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WAVELET REPRESENTATION OF THE NUCLEAR DYNAMICS

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Abstract

The study of transport phenomena in nuclear matter is addressed in a new approach named DYWAN, based on the projection methods of statistical physics and on the mathematical theory of wavelets. In this framework the derivation of the equations of motion and the resolution schemes are consistently defined, following a precise hierarchy of approximations. The latter especially rely on scales separation and on the decomposition of the information on orthogonal subspaces. Strongly compressed representations of the nuclear systems are obtained with an accurate description of the wave functions and of their antisymmetrization. The results of the approach are illustrated for the ground state description as well as for the dissipative dynamics of nuclei at intermediate energies.

1 Introduction

The appearance of new and efficient experimental facilities, especially of high quality 4π detectors, has made possible the study of new physical processes observed in heavy-ion physics at intermediate energy, such as multifragmentation, vaporization and collective flow. The highly excited nuclear systems produced in these reactions constitute an appropriate framework for studying the properties of hot and condensed nuclear matter and for extracting information on the nuclear equation of state. These nuclear systems are constituted of a finite number of mutually interacting particles, involving a huge number of degrees of freedom, and consequently their analysis is rather complex. Different theoretical investigations exist, in particular quite successful statistical descriptions at equilibrium [1] [2]. However, the dynamics is expected to play a significant role since the experimental data results from violent and far from equilibrium processes. The characterization of the role of the dynamical mechanisms requires the development of modelization and simulation technics. In accordance with the quality of the experimental data, these models have to be improved to take into account more completely the underlying quantal physical properties. Some advances have recently been made either by the treatment of density fluctuations through stochastic approaches [3] [4], or by taking explicitly into account the antisymmetrization and the fermionic nature of the problem [5] [6]. These methods suffer from high calculation cost which makes their application to the study of massive systems difficult. It is then worthwhile to seek other frameworks well suited to improve the physical description, while keeping a fast numerical treatment by an adapted optimization of the representation space.

In a previous work [7], it has been shown how the wavelet theory permit the definition of this kind of representation. The advantages of this method relies on the definition of a precise hierarchy of approximations and on the optimization of the description by taking into account the geometrical and symmetry properties of the problem. The simulation of dynamical processes rely on the quality of the resolution techniques as well as on the approximations performed to derive the evolution equations. They are in fact intimately intertwined. In order to determine the scope of validity of the simulation, it is necessary to estimate precisely the amount of information which is lost in the different approximation processes. The huge number of degrees of freedom requires a reduction of the description to a restricted but convenient set of variables, adapted to the phenomena under study. It is essential to define a framework allowing a description of the time evolution of the system

through a precise control of the information. The natural context for this consists in using the projection methods of statistical physics [8] combined with the mathematical theory of wavelets [9]. In this framework the approximation schemes defined for the description of the physical observables and their mathematical representations present the advantage that they rely on common concepts. The purpose of the present paper is to illustrate the DYWAN (DYnamical WAvelet in Nuclei) model using the specific exemple of heavy-ion reaction at intermediate energies [10]. The realization presented here does not exhausts all the potentialities of the approach, thus giving the possibility for substantial improvements and developpements in the future. The application shown here, constitutes only a simplified version conceived to easly demonstrate the elementary principles of the model.

The paper is organized as follows. In section 2 the theoretical principles of the projection methods and wavelet theory are presented. Section 3 is devoted to the description of nuclei in the wavelet representation. Section 4 and 5 respectively address the description of the mean field and dissipative dynamics within the DYWAN approach.

2 Theoretical background of the DYWAN model

The dynamical description of systems which consist of a great number of mutually interacting particles in terms of the N body density matrix evolution, is formally given by the Liouville or Von-Neumann equations. Their resolution in principle, leads to a complete knowledge about the evolution of the system. However, because of the large number of degrees of freedom involved, the complexity makes their resolution unfeasible. Thus, it is necessary to define physical approximations to obtain tractable equations suited to describe the nuclear dynamics. In addition, the solutions generally cannot be expressed analytically, and then the approximation related to the resolution algorithm must be consistent with those required by the theoretical derivation. In any case, the predominant feature is the loss of information. The framework to take this into account is provided by the projection methods of statistical physics. Different formulations of these methods exist in references [8] [11] and references therein. In this paper the notation of Balian [8] will be adopted, since it relies on an elegant geometrical presentation.

The description of a many-component physical system at a microscopic level, relies on the knowledge of the density operator D , which contains all the information

necessary to determine its time evolution. Making the problem intelligible requires a description in term of a reduced set of variables considered as relevant for the phenomena under study, and to discard the others through suitable approximations. The determination of these relevant variables is most usually dictated by the scale properties of the system. The time evolution of the selected variables describe the dynamics of the system, while their coupling with irrelevant variables introduces non-linear dissipative effects.

At the microscopic level relevant variables are denoted by ω^i , while the observables which belong to the complete space will be denoted by A . In an incomplete description only the subset of observables ω^i is known. Their statistical averages ρ_i define the macroscopic relevant observables:

$$\rho^i = \text{Tr} D \omega^i = (\omega^i, D) \quad (1)$$

The fact that no information is available about the irrelevant observables makes the exact determination of the state D of the system impossible. Nevertheless the amount of missing information can be measured by defining the entropy $S(D)$ [8]:

$$S(D) = -\text{Tr} D \ln D \quad (2)$$

Among all the possible states compatible with the available information, we choose the one in which no hypothesis about the unknown degrees of freedom is made. This means the state D_0 which contains the least information on the irrelevant observables and which consequently maximizes the entropy. It contains the same information about the relevant variables that the exact states D do, but gives the least biased description of the irrelevant observables.

From the definition of the entropy, a scalar product $(A_1, A_2)_D$ of two observables can be defined, and then, a Riemanian structure in the space of observables is obtained. This makes possible the construction of two orthogonal projectors $\mathcal{P}$ and $\mathcal{Q} = I - \mathcal{P}$, which project the overall state space onto the relevant and irrelevant subspaces, respectively. Any observable A can thus be decomposed in the following way:

$$A = A\mathcal{P} + A\mathcal{Q} \quad (3)$$

The superoperator $\mathcal{P}$ acts either in the space of states or in the space of observables, and consequently any density operator can also be splitted in two parts:

$$D = \mathcal{P}D + \mathcal{Q}D = D_0 + D_1 \quad (4)$$

The exact density matrix of a quantum system evolves according to the Liouville-Von-Neumann equation:

$$\frac{dD}{dt} = -i[H, D] = -i\mathcal{L}D \quad (5)$$

where H and $\mathcal{L}$ are respectively the Hamiltonian and the Liouvillian of the system.

The projection of this equation onto the relevant and irrelevant subspaces leads to the coupled evolution equations of each component:

$$\left(\frac{d}{dt} + i\mathcal{P}\mathcal{L}\mathcal{P} - \mathcal{Q}\dot{\mathcal{P}} \right) D_0 = (-i\mathcal{P}\mathcal{L}\mathcal{Q} + \mathcal{P}\dot{\mathcal{P}}) D_1 \quad (6)$$

$$\left(\frac{d}{dt} + i\mathcal{Q}\mathcal{L}\mathcal{Q} + \mathcal{P}\dot{\mathcal{P}} \right) D_1 = (-i\mathcal{Q}\mathcal{L}\mathcal{P} - \mathcal{Q}\dot{\mathcal{P}}) D_0 \quad (7)$$

To close this set of equations, a supplementary condition must be added, a natural choice is provided by the maximal entropy condition characterized by the generalized canonical form:

$$D_0 = \exp\left(-\sum_i \lambda_i \omega^i\right) \quad (8)$$

The equation of motion of the relevant component D_0 can be expressed formally by elimination of the irrelevant component. The resulting time evolution of the relevant variables ρ_i , is driven by the generalized transport equation:

$$\begin{aligned} \dot{\rho}^i = (\varphi^i, D_0) &+ \int_{t_0}^t dt' (\varphi^i(t) \mathcal{W}(t, t'), \varphi^i(t'))_{D_0} \lambda_i(t') \\ &+ (\varphi^i(t); \mathcal{W}(t, t_0) D_1(t_0)) \end{aligned} \quad (9)$$

where $\mathcal{W}$ plays the role of an evolution superoperator in the irrelevant space. It is defined as the solution of equation (7), with vanishing right hand side. The observable φ^i given by:

$$\varphi^i = \dot{\omega}^i - i[\omega^i, H]$$

represents the force associated to the observable ω^i . Equation (9) clearly points out the characteristic contributions to transport phenomena. The first term of the right

hand side evidences the propagation due to the instantaneous mean field produced among the relevant observables. The second term displays the effects of the dynamical correlations built up during the time evolution of the system. It describes the dissipative behavior coming from the coupling with the irrelevant variables. The third term is related to the propagation of the initial correlations. This contribution will not be directly addressed here, since we will be concerned with systems assumed to be initially uncorrelated.

