Integral Relations for Twist 2 and Twist 3 Contributions to Polarized Structure Functions\textsuperscript{1}

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Abstract. We discuss the relations between the twist 2 and twist 3 contributions to polarized deep-inelastic scattering structure functions both for neutral and charged current interactions which are predicted by the operator product expansion in lowest order in QCD.

INTRODUCTION

In the case of polarized deep inelastic scattering the cross sections depend on (up to) three unpolarized $F_i(x, Q^2)\textsuperscript{3}_{i=1}$ and five polarized structure functions $g_j(x, Q^2)\textsuperscript{5}_{j=1}$ in the limit of vanishing fermion masses. The lowest twist contributions are those of twist 2 for the structure functions $F_i(x, Q^2)$ and $g_{1,4,5}(x, Q^2)$. The structure functions $g_2(x, Q^2)$ and $g_3(x, Q^2)$ contain as well twist 3 terms \textsuperscript{[1,2]}. In this note we give a summary on the relations between the twist 2 and twist 3 contributions to the different structure functions in lowest order in QCD. We also comment on a sum rule which has been derived in ref. \textsuperscript{[3]} recently.

TWIST 2

For the twist 2 contributions to the structure functions one may seek a partonic interpretation. However, in lowest order in QCD only two generic parton combinations exist to describe the polarized structure functions:

\begin{equation}
g_1(x, Q^2) \propto \Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2) \quad \text{and} \quad g_5(x, Q^2) \propto \Delta q(x, Q^2) - \Delta\bar{q}(x, Q^2).
\end{equation}
The remaining three structure functions are therefore related to this basis by three linear operators. Two of the corresponding relations have been known for several years already, the Dicus relation [4]

\[ g_4^i(x, Q^2) = 2x g_5^i(x, Q^2), \]  

(2)

and the Wandzura-Wilczek relation [5]

\[ g_2^i(x, Q^2) = -g_1^i(x, Q^2) + \int_1^x \frac{dy}{y} g_1^i(y, Q^2). \]  

(3)

The third relation has been found only recently in ref. [6]

\[ g_3^i(x, Q^2) = 2x \int_1^x \frac{dy}{y^2} g_4^i(y, Q^2). \]  

(4)

Eqs. (2–4) can either be obtained analyzing the polarized structure functions by means of the operator product expansion [6,2] or in applying the covariant parton model [7], cf. [6]. In the operator product expansion they result from equating different expressions for the matrix element \( a_n \) of the symmetric part of the quark operators in lowest order QCD, see e.g. [2]. These relations between the different contributions to the longitudinal and transverse spin projections of the hadronic tensor are illustrated in Figure 1.

\[
\begin{align*}
W_{\mu\nu}^\|$ & = \, i\varepsilon_{\mu\nu\alpha\beta} \frac{g_\alpha P_\beta}{\nu} g_1(x) + \frac{P_\mu P_\nu}{\nu} g_4(x) \quad -g_{\mu\nu} g_5(x) \\
\uparrow & \quad \uparrow \\
W_{\mu\nu}^\perp & = \, i\varepsilon_{\mu\nu\alpha\beta} \frac{g_\alpha S_\beta^\perp}{\nu} [g_1(x) + g_2(x)] + \frac{P_\mu S_\nu + P_\nu S_\mu^\perp}{2\nu} g_3(x) \\
\Delta q & + \Delta q^\perp \\
\Delta q & - \Delta q^\perp
\end{align*}
\]

Figure 1: Relations between the twist 2 contributions of the polarized structure functions

Whereas the corresponding parts of \( W_{\mu\nu}^\|$ and \( W_{\mu\nu}^\perp \) are connected by integral relations, those acting in either part are just multiplications by a factor. For the valence parts this holds as well for the two contributions to \( W_{\mu\nu}^\perp \).

2) A derivation of eq. (3) using the latter method was given in refs. [8] before.

