partial to hadronic widths for b and c quarks, R_b^0 and R_c^0 , and the pole values of the forward-backward asymmetries for b and c quarks, $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$. The anticipated improvements from the current errors to those which are expected when all LEP 1 data, and also the expected future data on A_b and A_c at SLC, are analysed are given in table 18. Again the estimated future errors attempt to take into account the expected improvements in both the statistical and systematic errors.

At present, there is still a possible discrepancy in the value of R_b^0 between the

experimental value and that expected in the Standard Model from knowledge of the direct top quark mass measurement of $m_t = 175 \pm 6$ GeV [13–15], and from fits to other precision electroweak data. Although this is clearly of great importance if confirmed by further analysis, the impact of the precision of these measurements on the fits discussed below is limited.

Also used in the fits below is the value of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ obtained from studies of the inclusive forward-backward charge asymmetry in hadronic events, $\langle Q_{\text{FB}} \rangle$. The present error is $\delta \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.0010$, and the expected future error is taken to be 0.0008.

8.5 Left-right asymmetry A_{LR}

The measurement of the left-right asymmetry A_{LR} by the SLD Collaboration at the SLC, exploiting the high degree of polarisation obtained with the electron beam, provides the single most accurate value of $\sin^2\theta_{\text{eff}}^{\text{lept}}$. The present error is $\delta \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.00047$. The projected future error depends on how large the future integrated luminosity will be, but is expected to be in the range 0.00021 to 0.00029 [48]. Here the error is taken to be $\delta \sin^2\theta_{\text{eff}}^{\text{lept}} = 0.00025$.

quantity	current error	estimated future error
δR_b	0.0011	0.0008
δR_c	0.0056	0.0042
$\delta A_{\text{FB}}^{0,b}$	0.0023	0.0020
$\delta A_{\text{FB}}^{0,c}$	0.0048	0.0040
δA_b	0.049	0.027
δA_c	0.084	0.044

Table 18: Current and estimated future precision of the LEP Z heavy quark parameters.

8.6 Expected constraints on Standard Model fits

The expected errors on electroweak quantities discussed in the previous sections have been used to investigate the corresponding precision on the SM parameters. In these fits it is further assumed that m_t will be measured at hadron colliders to an accuracy of $\delta m_t = 3$ GeV, and that the error on $\alpha^{-1}(m_Z)$ will be reduced from 0.09 to 0.05.

In order to perform these fits the values of all the electroweak variables are fixed at their SM values corresponding to a certain choice of m_t , m_H and $\alpha_s(m_Z)$. The values $m_t = 175$ GeV and $\alpha_s(m_Z) = 0.123$ are used throughout; but several values of m_H are used. It is, of course, the impact of the improved precision on the indirect determination of m_H which is of particular interest.

	$m_H = 100 \text{ GeV}$	$m_H = 300 \text{ GeV}$	$m_H = 500 \text{ GeV}$
standard assumptions	100^{+49}_{-35}	300^{+122}_{-89}	500^{+199}_{-143}
$\delta m_W = 20 \text{ MeV}$	100^{+46}_{-34}	300^{+108}_{-83}	500^{+175}_{-131}
$\delta \sin^2 \theta_{\text{eff}}^{\text{lept}} (A_{\text{LR}}) = 0.00015$	100^{+45}_{-33}	300^{+111}_{-83}	500^{+182}_{-134}
$\delta m_t = 0.5 \text{ GeV}$	100^{+39}_{-30}	300^{+90}_{-71}	500^{+146}_{-114}
$\delta \alpha^{-1} = 0.09$	100^{+62}_{-42}	300^{+141}_{-101}	500^{+229}_{-159}

Table 19: Results of SM fits to m_H , assuming the expected future accuracy on electroweak quantities, as discussed in the text. These are referred to as the standard assumptions. The value of each variable is the SM value for $m_t = 175 \text{ GeV}$, $\alpha_s(m_Z) = 0.123$ and either $m_H = 100, 300$ or 500 GeV . The fitted error on m_t is $\simeq 3 \text{ GeV}$ and that on $\alpha_s(m_Z)$ is $\simeq 0.002$. In the subsequent fits the errors assigned are the same as for the standard assumptions, except on the quantity shown.

The results of these fits are given in table 19. It can be seen that, particularly in the case of a light Higgs, a precise constraint on m_H should be obtained.

It is of interest to investigate the relative sensitivities of the precision electroweak variables used. To see this the fits in table 19 have been repeated, changing the error on just one quantity at a time from the standard assumptions. This is repeated separately for each value of m_H . The results are also given in table 19. Further improvements in either the accuracy of m_W or A_{LR} will improve the indirect determination of m_H . A more substantial improvement is obtained if the top-quark mass could be measured to 0.5 GeV or better, as might be expected at the Next Linear Collider. However, if the accuracy on $\delta \alpha^{-1}$ is not improved from its present value of 0.09 , then the limits on m_H will be substantially worse. This again emphasises the importance of the low energy e^+e^- measurements. No account of the theory error has been included in these estimates. Given the long timescale it is hoped that the theoretical uncertainties can be significantly reduced !

The expected precision on the indirect versus direct determinations of m_t and m_W has also been estimated. Using the expectations discussed above, a comparison of the probable direct accuracy on m_W (from hadron colliders and LEP 2) and m_t (from hadron colliders), compared to the indirect accuracy on these quantities (from LEP and SLC), is shown in Fig. 21. The central values used for this figure are for $m_t = 175 \text{ GeV}$ and $m_W = 80.39 \text{ GeV}$. It can be seen that the expected accuracy on m_W for the direct and indirect determinations are comparable, but the expected precision on m_t from the indirect determination is considerably worse.

Figure 21: A comparison of possible future precision of the direct versus indirect determinations of m_t and m_W . The contour corresponds to the 70% confidence level.

8.7 Improvements in model independent variables

The expected improvements in the quasi model independent ϵ variables from improvements in the precision of electroweak variables has also been investigated. Fig. 22 shows the expected future precision compared to the present precision as discussed in section 7. For the sake of comparison the central values of all quantities have been set to correspond to the SM values $m_t = 175$ GeV, $m_H = 100$ GeV and $\alpha_s(m_Z) = 0.123$. The impact of the expected improvements in the precision of m_W on ϵ_2 can be clearly seen.

These precision electroweak data will remain very important in our understanding of Nature. If and when new particles are discovered these data will help constrain the scope of models beyond the SM. It is therefore imperative that they are obtained with the best possible accuracy.

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Figure 22: Results of fits to ϵ_1 , ϵ_2 and ϵ_3 , showing the 70% confidence level contours for the present data (outer contour) and for the expected future precision (inner shaded contour). In both cases the values of all quantities have been set to the SM values corresponding to $m_t = 175$ GeV, $m_H = 100$ GeV and $\alpha_s(m_Z) = 0.123$.

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