In its present form equation (9) is currently intractable, additional assumptions have to be implemented, as for example the short memory approximation. This approximation can be applied when the irrelevant variables change much more rapidly in time than the relevant observables. In the framework of nuclear reaction at low energies, the time interval between two successive nucleon collisions is commonly assumed to be greater than the interaction time of the colliding particles. Therefore the propagator $\mathcal{W}(t, t')$ involves rapid oscillations while φ varies much more slowly. The dominant contributions of the integrand are then obtained for $(t \simeq t')$, since oscillating contributions which interfere destructively appear as the interval $(t - t')$ increases. According to the previous considerations, the short memory or Markov approximation can then be carried out, and in that case the evolution superoperator in the irrelevant space can be cast in the following simple form:

$$\mathcal{W}(t, t') = \exp(-i(t - t')\mathcal{Q}\mathcal{L}\mathcal{Q}) \mathcal{Q}(t) \quad (10)$$

Another convenient step towards a more tractable evolution equation consists in doing a perturbative approximation taking into account the splitting of the Hamiltonian (as well as the Liouvillian) of the system into two parts, according to the previously defined projection superoperator:

$$H = H_0 + H_1 = H\mathcal{P} + H\mathcal{Q} \quad (11)$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 = [\mathcal{H}_0,] + [\mathcal{H}_1,] \quad (12)$$

The perturbative expansion relies on the assumption that H_0 dominates the evolution of the relevant observables while H_1 acts as a perturbation. In addition, making the assumptions that $\mathcal{L}_0$ has no off-diagonal elements and that H_0 keeps the relevant subspace unchanged, leads to a reduction of equations (6) and (7) to the following form:

$$\left(i \frac{d}{dt} - \mathcal{L}_0 - \mathcal{P}\mathcal{L}_1 \right) D_0 = \mathcal{P}\mathcal{L}_1 D_1 \quad (13)$$

where the irrelevant component D_1 of the density operator can be written formally as:

$$D_1(t) = W(t; t_0)D_1(t_0) - i \int_{t_0}^t ds W(t, s) \mathcal{Q}(s) L_1 D_0(s) \quad (14)$$

Combining the perturbative expansion and the Markovian approximation, the propagator W can be approximated by:

$$W(t, t') = \exp(-i[\mathcal{L}_0(t) + \mathcal{Q}\mathcal{L}_1(t)\mathcal{Q}](t - t')) \mathcal{Q}(t) \quad (15)$$

Considering an interacting fermionic system with at most two body interactions, the Hamiltonian can be written explicitly as:

$$H = \sum_{\alpha\gamma} e_{\alpha\gamma} a_{\alpha}^{\dagger} a_{\gamma} + \frac{1}{4} \sum V(\alpha\beta\gamma\delta) a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta} = H_{sp} + V \quad (16)$$

where a and $a^{\dagger}$ are respectively the creation and annihilation fermion operators, $e_{\alpha\gamma}$ the matrix elements of the kinetic part and $V(\alpha\beta\gamma\delta)$ the elements of the two body interaction matrix. The previous approximations are intimately related to the assumption that the dynamics is mainly governed by the mean-field. The natural set of relevant observables is thus constituted by the single particle operators:

$$\omega^i = \omega^{\alpha\beta} = a_{\alpha}^{\dagger} a_{\beta} \quad (17)$$

Therefore, using an extended form of Wick's theorem [12] and denoting by $:$ the normal product, the splitting of the Hamiltonian in its relevant and irrelevant part can be explicitly written as:

$$H_0 = H\mathcal{P} = E_{HF}[\rho] + : \hat{h}[\rho] : \quad (18)$$

$$H_1 = H\mathcal{Q} = : V : \quad (19)$$

with:

$$E_{HF}[\rho] = \text{tr}(e\rho) + \frac{1}{2} \text{tr} \text{tr} \rho V \rho = \text{Tr} H D_0 \quad (20)$$

$$\hat{h}[\rho] = \sum_{\alpha\gamma} h_{\alpha\gamma}[\rho] a_{\alpha}^{\dagger} a_{\gamma} \quad (21)$$

$$h_{\alpha\gamma}[\rho] = e_{\alpha\gamma} + \omega_{\alpha\gamma}[\rho] \quad (22)$$

$$\omega_{\alpha\gamma}[\rho] = \text{tr} V \rho = \sum_{\beta\delta} V(\alpha\beta\gamma\delta) \rho_{\beta\delta} \quad (23)$$

The relevant part of the Hamiltonian corresponds to the single particle Hamiltonian while the irrelevant part is given by the correlation : V : or residual interaction. Multiplying equation (13) by $\omega_{\alpha\gamma}$ and taking the trace one gets the dissipative evolution equation of the one body density matrix:

$$i\dot{\rho} = [h(\rho), \rho] + iI(\rho) \quad (24)$$

The first term describes the evolution of the density matrix under the action of the mean-field $h(\rho)$, while $I(\rho)$ stands for the collision term. This can be expressed by the Markov approximation and by using Wick's theorem. In the basis $|\alpha\rangle$ which diagonalizes the Hamiltonian, the collision term takes the form:

$$\begin{aligned} I_{\bar{\gamma}\bar{\alpha}} &= \frac{1}{2}i \sum V(\alpha\beta\bar{\alpha}\delta)\delta(\varepsilon_{\bar{\gamma}} + \varepsilon_{\delta} - \varepsilon_{\alpha} - \varepsilon_{\beta})V(\alpha'\beta'\gamma'\delta') \\ &\quad \cdot [\bar{\rho}_{\bar{\gamma}\alpha'}\bar{\rho}_{\delta\beta'}\rho_{\delta'\beta}\rho_{\gamma'\alpha} - \rho_{\bar{\gamma}\alpha'}\rho_{\delta\beta'}\bar{\rho}_{\delta'\beta}\bar{\rho}_{\gamma'\alpha}] \\ &- \frac{1}{2}i \sum V(\bar{\gamma}\beta\gamma\delta)\delta(\varepsilon_{\gamma} + \varepsilon_{\delta} - \varepsilon_{\bar{\alpha}} - \varepsilon_{\beta})V(\alpha'\beta'\gamma'\delta') \\ &\quad \cdot [\bar{\rho}_{\gamma\alpha'}\bar{\rho}_{\delta\beta'}\rho_{\delta'\beta}\rho_{\gamma'\alpha} - \rho_{\gamma\alpha'}\rho_{\delta\beta'}\bar{\rho}_{\delta'\beta}\bar{\rho}_{\gamma'\alpha}] \end{aligned} \quad (25)$$

with

$$\bar{\rho}_{\alpha\gamma} = \delta_{\alpha\gamma} - \rho_{\alpha\gamma}$$

Equations (24) and (25) constitute the quantum generalization of the semi-classical Boltzmann equation [13].

A similar collision term can be derived from either the Green's functions techniques or from the reduced density matrices [14] [15]. Let us consider the latter case. It provides a complementary point of view to the projection method. As a matter of fact equation (25) can also be obtained [16] from the framework of reduced matrices techniques from the collision term:

$$I(1, t) = \text{tr}_2 L(1, 2) \int_0^t d\tau U_2(1, 2, \tau) [C_2 \rho_2^0(t - \tau) + C_3 \rho_3^0(t - \tau)] \quad (26)$$

where $L(1, 2)$ corresponds to the Lagrangian operator associated to the residual interaction, $U(1, 2, \tau)$ is the two particle propagator, $C_2 \rho_2^0(t - \tau)$ and $C_3 \rho_3^0(t - \tau)$ describe respectively the creation of the two and three body correlations from the vacuum of correlation.

Another interesting feature concerning the collision term has to be stressed. The full N body density matrix D describes a system with a fixed number of particles

even though the density matrix D_0 associated with the relevant space and leading to (24) is defined in the Fock space and has the canonical form (8) , and preserves the particle number only on the average.

As it stands, the numerical treatment of the collision term (25), is a prohibitive task. Despite the fact that, mainly at low energy regimes the off-diagonal terms can play an important role, a great reduction of the description is achieved by neglecting these terms, which decay faster than the diagonal ones [15]. These supplementary approximations lead to a master equation [15] for the occupation numbers of single particle states. This master equation consists in restricting the set of relevant observables to occupation number operators $\hat{n}_\mu = c_\mu^\dagger(t)c_\mu(t)$ [12]. These operators are built from the creation and annihilation operators $c_\mu^\dagger(t)$ and $c_\mu(t)$ related to a set of states $|\mu(t)\rangle$, solutions of a TDHF-like equation:

$$i\frac{\partial}{\partial t}|\mu(t)\rangle = \hat{h}[\rho]|\mu(t)\rangle \quad (27)$$

The associated density matrix is then given by:

$$\rho_{\alpha\gamma}(t) = \sum_{\mu} \langle \gamma|\mu(t)\rangle n_{\mu}(t) \langle \mu(t)|\alpha \rangle \quad (28)$$

and the corresponding occupation numbers by:

$$n_{\mu} = Tr \hat{n}_{\mu} D_0 \quad (29)$$

In this context, the relevant variables become explicitly time dependent, and the space of the relevant variables are no longer conserved in the time. Using the projection technique described above, with the occupation numbers playing the role of the relevant observables, it can be shown that the unperturbed Hamiltonian H_0 is [12]:

$$H_0 = H\mathcal{P} = E_{HF} + : \mathcal{H}_0 : \quad (30)$$

$$\mathcal{H}_0 = \sum_{\nu} \varepsilon_{\nu}(t) \hat{n}_{\nu}(t) \quad (31)$$

$$\varepsilon_{\nu}(t) = \langle \nu(t) | \hat{h}[\rho] | \nu(t) \rangle$$

The residual interactions contain in addition to the correlation : V :, the off diagonal component of the single particle Hamiltonian:

$$H_1 = H\mathcal{Q} = \hat{h}[\rho]\mathcal{Q} + :V: \quad (32)$$

$$\hat{h}[\rho]\mathcal{Q} = \sum_{\mu \neq \nu} h_{\mu\nu}[\rho] c_\mu^\dagger c_\nu \quad (33)$$

The time derivative of equation (29), according to the previous assumptions, provides the master equation of the occupation numbers of mean field single particle states [12]:

$$\begin{aligned} \dot{n}_\mu = & \pi \sum |V(\mu\kappa\nu\lambda)|^2 \delta(\epsilon_\mu + \epsilon_\kappa - \epsilon_\nu - \epsilon_\lambda) \\ & [\bar{n}_\mu \bar{n}_\kappa n_\nu n_\lambda - n_\mu n_\kappa \bar{n}_\nu \bar{n}_\lambda] \end{aligned} \quad (34)$$

where ϵ_μ are the single particle energies. Equations (27), (28) and (34) constitute description of the dissipative evolution equation for the single particle density matrix.