3) The lowest moments of \( g_i(x) \) \( i=1 \) were studied in ref. [9] and agree with the corresponding relations derived directly from eq. (2–4).
TWIST 3

For the twist 3 contributions to the structure functions $g_2(x, Q^2)$ and $g_3(x, Q^2)$, which emerge in the different neutral and charged current reactions, the operator product expansion \cite{2} implies the relation:

$$\int_0^1 dx x^n \{ 4g_5 - \frac{n + 1}{x}g_3 \}^{\nu_\nu} = \frac{12(n-1)}{n} \int_0^1 dx x^n \{ ng_1 + (n+1)g_2 \}^{\gamma p-\gamma n}, n = 2, 4 \ldots . \quad (5)$$

It results from equations between differences of the matrix elements $d_n$ of the non-symmetric part of the quark operators in lowest order QCD. Since one may express the twist 3 contributions to $g_2$ and $g_3$ by

$$g_2^{III}(x, Q^2) = g_2(x, Q^2) + g_1(x, Q^2) - \int_x^1 \frac{dy}{y} g_1(y, Q^2), \quad (6)$$

$$g_3^{III}(x, Q^2) = g_3(x, Q^2) - 4x \int_x^1 \frac{dy}{y} g_5(y, Q^2), \quad (7)$$

the analytic continuation of eqs. (6,7) in $n$ can be rewritten by

$$g_3^{III, \nu_\nu}(x, Q^2) = 12 \left[ x g_2^{III}(x, Q^2) - \int_x^1 dy g_2^{III}(y, Q^2) \right]^{\gamma p-\gamma n} \quad (8)$$

as a relation between twist 3 contributions only.

Recently, a sum rule for the valence part of the structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$

$$\int_0^1 dx x (g_1^V(x) + 2g_2^V(x)) = 0 \quad (9)$$

was discussed in ref. \cite{3}. We would like to investigate the relation of eq. (9) to the operator product expansion. Here one firstly meets the problem that the valence parts $g_1^V(x)$ and $g_2^V(x)$ cannot be isolated for electromagnetic interactions from the complete structure functions and a formulation of eq. (9) with the help of the local operator product expansion is thus not straightforward.

On the other hand, one may consider

$$\int_0^1 dx x^n (g_1^-(x, Q^2) + 2g_2^-(x, Q^2)) = \sum_q \frac{((g_q^V)^2 + (g_q^A)^2)(nd_n^q - (n-1)a_n^q)}{4(n+1)}, \quad n = 1, 3 \ldots , \quad (10)$$

\footnote{This sum rule was found firstly in ref. \cite{10} for a specific flavor combination.}
which results from eqs. (61,62), ref. [2], in the charged current case. It is
easily seen that the left-hand-side of eq. (10) includes only valence quark
contributions and one may even rewrite eq. (10) for individual quark flavors
separately, denoting the valence parts of the corresponding matrix elements
by $a^V_n$ and $d^V_n$, respectively. For the first moment one obtains

$$
\int_0^1 dx (g_1^V(x, Q^2) + 2g_2^V(x, Q^2)) = \frac{e_q^2}{8} d_1^V(q).
$$

The right-hand-side of eq. (11) vanishes in the case of massless quarks, because

$$
d_1^V(S^\beta P^\mu - S^\mu P^\beta) = m_q \langle PS | \bar{q} i \gamma_5 \sigma^{\beta \mu} q | PS \rangle 
\equiv \frac{m_q}{M} (P^\beta S^\mu - P^\mu S^\beta) \int_0^1 dx (h_1^q(x) - \bar{h}_1^q(x)).
$$

For the latter equation, see [11]. $h_1^q(x)$, $\bar{h}_1^q(x)$ are the quark and antiquark
transversity functions, respectively, which can be measured in the Drell–Yan
process. The right-hand-side of eq. (12) vanishes in the limit $m_q \to 0$, which
yields $d_1^V = 0$. Due to this, eq. (12), similar to the case of the Burkhardt–
Cottingham sum rule [12],

$$
\int_0^1 dx g_2^V(x, Q^2) = 0,
$$

is not described by the operator product expansion, but is formally consistent
with it.

REFERENCES

441 (1976).
therein.
443.