Despite the set of strong assumptions made to derive the equation of evolution (34), the latter is still complex and its numerical resolution has not yet been fully addressed. Nevertheless, one can expect that the resolution schemes will gain in efficiency if their mathematical framework is strongly connected with the concepts used to set up the evolution equations of the physical systems. The wavelet theory offers the mathematical background from which this approach can be developed.

The wavelet theory provides efficient and adaptative representations of the physical space. Suppose we are concerned with a physical quantity f which is a function of a continuous variable x , then the continuous wavelet transform of f is characterized by the set of coefficients D_{ab} defined by:

$$D_{a,b} = \int_{-\infty}^{\infty} f(x) \overline{\psi_{a,b}}(x) dx = \langle f, \psi_{a,b} \rangle, \quad (35)$$

where $\bar{\psi}$ is the complex conjugate of the generating function ψ . The family of wavelets $\psi_{ab}(x)$ is generated by dilating and translating the function of reference ψ , called the mother wavelet, in the following way:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right), \quad (36)$$

where $a \in \mathbb{R}^{+*}$ is a dilation parameter and $b \in \mathbb{R}$ a translation parameter. The resulting functions given by equation (36) belong to the Hilbert space $L^2(R)$, and in the same way as the reference function, they are oscillating functions with vanishing momentum. They take significant values on an interval located around the position given by the translation parameter. The size of this interval is proportional to the dilation parameter a . As a consequence, the coefficients D_{ab} provide information over the behavior of the analyzed function on the corresponding interval. An important property of wavelet analysis is that all the information contained in the analyzed signal is conserved in the set of coefficients D_{ab} , and from their knowledge it is possible to reconstruct, in an exact way, the initial function. Nevertheless, calling $\hat{\psi}$ the Fourier transform of the wavelet ψ , the following condition must be satisfied:

$$D_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \quad (37)$$

In that case the continuous wavelet transform is invertible, and the inverse transform is given by the relation:

$$f(x) = \frac{1}{D_\psi} \int_0^\infty \frac{da}{a^2} \int_{-\infty}^\infty D_{a,b} \psi_{a,b}(x) db \quad (38)$$

Wavelet families are generated by the unique unitary irreducible representation $U(a, b)$ [17]:

$$(U(a, b)\psi)(x) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right), \quad (39)$$

of the affine group G_{aff} characterized by the action:

$$x \longrightarrow ax + b, \quad (40)$$

and the group law

$$(a, b)(a', b') = (aa', b + ab') \quad (41)$$

The wavelet analysis can then be fully addressed from the group theoretic point of view [18]. Actually, wavelets form a class of generalized coherent states resulting from the Gilmore Perelomov theory of generalized coherent states over homogeneous spaces. The generalization to similitude groups in n -dimensional spaces straightforwardly yields multidimensional wavelets.

An essential property of wavelets is that it provides an efficient way to optimize the representation spaces. As a matter of fact, it defines mathematical tools suited to

determine bases of representations where the information is compressed. It means that the number of relevant coefficients is as restricted as possible. This can be obtained with wavelets which have a great number N of vanishing moments defined by the condition:

$$\int_{-\infty}^{\infty} x^n \psi(x) dx = 0 \quad \text{for } n = 0, \dots, N \quad (42)$$

The number of characteristic coefficients decreases as the number of vanishing moments increases and consequently the greater is N , the more efficient is the analysis. A great advantage of wavelet analysis is that it makes possible the definition of discrete analysis and especially a class of them using orthogonal bases. This is achieved in the following parameterization of the dilation and translation parameters:

$$a = 2^m \quad b = n2^m \quad (43)$$

where m, n range over Z . With this parameterization the associated wavelets family is composed of the elements

$$\psi_{m,n}(x) = 2^{-m/2} \psi(2^{-m}x - n) \quad (44)$$

The discrete wavelet transform is then defined by [9]:

$$D_{m,n} = \int_{-\infty}^{\infty} f(x) \overline{\psi_{m,n}(x)} dx = \langle f, \psi_{m,n} \rangle \quad (45)$$

All the information given by the initial signal is contained in this set of coefficients and the reconstruction formula is given by:

$$f = \sum_{m,n} \langle f, \psi_{m,n} \rangle \psi_{m,n} . \quad (46)$$

A striking interest of wavelet analysis relies on its multiresolution analysis [9]. As a matter of fact, it introduces a well defined scheme of decomposition of the information on complementary and orthogonal subspaces, following a suited criteria. In this sense, the multiresolution analysis and the projection methods in statistical physics, both rely on the same principle in which a given space can be split into orthogonal subspaces, according to the information criteria. The principal idea consists in defining successive approximation f_j of the analyzed function f by projecting it onto a set of subspaces V_j embedded in the following way:

$$\dots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset \dots$$

The approximation of f at the different levels j is given by the orthogonal projection of f onto the subspace V_j

$$f_j = A_j f = \sum_k \langle f, \phi_{j,k} \rangle \phi_{j,k} = \sum_k C_{jk} \phi_{j,k} , \quad (47)$$

A_j being the orthogonal projector onto V_j , and the set of functions

$$\phi_{j,k}(x) = 2^{-j/2} \phi(2^{-j}x - k) \quad (48)$$

called scaling functions. They constitute an orthonormal basis of this subspace. The successive coefficients C_j give information about the signal at different scales j . The number of coefficients decreases as the scale j increases, and consequently the successive approximations f_j correspond to lower and lower resolution versions of the original function. The loss of information between two successive approximations will then be recovered by defining the space W_j which complements V_j in V_{j-1} , i.e. a space that satisfies:

$$V_{j-1} = V_j \oplus W_j \quad (49)$$

The difference between two successive approximations can be fully expressed in terms of wavelets

$$f_{j-1} - f_j = \sum_k \langle f, \psi_{j,k} \rangle \psi_{j,k} = \sum_k D_{jk} \psi_{j,k} \quad (50)$$

The set of coefficients $C_{j,k}$ and $D_{j,k}$, contains exactly the same amount of information as the scale coefficients $C_{j+1,k}$ defined at the previous scale. The original information is then distributed on orthogonal and complementary subspaces in a fully ordered way. Scaling functions will focus on the slow oscillating components of the analyzed signal while wavelets will give information about the fast oscillations. This representation makes possible to define a hierarchy of approximations of the original function and to separate its high and low frequency contributions on different scales.

Since the model is dealing with the time evolution of nuclear systems in a wavelet representation, the dynamical properties can be conveniently incorporated through the affiliation of wavelet to group theory formalism [19]. As mentioned previously, wavelets can be viewed as generalized coherent states, in the framework of the Gilmore-Perelomov theory [20] [21]. The dynamics of a system is driven by its Hamiltonian which in general, for complex systems, needs to be conveniently approximated and expressed in terms of a given set of observables. Through a suitable

scheme of approximations, these operators define a dynamical group G which spans the related algebra.

As it is well known from the coherent states theory, the construction of generalized coherent states relies on the choice of the dynamical group G and a reference state $|\Phi_0\rangle$. This state defines a maximum stability subgroup H which contain all the elements h which leave the reference state unchanged except for a phase factor

$$h|\Phi_0\rangle = e^{i\Phi(h)}|\Phi_0\rangle \quad (51)$$

Each element g which belongs to G can be expressed in a unique way by a product of an element h belonging to H and an element Ω of the coset space G/H :

$$g = \Omega h \quad (52)$$

Coherent states are then defined by the action of the coset element Ω on the reference state. Then the coherent states $|\beta\gamma\rangle = \Omega|\Phi_0\rangle$ preserves the algebra and topological properties of the coset.

The choice of the group and the reference state leads to different families of generalized coherent states. Canonical coherent states are defined in the special case where the dynamical group G is the Weyl Heisenberg group associated to the set of generators $\{n = a^\dagger a, a^\dagger, a, I\}$. The reference state has a stability subgroup H related to the generators $\hat{n}$ and I , and it exhibits a gaussian structure in coordinate representation. The canonical coherent states are generated by a translation operator $D(\beta)$ on the reference state. The corresponding map is the windowed Fourier transform or Gabor transform. Another class of generalized coherent states called correlated coherent states are introduced by considering a dynamical group related to the set of operators $\{n = a^\dagger a, a^{\dagger 2}, a^2, a^\dagger, a, I\}$. The resulting correlated coherent states are generated by a complementary operator $S(\gamma)$ which consists in applying a dilation operation on the reference state [7]:

$$|\beta\gamma\rangle = S^{-1}(\gamma)D(\beta)|0\rangle \quad (53)$$

Defining the complex parameters α_1 and α_2 :

$$\alpha_1 = \sqrt{m\omega}(1 - i|\gamma|^2 + i\gamma^2) \quad (54)$$

$$\alpha_2 = \frac{1}{\sqrt{m\omega}}(1 - i|\gamma|^2 - i\gamma^2) \quad (55)$$

the translation and dilation of these wavelets can be expressed through the first and second moments of the correlated coherent states in the configuration space representation:

$$\xi = \langle x \rangle = \sqrt{2\hbar} \operatorname{Re}(\beta\alpha_2^*) \quad \chi = \langle (x - \xi)^2 \rangle = \frac{\hbar}{2} |\alpha_2|^2 \quad (56)$$

or equivalently in the spectral space representation:

$$\pi = \langle p \rangle = \sqrt{2\hbar} \operatorname{Im}(\beta\alpha_1^*) \quad \phi = \langle (p - \pi)^2 \rangle = \frac{\hbar}{2} |\alpha_1|^2 \quad (57)$$

the momentum-position correlation satisfying the generalized uncertainty relation:

$$\sigma = \langle (x - \xi)(p - \pi) \rangle = -\frac{\hbar}{2} \operatorname{Im}(\alpha_1\alpha_2^*) \quad (58)$$

which are bounded by the generalized uncertainty relationship

$$\Delta = \chi\phi - \sigma^2 = \frac{\hbar^2}{4} . \quad (59)$$

Even if the correlated coherent states are a special case of wavelets, they present interesting properties, especially they can be expressed analytically and then they offer the possibility to control very precisely the efficiency and the quality of numerical treatments involved in the simulation of the dynamics.

3 Static description

One of the main purposes here, is to establish an efficient and reliable treatment of the evolution equations for the relevant states or observables displayed in section 2, in the context of the wavelet representation. It is obvious that the quality of the dynamical description will depend on the initial representation of the studied system. The properties of the static description must be preserved in time as well as possible, at least on the characteristic times of the processes we intend to study. Moreover, the determination of the representation basis is essential for the optimization of the numerical treatment. Let us focus on the description of nuclei in their initial configuration. In a first step, our approach will be addressed in some special cases where analytical solutions are available in order to check the quality of the model. Afterwards it will be applied to the case of many body systems in a selfconsistent mean-field.

The nuclear mean-field potential is built up by the mutual action of all the nucleons, but well known analytical ansatz exists which reproduce well the nuclear potential [22]. Due to possible comparisons with analytical solutions a preliminary study of the mean-field description can be carried out in the case of a nuclear potential approximated by an harmonic one. The harmonic potential is expressed as follows:

$$V(r) = \frac{1}{2}m\omega_0^2 r^2 \quad (60)$$

where ω_0 is the frequency of the oscillator which is connected to the number of nucleons A of the system by [22]:

$$\hbar\omega_0 \simeq \frac{5}{4} \left(\frac{3}{2}\right)^{1/3} \frac{\hbar^2}{mr_0^2} A^{1/3} \quad (61)$$

This potential exhibits a discrete energy level structure which in this case consists of equidistant levels. In the one dimensional case, the Hamiltonian associated with such a system is given by:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2. \quad (62)$$

The associated eigenvectors can be analytically expressed in the configuration space through the well known expression:

$$\varphi_n(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} \frac{e^{-\beta^2 \frac{x^2}{2}}}{\sqrt{2^n n!}} H_n(\beta x) \quad \text{with} \quad \beta = \sqrt{\frac{m\omega}{\hbar}}, \quad (63)$$

H_n being the Hermite polynomials.

The starting point of the wavelet representation relies on the expansion of the single particle wave functions of nucleons $|\varphi^\lambda\rangle$ onto an orthogonal wavelet basis $|\psi_i\rangle$:

$$|\varphi^\lambda\rangle = \sum_i \omega_i^\lambda |\psi_i^\lambda\rangle \quad (64)$$

where the index i stands for both the dilation and translation parameters. A convenient mean field description of fermionic systems is performed through density operators ρ , which can be explicitly expressed in term of the one body particle wave functions $|\varphi^\lambda\rangle$ as :

$$\rho = \sum_{\lambda=0}^N |\varphi^\lambda\rangle \langle \varphi^\lambda| \quad (65)$$

With the summation of the different wave functions, the oscillations exhibited by the single particle wave functions are progressively smoothed. Taking into account all the contributions of the projection basis leads to an exact representation of the density matrix:

$$\rho = \sum_{\lambda=0}^N \sum_{i,j} \beta_{i,j}^{\lambda} |\psi_i^{\lambda}\rangle \langle \psi_j^{\lambda}| \quad (66)$$

with

$$\beta_{i,j}^{\lambda} = \omega_i^{\lambda*} \omega_j^{\lambda} \quad (67)$$

The characteristic feature of the wavelet analysis is the structuration of the information which enables one to define a controlled hierarchy of approximations. The use of the properties of wavelets such as orthogonality, symmetry and vanishing moments, gives the ability to concentrate the information on a reduced number of coefficients. These features have been investigated in reference [7], where a biorthogonal spline basis has been used [23]. This provides the advantage for dealing with symmetric functions which are in addition compactly supported. In the present work we have used an orthogonal wavelet basis. These functions preserve the symmetry properties and even if they are not compactly supported, they decay exponentially and they are numerically compactly supported. The resulting wavelet reconstruction of the density matrix in this case takes a quasi diagonal structure. According to the decomposition in scaling and wavelet functions, the analysis of the one-particle wave functions lead to an adaptative determination of the reference scale of the description. This reference level defines the subspace V_0 of the multiresolution analysis, related to the smoothest approximations of the single particle wave functions. The refinements of the reconstructions are introduced through the wavelet coefficients associated with the successive finer scales of the description. It has been shown [7], that the reference level provides a strongly compressed and reliable first approximation, characterized by a well defined separation of the frequency components and their optimized localisation in phase space. It has also be pointed out that the diagonal elements of the density operator contain the essential part of the global information of the particle system. The off-diagonal terms interfere destructively, exhibit high frequencies and vanishing average values. It is then convenient to define a first approximation level by restricting the index of equation (66) to $i = j$, as described in a previous work [7].

In addition the spline scaling and wavelet functions can be very accurately fitted by correlated coherent states [24], which have simple analytic expression. This

provides another level of description obtained by expressing the density matrix in term of correlated coherent states $|\alpha_i\rangle$:

$$\rho \simeq \sum_{\lambda=0}^N \sum_i \beta_i^\lambda |\alpha_i^\lambda\rangle \langle \alpha_i^\lambda| \quad (68)$$

This last approximation gives the opportunity to make a more direct link with current dynamical models of heavy ion collisions, which use gaussian states as elements of projection. Since it constitutes the lowest approximation level of the DYWAN approach, it will be taken as the reference scale and the results described in the rest of this paper will refer mainly to this description.

In order to illustrate the quality of the approximations in the configuration space as in the associated frequency space, a distribution function can be expressed by applying the Wigner transformation on the reference approximation:

$$f^w(x, p) = \frac{1}{\pi\hbar} \int_{-\infty}^{\infty} ds \psi(x+s)\psi(x-s)e^{\frac{2ips}{\hbar}} \quad (69)$$

The integrations over the position and momentum variables give respectively the expression of the density in momentum or position. The obtained densities in the case of a unidimensional system consisting of eight fermions enclosed in an harmonic potential well are displayed on Fig.1. Although quantal oscillations at the top of the density are not reproduced at the lowest level of description, the obtained distributions are in good agreement with the exact ones. The coherent states adjust their widths to the structure of the wave functions associated with each energy level. As a consequence of this adaptative procedure, the surface is well described. Going up at the previous level in the hierarchy of reconstruction, that is the crudest reconstruction of the system in term of spline wavelets, the quantum oscillations are well reproduced and hardly distinguishable from the exact ones. At this level the lack of information is not clearly visible in the configuration space, since it lies in the frequency space. This is illustrated in Fig.1, where the frequency response is displayed in terms of correlated coherent states. It appears that the frequency distribution has a lower quality than the one in the configuration space. It stems from the fact that the analysis has been done on the configuration representation and, consequently, the approximated reconstructions tend to favor this configuration description compared to the frequency one. Different possibilities exist to improve the description but will not be done here.

The quality of the reconstruction can also be tested by comparing the semi-classical distribution function with the quantum one for which an analytical exact expression is available [25]:

$$f(\varepsilon) = \sum_{n=0}^N \frac{(-1)^n}{\pi \hbar} e^{-\varepsilon} L_n(2\varepsilon) \quad (70)$$

L_N denotes here the Laguerre polynomials and the dimensionless parameter ε is given by:

$$\varepsilon = \frac{2E}{\hbar\omega} = \frac{p^2}{m\hbar\omega} + \frac{m\omega}{\hbar} r^2 \quad (71)$$

The exact distribution displayed in Fig.2, is isotropic in the phase space and exhibits two regions which have different qualitative behavior. Denoting by ε_F the value of the parameter ε associated with the Fermi energy, the distribution goes to zero exponentially if $\varepsilon > \varepsilon_F$, while if $\varepsilon < \varepsilon_F$, the distribution oscillates around its mean value [26] [27]. Let us mention that since the reconstruction stems from the analysis of the wave functions in the configuration space, this representation is favored compared to its spectral response. The corresponding distribution in the phase space displays a small anisotropy but is in good agreement with the exact one, in particular the mean value as well as the diffusivity are well reproduced. The calculated density behaves as if it were the result of a smoothing procedure of the exact density. At this level of reconstruction, there are strong analogies with semiclassical approximations, as for exemple with the Extended Thomas-Fermi approximation [22].

The investigations of the one dimensional harmonic oscillator is straightforwardly extended to three dimensions, using a basis built from the tensor product of the basis elements of the one dimensional case. Due to the summation of the one particle wave functions in the one body density matrix, the quantal oscillations are smoothed as the number of involved energy levels increases. The results of the reconstruction approximation in term of correlated coherent states is illustrated in Fig.3 for a three dimensional isotropic harmonic oscillator. This approximation (on the left) differs from the exact result (on the right), only in the central region, where small amplitude quantal oscillations are washed out.

Let us now address the description of fermions enclosed in a three dimensional selfconsistent nuclear potential. As stated before the mean field potential resulting from the bare interactions between particles is excessively difficult to calculate. A usual way to remedy this problem, is the resort to phenomenological interactions,

expressed in term of the one body density operator. They give an appropriate parameterization of the nuclear potential, and reproduce the main properties of nuclei in particular their binding energies, mean square radius or the fission barriers of heavy nuclei [28]. The effective interaction used in a first step, is a simplified version of the zero-range Skyrme effective force, the so called Zamick interaction [29]. With this force the one body potential reads:

$$U(r) = t_0 \frac{\rho(r)}{\rho_0} + t_3 \left(\frac{\rho(r)}{\rho_0} \right)^{1+\gamma} \quad (72)$$

Here we work with a soft equation of state, with the following parametrization:

$$t_0 = -356 \text{ MeV} \quad t_3 = 303 \text{ MeV} \quad \gamma = 1/6$$

This force gives a saturation density value $\rho_0 = 0.145 \text{ fm}^{-3}$, and an incompressibility modulus of $K_\infty = 200$. In order to focus on the specific characteristic of the approach, the calculations have been restricted to the simplified interaction even if more realistic interaction are available. These latter forces include supplementary terms which take into account surface effects [30] or isospin dependence [31]. More sophisticated interaction can also be implemented, particularly those incorporating the non local character of the nuclear interaction [32].

The algorithm to calculate the self consistent nuclear potential has the following structure. A first assessment of the energy levels and the definition of a reference discrete wavelet basis is obtained from a harmonic approximation of an initial guess of the potential. The decomposition of the wave functions in terms of these wavelets is then carried out. The matrix density is approximated according to the chosen reconstruction approximation. It generates a new potential from which the previous process is applied once again. This self consistent procedure ends when the variations of the Fermi level between two successive iterations becomes negligible. A lead nucleus has been prepared in this way. The elements of the discrete basis retained by this procedure are displayed in Fig.4 where each circle indicates the location of a wavelet center projected in the plane (x, k_x) of the phase space. The underlying level structure of the description is clearly visible. Nevertheless, this representation of a six dimensional configuration must not be misinterpreted, since on one hand, a projection on a plane is performed and, on the other hand, other information is lacking such as the one related to the extension of the wavelets. Let us recall that these widths play an essential role in the description of the nuclear surface and the spectral response. They are completely determined by the analysis of the wave

functions in the initial conditions, and cannot be viewed as parameters of the model.

Some typical density profiles are shown in Fig.5. For a calcium nucleus (left part), one observe that there is no saturated bulk region as in heavier nuclei like lead (right part). On the average, the density profiles are reasonably reproduced by the model. Obviously at the currently chosen level of reconstruction, the treatment of the states will not provide as precise a result as a pure Hartree-Fock calculation [33]. The algorithmic structure is similar, and offers the possibility of successive upgrading, either through more precise density reconstructions or with the implementations of more sophisticated effective interactions. The results concerning binding energies and mean square radius are listed in table (1). They are consistent with experimental values as well as with Hartree-Fock calculations [28], owing to the fact that the effective interaction used here is not the best suited one. The previous results stress that even in the case of a strong compression of the information associated with our crudest level of description in the hierarchy of approximations, the global properties of the nuclei are still well reproduced, keeping track of the quantal structure of the system. We then have initial conditions suited to the dynamical description of heavy-ion collisions, since the representation basis has been optimized, while preserving a good description of nuclei.

4 Mean-field evolution

The modeling framework of heavy-ion collisions at intermediate energy, is based on the hypothesis that the dissipative and mean-field processes can be distinguishable from time scale considerations. Let us recall that the mean field dynamics is governed by the TDHF evolution, which can be described through the time dependence of the one body density matrix.

$$i \frac{d}{dt} \rho = [h(\rho), \rho]$$

This equation is also relevant to describe fermionic systems at finite temperature. Expressing the density matrix in terms of the eigenfunctions $|\varphi_\lambda\rangle$ which diagonalizes the one body Hamiltonian, one gets the following time evolution of the one particle wave function:

$$i \frac{d}{dt} |\varphi_\lambda\rangle = h(\rho) |\varphi_\lambda\rangle \quad (73)$$

The usual resolution methods of the TDHF equation use lattice numerical technics

[34]. These methods are reliable but involve an important numerical cost. As a consequence, it is practically impossible to include dissipative effects in this framework. However, experimental results at intermediate energies evidenced a clear signature of dissipative processes. Consequently, these effects must be taken into account, while keeping a good description of the mean-field dynamics. A realistic representation of the dynamics then requires the introduction of a controlled approximations scheme, defining a path from an exact TDHF treatment to a succession of approximated solutions.

The determination of the projection basis is essential for compacting the information on a restricted set of coefficients. Orthogonal wavelet bases are especially well suited for this optimization. An appropriate representation can considerably restrict the range of indices (i,j) in the relation (66), while preserving a good description of the density matrix. The wavelet analysis of the nuclear phase space structure, points out the existence of rapid and slow oscillating components. The predominant part of the mean field information is concentrated in the low frequency component. For systems consisting of more than ten nucleons, the highest frequency contributions interfere destructively and then the diagonal contributions contain the main part of the global relevant information in the configuration space.

According to the decomposition of the density matrix (66), the full TDHF evolution can be expressed in terms of the elements of a reference basis where the eigenstates of the Hamiltonian are projected. The time evolution is then represented through the evolution of the projection coefficients or through the one of the elements of the representation basis:

$$\frac{d}{dt}|\psi_i\rangle = \sum_{\lambda} \left(\frac{d}{dt}|\varphi_{\lambda}\rangle \right) \langle \varphi_{\lambda}|\psi_i\rangle + \sum_{\lambda} |\varphi_{\lambda}\rangle \left(\frac{d}{dt} \langle \varphi_{\lambda}|\psi_i\rangle \right) \quad (74)$$

Constraining the description to constant projection coefficients, the functions of the representation basis evolve according to the equation:

$$i \frac{d}{dt}|\psi_i\rangle = h(\rho)|\psi_i\rangle \quad (75)$$

where the solution cannot be solved if boundry conditions are not explicitly given. In general, the resolution of this problem can not be done analytically, it requires a numerical treatment. Consequently, the most reliable numerical representation of this evolution must be expressed through the time dependence of the set of parameters which most completely characterize the elements of the representation basis.

The optimization of this process compels one to find a representation basis associated with a smaller number of parameters, compatible with the extraction of the suited approximated solutions. The group properties of wavelets provide the best suited mathematical framework. Although in principle this approach can be applied to any kind of wavelets, in accordance with the approximation scheme of reference, the case of correlated coherent states will be specifically addressed. The latter are well adapted to analytic formulations. In addition they will allow one to get a connexion with other treatment of transport phenomena using gaussian like functions.

One of the reliable research methods of numerical solutions of equations of the type (75), is based on the use of a variationnal principle [35] [36] [37]. The approximated solutions which minimize the error with respect to the exact ones, are determined by the implementation of the trial wave functions in the variationnal principle. Their agreement with the exact solutions are best when the trial functions reproduce the symmetries of the problem. This resolution technique gives the evolution equations of the parameter associated with the projection states. They are obtained by minimizing the value of the action A defined in the general case of a wavelet basis $|\psi\rangle$, by the following relation:

$$A = \int_{t_1}^{t_2} \langle \psi | i \frac{\partial}{\partial t} - h | \psi \rangle dt \quad (76)$$

In general the one body Hamiltonian contains a non local potential. The latter can be explicitly exhibited in the configuration space representation

$$\mathcal{V} = \langle \psi | V | \psi \rangle = \int dx dx' \langle \psi | x' \rangle \langle x' | V | x \rangle \langle x | \psi \rangle \quad (77)$$

where we have restricted the notation to a one-dimensional space, for the sake of simplicity. This expression is nothing else than the wavelet transform of the potential. Considering a finite range interaction, the evolution equations of the correlated coherent states parameters are given by:

$$\dot{\xi} = \frac{\pi}{m} + \frac{\partial}{\partial \pi} \mathcal{V} \quad (78)$$

$$\dot{\pi} = -\frac{\partial}{\partial \xi} \mathcal{V} \quad (79)$$

$$\frac{d\chi}{dt} = \frac{4\gamma\chi}{m} - \frac{\partial \mathcal{V}}{\partial \gamma} \quad (80)$$

$$\frac{d\gamma}{dt} = \frac{1}{8m\chi^2} - \frac{2\gamma^2}{m} - \frac{\partial}{\partial\chi}\nu \quad (81)$$

with

$$\gamma = \frac{\sigma}{2\chi} \quad (82)$$

The following product is constrained to its minimal value

$$\chi\phi - \sigma^2 = \frac{1}{4} \quad (83)$$

According to the point of view adopted in the investigation of the statics in section 3, we will now restrict applications related to local or zero range interactions. In that case the derivative with respect to $\langle p \rangle$ and γ disappear in equations (79) (80).

Since the evolution equation of the states $|\alpha_i\rangle$ are of the TDHF type, the resulting time dependence of the scalar product are:

$$\frac{d}{dt} \langle \alpha_i(t) | \alpha_j(t) \rangle = -i \langle \alpha_i | h^+(\rho) - h(\rho) | \alpha_j \rangle \quad (84)$$

If the Hartree-Fock potential is hermitic, i.e. $h^+(\rho) = h(\rho)$, the scalar products in equation (84) are independent in time. Then an interesting property is that the solutions of the TDHF type equation conserve the orthogonality property when it has been introduced in the initial conditions. Even if the correlated coherent states constitute a class of approximated solutions, they are in very good agreement with the exact ones. In addition, in the numerical implementation, the basis functions are in fact compactly supported and an extremely good conservation of the orthogonality has been observed.

Before studying the mean-field dynamics, let us investigate the evolution of a fermionic system enclosed in an harmonic potential in terms of correlated coherent states. According to the evolution equations (80) (81), the correlated coherent states are characterized by momentum position correlation, which in general, is different from zero. The translation and dilation parameters are continuously adapting their values as the wavelets cover closed trajectories in phase space [38]. Their spreading increases as they reach the center of the distribution and decreases when they are located at the periphery. The trajectory of the center of wavelets are displayed in Fig.6 in the case of a system consisting of 20 fermions enclosed in a three dimensional harmonic potential and for three different energy levels. The chosen representation consists of a projection of the trajectory onto the plane (x, k_x) of the phase space.

The evolution of the widths of a correlated coherent state along three directions is depicted on Fig.7. The amplitude of oscillations change with the direction. The trajectories are stable and the coherent states take identical values at the end of a period [39].

In order to analyze the particle distribution as a function of the energy, the quantity $n(\varepsilon)$ is introduced. It represents the number of particles with energy lower than ε [40]. The spectrum exhibits an increasing tendency with the energy and its derivative gives the corresponding level density $g(\varepsilon)$. This quantity is displayed on the left part of Fig.8 for an isotropic time independent harmonic oscillator potential. Three distinct energy levels are clearly exhibited which stay in the same configuration completely invariant with the time. The Fermi-Dirac statistics is conserved in an exact way. The right part of this Fig.8 represents the same observables in the case of an anisotropic harmonic oscillator. In that case the energy distribution exhibits a more continuous behavior. Three peaks still remain in the density level spectra but their widths tend to overlap.

Let us now address the behavior of a time dependent anisotropic harmonic oscillator. The quantities $n(\varepsilon)$ and $g(\varepsilon)$ are illustrated in Fig.9. One observes level crossings which occur periodically, and clear modifications of the distribution along the time. The initial distinct peaks melt in time and reappear periodically. Although the energy levels change in time, their occupation rates keep the same values. As a consequence Fermi-Dirac statistics is conserved in time. Generally, taking into account the overall wave function structure, we see that it favors the conservation of the Fermi-Dirac statistic. The gaussian packets spreading reproduce the wave packets extension even if the center of these states travel along classical trajectory [41] [42].

It has been shown in the previous section that the DYWAN model gives a relevant description of nuclei in their ground states, and then it defines initial conditions of the nuclear dynamics. In contrast to the case of the external harmonic potential previously studied, the mean field nuclear dynamics is governed by the evolution of a self consistent potential. The time dependence of the matter distribution is therefore describe by evolving wavelets in a mean field potential. Each of the wavelets contribute to building the mean-field potential. Let us now point out some characteristic properties of the current modeling approach in this context. The first relevant investigation is to study the response of a selfconsistent system under conditions

slightly different from its ground state. This is illustrated in Fig.10 where centroids of the moving wavelets related to three different energy levels are projected onto a position wave number plane. The nuclear system is mainly perturbed by low level energy excitations of an oscillating type. This can be clearly observed on outermost trajectories in the position wave number plane. These trajectories exhibit smooth variations driven by the oscillation of the self consistent potential. For the levels with lower energy and therefore more central locations in configuration space, the shift with respect to the non perturbed trajectories, decreases as is to be expected. In addition the inner trajectory gives an indication of the quality of the simulation, since its spreading is practically negligible on a time interval of $200 fm/c$. Despite the unavoidable numerical errors and the fact that we are dealing with approximate solutions, the life time associated with the undetermination of the energy on the discrete level is so long, that no spurious emission of particles have been observed over durations of several thousand fm/c . This is not the case with current simulations based on pseudo particle methods [43].

The time locations of the translation parameters of wavelets contain only a part of the information on the dynamics. In fact a crucial part is involved in the time evolution of the dilation parameters since they greatly determine the spectral response of the system. The evolution of these dilation parameters is displayed in Fig.11. they exhibit an oscillating behavior with a maximal amplitude which is quite regular in time. The associated time evolution of the level density is shown in Fig.12. The behavior exhibits strong similarities with the one observed in the case of the time dependent anisotropic oscillator. The levels periodically overlap and separate, as can be expected from the oscillating induced excitation of the system.

Let us now study the collision of two nuclei within the DYWAN model. The mean field description is performed in the case of the $C(84MeV) + C$ reaction at $b = 1fm$, and the time evolution of the Wigner distribution function projected on the plan (z, p_z) of the phase space as depicted on Fig.13. As the nuclei approach to each other the potential wells merge into a common well of roughly the same depth. The dynamics of the collision has been represented in Fig.13 through the projection of the Wigner distribution on a plane in the phase space related to positions and momenta along the beam axis. It can be observed that during the phase of strong overlap in position space, the distribution of each nucleus becomes wider along the beam axis and narrower along the momentum axis. Due to the strong modification of the potential shape along the beam axis, the widths of the coherent

states increase along this direction. On the contrary along the other directions they are quite unchanged, since in this case the potential remains practically the same. This fact is more clearly evidenced in Fig.14 where the time evolution of the widths corresponding to three orthogonal directions are represented. The modification of the correlated coherent states extension is characteristic of the spreading of the wave packets under the influence of the potential. After their separation, the nuclei contract themselves along the beam direction and their velocity increases. The width of the coherent states take values close to those related to the initial conditions. The lack of the dissipative effects is evidenced by a transparency process. Let us now compare the results of the simulation with a TDHF calculation. Such a comparison is shown on Fig.15 through the time evolution of the density profiles projected in the reaction plane. The results of the DYWAN model and TDHF approximation exhibit similar behaviors. They are characterized by a long shaped structure prior to the separation into fragments. In both cases the outgoing fragments display a transition from prolate to oblate type shapes.

Let us also briefly comment on other simulation approaches of nuclear collisions, starting with the Fermionic Molecular Dynamics [5] [44]. The FMD model has been recently developed. It constitutes an appropriate framework to take explicitly into account the fermionic nature of the nuclear systems. As a matter of fact the FMD approach describes the N body wave function $|\Psi_N\rangle$ by a Slater determinant built from N different one body gaussian wave packets $|q_i\rangle$:

$$|\Psi_N\rangle = \mathcal{A}(\pi_{i=1}^A |q_i\rangle) \quad (85)$$

where $\mathcal{A}$ stands for the antisymmetrization operator. The single particle states are parameterized in terms of time dependent mean positions, momenta and complex widths. The time evolution of these wave packets is obtained by applying the quantum variational principle to the antisymmetrized many body trial state. The parameters of the gaussian wave packets do not represent nucleon positions or momenta due to the explicit antisymmetrization. The widths of the wave packets evolve while they remain isotropic. In this framework, the Fermi statistics of the system is treated exactly, although the trial N -body wave function is an approximate solution of the full N -body problem. In this model the one body density matrix is given by:

$$\rho^{(1)} = \sum_{k,l=1}^A |q_k\rangle O_{kl} \langle q_l| \quad (86)$$

where O_{kl} is defined as the inverse of the overlap matrix:

$$(O^{-1})_{kl} = \langle q_k | q_l \rangle \quad (87)$$

Since the single particle trial states are not orthogonal, the calculation of the density matrix requires to take into account all the elements of the overlap matrix. In principle, the calculation of the potential requires numerical algorithms of order A^4 , where A is the number of nucleons of the system. Consequently it restricts the application of this model to the study of rather light nuclei. The DYWAN model follows a complementary but different point of view. It aims a reduction of the algorithmic complexity. The representation space of FMD associates with the A nucleons of the system, A trial one-body functions, whereas the DYWAN model deals with NA trial functions, with $N \leq 10$. Nevertheless the properties of the wavelet basis provide a precise structuration of the density matrix and a scheme of approximations characterized by an efficient compression of the information. As a consequence the numerical treatment of DYWAN is of the order $(NA)^2$, which makes possible the study to heavy systems.

The models mentionned up to now, rely on quantum flesh even if the skeleton is somewhat classical. Let us now refer to descriptions based on the evolution of the one body distribution function in phase space [45] [46]. The Wigner transform of the TDHF equation of motion for the one body density matrix leads to the equivalent equation:

$$\frac{\partial}{\partial t} f(\vec{r}, \vec{p}, t) + \frac{\vec{p}}{m} \nabla_{\vec{r}} f(\vec{r}, \vec{p}, t) + \left[\frac{2}{\hbar} U(\vec{r}, \vec{p}, t) \sin\left(\frac{\hbar}{2} \Lambda\right) \right] f(\vec{r}, \vec{p}, t) = 0 \quad (88)$$

with

$$\Lambda = \nabla_{\vec{r}} \nabla_{\vec{p}} - \nabla_{\vec{p}} \nabla_{\vec{r}} \quad (89)$$

and where f is the Wigner transform of the one body density matrix. The Vlasov equation results from an expansion in term of $\hbar$ and by a truncation at second order. The Vlasov equation can be written in a condensed form as:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, h\} = 0 \quad (90)$$

it describes the time invariance of the density in phase space. The details of the underlying microscopic structure are for the most part lost. This equation keeps only the fingerprints of the antisymmetrization and its conservation in time from the TDHF equation. According to the approximations performed to obtain Vlasov equation, dissipative effects are included through the Uehling-Uhlenbeck collision term [47]. The Landau-Vlasov equation is then obtained:

$$\frac{\partial f}{\partial t} + \{f, h\} = I_{coll} \quad (91)$$

This Equation remains difficult to solve and commonly requires resort to pseudo-particle methods [46].

The one-body distribution function is described through its projection on a basis of reduced coherent states. The latter are normalized gaussians with fixed widths both in coordinate and momentum space. The time dependence of the distribution function is then described in terms of evolving reduced coherent states which keep fixed widths and weights. These basis functions constitute a continuous non-orthogonal overcomplete set. Consequently the observables are expressed as continuous integrals. Their evaluation requires the resort to Monte-Carlo techniques. The related sampling of the phase space associates to each nucleon a number N_g of gaussians. This semi-classical model is able to describe the global properties of nuclei in their ground states, as well as the reaction mechanisms observed in heavy ion collisions at intermediate energies. To give some insights into the differences between the DYWAN and Landau-Vlasov calculations, the same effective interaction has been taken in both models. A comparison of the two models is performed for the reaction $Ca(50MeV) + Ca$ at $b = 2fm$. Let us stress that the dissipative effects have not been included in these calculations. The time dependence of the density profiles projected onto the reaction plane are represented in Fig.16a for a DYWAN calculation and in Fig.16b for a Vlasov calculation. With the DYWAN model the final fragments exhibit an important spreading along the beam axis just after their separation. It is closely connected to the extension of the wave packets in the overlapping mean-fields. This effect is completely absent in the Landau-Vlasov simulation.

Although the comparison between the density gives quite similar shapes in the exit channel, the underlying information is very different in these two models. In Landau-Vlasov the main part of the information is contained in the coordinates of the center of the gaussian states. The sampling of the phase space involve a lost of the memory of the underlying microscopic structure. On the contrary with DYWAN the location of the wavelets is structured according to the quantal properties of the system and a preponderant part of the information is contained in the widths of the wavelets. The energy level structure contained by the DYWAN model is completely absent in the Landau-Vlasov model.

5 Dissipative Dynamics

The existence of dissipative effects in experimental data, requires to take them into account. The description of the mean-field dynamics has then to be performed as precisely as possible, while allowing the inclusion of the dissipative processes. A coherent way to introduce these effects with respect to the mean field consists in using the master equation of occupation numbers:

$$\begin{aligned} \dot{n}_\mu &= \pi \sum_{\kappa\nu\lambda} W(\mu\kappa\nu\lambda) \delta(\varepsilon_\mu + \varepsilon_\kappa - \varepsilon_\nu - \varepsilon_\lambda) \\ &\quad [\bar{n}_\mu \bar{n}_\kappa n_\nu n_\lambda - n_\mu n_\kappa \bar{n}_\nu \bar{n}_\lambda] \end{aligned} \quad (92)$$

where

$$\bar{n} = 1 - n.$$

Let us recall that this equation has been derived according to an important number of theoretical approximations. Among them, the residual interactions are considered as perturbation in comparison with the mean-field processes, the memory effects are neglected by adopting a Markovian approximation and the off-diagonal elements of the density matrix are neglected. This equation governs the variations of the occupation numbers n_μ of each energy level ε_μ . These modifications are made through the transition rate $W(\mu\kappa\nu\lambda)$, which are the antisymmetrized elements of the two body interaction matrix. The energy conservation is ensured by the Dirac distribution appearing in equation (92). This equation includes loss and gain terms which gives the population variations of the levels $(\mu\kappa, \nu\lambda)$. These terms take explicitly into account the Pauli principle. Solving this equation requires a knowledge of the entire set of energy levels, and of their occupation rates which take continuous values. The summation over the three indices $(\kappa\nu\lambda)$, even restricted by the term $W(\mu\kappa\nu\lambda)$, takes into account a huge number of possible configurations. Moreover the estimation of these transition rates requires a knowledge of the residual interactions. All these constraints make the full calculation of the master equation intractable. It is necessary to adopt numerical approximations in order to simulate this equation.

Starting from the fact that in the framework of the performed approximations the collision term is of second order, an additional approximation consists in considering that the density behaves locally as in an infinite homogeneous system. In that case the mean free path of a nucleon and the mean time between two successive collisions are long compared to the time and spatial scales of the collision. In this

approximation only the two body collisions are taken into account.

For an infinite system, the wave functions become plane waves labeled by a momentum k and spin and isospin quantum numbers, which are averaged. The summation over the states $(\kappa\nu\lambda)$ in the master equation can then be replaced by an integral over the momentum and the occupation numbers and the distribution function are connected by:

$$n(r, p, t) = \left(\frac{2\pi\hbar}{g}\right) f(r, p, t) \quad (93)$$

where g is the spin and isospin degeneracy. Taking this relation into account, the master equation (92) leads to:

$$\begin{aligned} \frac{\partial f_\mu}{\partial t} = & \frac{\pi}{\hbar} V^3 \int \frac{dk_\kappa dk_\nu dk_\lambda}{(2\pi)^9} \delta(\varepsilon_\mu + \varepsilon_\kappa - \varepsilon_\nu - \varepsilon_\lambda) \\ & \cdot [(1 - f_\mu)(1 - f_\kappa)f_\nu f_\lambda - f_\mu f_\kappa(1 - f_\nu)(1 - f_\lambda)] | < k_\mu k_\kappa | V | k_\nu k_\lambda > |^2 \end{aligned} \quad (94)$$

The main problem in solving this equation is to estimate the value of the transition rates. Since the bare interaction is not well known, it is difficult to extract realistic residual interactions. It thus compels us to use effective interactions. The diffusion matrix is connected to the scattering nucleon-nucleon cross section calculated in an infinite system and in the Born approximation it reads [48]:

$$| < k_\mu, k_\kappa | V | k_\nu, k_\lambda > |^2 = \frac{2^7 \pi^5 \hbar^4}{m^2 V^3} \delta(k_\mu + k_\kappa - k_\nu - k_\lambda) \frac{d\sigma}{d\Omega}(k, k') \quad (95)$$

with

$$k = \frac{1}{2}(k_\mu - k_\kappa) \quad k' = \frac{1}{2}(k_\nu - k_\lambda)$$

This equation is formulated in the framework of plane wave basis, the plane waves beeing considered as eigenfunctions of the Hamiltonian, which means that medium is homogeneous and infinite. In the case where only elastic collisions are taken into account, the differential cross section $\frac{d\sigma}{d\Omega}$ depends only on the angle between an incoming vector $\vec{k}$ and an outgoing vector $\vec{k}'$ and on the norm of $\vec{k}$. The Uehling-Uhlenbeck equation constitutes the usual reference equation for the treatment of two body collisions and is written:

$$\begin{aligned} \frac{\partial f_\mu}{\partial t} = & \frac{\hbar^3 g}{4\pi^3 m^2} \int dk_\kappa dk_\nu dk_\lambda \delta(\varepsilon_\mu + \varepsilon_\kappa - \varepsilon_\nu - \varepsilon_\lambda) \delta(k_\mu + k_\kappa - k_\nu - k_\lambda) \\ & \cdot \frac{d\sigma}{d\Omega}(k, k') [(1 - f_\mu)(1 - f_\kappa)f_\nu f_\lambda - f_\mu f_\kappa(1 - f_\nu)(1 - f_\lambda)] \end{aligned} \quad (96)$$

This equation is obtained in the semiclassical limit, $\hbar \rightarrow 0$, and considering that the homogeneous medium approximation is applicable locally. Like the master equation, the Uehling-Uhlenbeck equation cannot be fully solved. Therefore it is necessary to use simulation methods. The main goal here is to simulate the master equation, in accordance with the best mean-field approximations. This condition is required to keep the maximum ammount of microscopic information. In principle the solution of the master equation of occupation numbers requires the knowledge of the set of all the accessible energy levels, despite the fact that only the lowest levels are known in the initial conditions. Techniques exist which are able to determine with good accuracy the excited energy levels [49, 50]. Expressing the Hamiltonian of the system on a basis consisting of N elements, gives after diagonalization, an estimate of the first N energy levels. These techniques can be implemented, but require an additional numerical cost. As a first step an alternative procedure has been adopted by defining prescriptions based on the properties of wavelet basis.

During a collision, several exit channels of two final states (3,4) correspond to one intrance channel of two initial states (1,2). Taking into account all the possible combinations, involves a fourfold summation in the master equation. The complete calculation is not actually possible. The most accepted technique used for estimating it consists in having to resort to Monte-Carlo methods, consistent with the resolution schemes already used.

The projection coefficients ω_i^λ of the wave function onto a wavelet basis

$$|\varphi^\lambda\rangle = \sum_i \omega_i^\lambda |\alpha_i^\lambda\rangle$$

satisfy the normalization condition:

$$\sum_i \omega_i^\lambda = 1 \quad (97)$$

The one body density matrix corresponding to a statistical ensemble can be written in the projection basis as follows:

$$\rho = \sum_\lambda n^\lambda |\varphi^\lambda\rangle \langle \varphi^\lambda| = \sum_{\lambda i} n_i^\lambda |\alpha_i^\lambda\rangle \langle \alpha_j^\lambda| \quad (98)$$

The coefficients n_i^λ are the statistical weight associated with each basis state and are defined by:

$$n_i^\lambda = |\sqrt{n_\lambda} \omega_i^\lambda|^2 \quad (99)$$

They satisfy the following normalization condition:

$$\sum_i n_i^\lambda = \sum_i n_\lambda |\omega_i^\lambda|^2 = n_\lambda \quad (100)$$

In principle the occupation rates should take continuous values. Nevertheless the Monte-Carlo methods are based on a discretization scheme, which will be defined here in a consistent way with the mean-field description. According to this discretization principle, the occupation rates of a level labeled by μ is express in the following way:

$$n^\mu = \sum_i n_i^\mu \Theta(i) \quad (101)$$

If the states labeled i contributes to the level μ , then $\Theta(i) = 1$, else $\Theta(i) = 0$. The occupation numbers are then defined by the presence or the absence of the state labeled i in the considered level. The loss term can then be written:

$$\frac{dn^\mu}{dt} = \sum_i n_i^\mu \frac{d\Theta(i)}{dt} \quad (102)$$

where $\frac{d\Theta(i)}{dt} = 0$ takes a zero value if at the end of the time step dt , the state labeled i contributes to the level μ , otherwise it takes a value equal to one. In this framework, the loss term of the master equation can be rewritten as:

$$\frac{dn^\mu}{dt} = \sum_{iklm} W(iklm) n_i^\mu \Theta(i) n_k^\kappa \Theta(k) n_l^\nu (1 - \Theta(l)) n_m^\lambda (1 - \Theta(m)) \quad (103)$$

where

$$(1 - n^\nu) = \sum_l n_l^\nu - \sum_l n_l^\nu \Theta(l) = \sum_l n_l^\nu (1 - \Theta(l)) \quad (104)$$

The summation related to the indices i and k are performed through a sampling method with random variables related to a uniform probability density law. The cross-section is then weighted by each of these states. The summations over l and m are performed according to the importance sampling method with a random variable related to a non-uniform probability density law, defined from the occupation probabilities $n_l^\nu (1 - \theta(l))$. In the simulation it stands for the probability that a wavelet $|\alpha_l^\nu\rangle$ randomly chosen, is orthogonal to the wavelets which contribute to the construction of the state ν . The treatment of the master equation and the mean-field

is then carried out in a consistent way. The Pauli-blocking is strictly conserved by this procedure.

Taking the dissipative effects into account leads to a more realistic description of heavy ion collisions at intermediate energies. During a collision between two nucleons, there occur transitions between states. In order to illustrate this process a nucleus has been prepared with holes in the initial state so as to leave some regions of the phase space unoccupied. Examples of transitions which happen in this case are displayed in Fig.17. On the left side is drawn a transition from a given energy level to another one corresponding to a higher energy, and on the right side, an opposite transition is displayed.

Several calculations have been performed in the framework of central $Ca + Ca$ collisions, and compared with Landau-Vlasov results. The number of collisions which occur at each time step is plotted in Fig.18 for both models. In order to emphasize the differences between the models, Pauli-Blocking in Landau-Vlasov has been estimated by the local distribution function which comes from the sampling of the phase space. Technical improvements [51], which suppress the drawback of the local fluctuations have not been taken into account. As a result, collisions which contribute very little to the dissipation occur. On the contrary with the DYWAN model the basis functions are distributed according to the structure of the wave function which involve a homogeneous distribution in phase space. Therefore no transition is allowed when the nuclei are described in their ground states. They occur only during the overlapping phase of the two interacting nuclei. The number of collisions is lower with DYWAN but they are much more efficient than the ones which occur in Landau-Vlasov. The results obtained with both models for central $Ca + Ca$ collisions at different beam energies, are represented in Fig.19a and Fig.19b. They represent the time evolution of the average density. The results exhibit similarities, and go from a fusion process to a vaporization process.

The evolution of the density in the reaction plane for the same reactions within the DYWAN model is represented in Fig.20. In the first step the system is compressed, after which an expansion phase appears. For the lowest beam energy an incomplete fusion process is observed. As the energy increases binary type processes appear. As the energy reaches 90MeV , the system explodes, so that the density becomes very low as indicated on Fig.19. In a DYWAN calculation it involves a large increase of the wavelets widths, which reproduce the extension of the wave

packet associated with the free particles. The model foresees a transition from a fusion process to a fragmentation around 40MeV . This result is in good agreement with experimental data obtained with an analog system [52].

6 Conclusion

This paper is devoted to the formulation of the DYWAN model. It addresses the description of transport phenomena in heavy-ion collisions at intermediate energies. As is well known, the number of degrees of freedom involved is so large that a full and exact treatment is hopeless. The unavoidable theoretical and numerical approximations imply a loss of information, which thus requires the introduction of a suitable framework. Therefore the DYWAN model has been founded on the projection methods of statistical physics and on the mathematical theory of wavelets. It insures a consistency in the modeling process, since the derivation of the evolution equations and the resolution schemes use common concepts. The projection of the information on complementary and orthogonal subspaces, provide well defined schemes of approximations. The wavelet analysis is especially efficient in compressing the information in a numerical representation. It constitutes a convenient framework to extract an optimal set of relevant variables. An immediate consequence is the opportunity to strongly decrease the numerical costs as well as to improve the content of the treated information.

This approach takes explicitly into account the structure of the wave functions in order to preserve as precisely as possible the microscopic properties of the system. The orthogonality properties of the wavelet basis permit a full account for the antisymmetrization in the density matrix. The mean field time evolution preserves with an excellent numerical accuracy the antisymmetrization. The group properties of the wavelet provide a convenient framework to introduce the dynamical symmetries of the problem, and then to extract optimized approximated solutions. The dissipative effects, essential to describe the reaction processes in this range of energy, are included according to the master equation of the occupation numbers. In the latter case a Monte Carlo method can be performed consistently with the mean field treatment, and the orthogonality of the wavelet basis enforces the Pauli principle. The dilation parameters of the wavelets contain a predominant part of the information, they mainly contribute to preserve as much as possible the underlying quantum structure. Most of the calculations displayed in this paper have been performed in what we consider as the lowest level of approximation of the DYWAN model.

Nevertheless the analyticity of the correlated coherent states allowed to control very precisely the quality of the numerical procedures. It proves that the simulation is accurate, fast and the consequently suitable for the investigation of heavy nuclear systems. We are convinced that only a part of the potentialities of the approach have been exploited, and can be strongly improved either from the physical point of view, or in the resolution schemes.

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Figure Captions

- Figure 1: Comparison of the scalar density (solid line) with that obtained from equ (68) (dashed line) for position (left side) and for momentum (right side), for a one dimensional system comprising eight nucleons enclosed in an harmonic potential well.
- Figure 2: Comparison between the exact distribution function $f(\epsilon)$ (full line) and those obtained with the DYWAN model, for an eight nucleons system enclosed in an harmonic potential well. The variable ϵ is expressed in $\frac{\hbar\omega}{2}$ units.
- Figure 3: Comparison between the exact density (right side) and that obtained with the DYWAN model (left side), in the case of a three dimensional harmonic potential.
- Figure 4: Coordinates of wavelet centroids in the (x, k_x) plane of the phase space for a lead nucleus.
- Figure 5: Density profiles, expressed in *nucleons/fm³*, obtained with the DYWAN model for a calcium and a lead nucleus.
- Figure 6: Trajectories in the (x, k_x) plane of the phase space followed by the center of the coherent states associated with three nucleons located on three different energy levels. This calculation was performed in the case of 20 nucleons enclosed in an harmonic potential well.
- Figure 7: Time evolution of the widths along three orthogonal directions of a wavelet in the case of an harmonic potential. Same case as in figure 6.
- Figure 8: Upper part: Number of particles with energy lower than ϵ $n(\epsilon)$, in the case of an isotropic harmonic oscillator (on the left) and in the case of an anisotropic one (on the right). Lower part: density level $g(\epsilon)$.

- Figure 9: Time evolution of $n(\epsilon)$ and $g(\epsilon)$ for an anisotropic harmonic potential which evolves in time.
- Figure 10: Trajectories of the centers of three nucleons located on three different energy level on the (x, k_x) plane in phase space. This calculation was performed in the case of a nuclear potential.
- Figure 11: Time evolution of the widths along three orthogonal directions of a wavelet in the same case as Figure 10.
- Figure 12: Time evolution of the density level of the nucleons. Same case as Figure 10.
- Figure 13: Evolution of the one body distribution function projected on a plane of the phase space during a $C(84MeV) + C$ reaction at $b = 1fm$.
- Figure 14: Time evolution of the widths along three orthogonal directions of a wavelet in the case of the $C(84MeV) + C$ reaction at $b = 1fm$.
- Figure 15: Comparison between the density projected onto the reaction plane for the reaction $C(84MeV) + C$ reaction at $b = 1fm$. The upper part gives the density obtained with TDHF and the lower part with DYWAN.
- Figure 16: The upper part (a) represents the density projected in the reaction plan for a $Ca(50MeV) + Ca$ reaction at $b = 2fm$, obtained with a Vlasov calculation. On the lower part (b) is depicted the density obtained with DYWAN for the same reaction.
- Figure 17: Illustration of the trajectory changes which occur during a collision.
- Figure 18: Comparison between the number of collisions occurring at each step of the reaction calculated with Landau-Vlasov and DYWAN.

- Figure 19: Comparison of the mean density obtained in time for $Ca + Ca$ reactions for three different energy beams obtained with the DYWAN model.
- Figure 20: Density projected in the reaction plane obtained with the DYWAN model for $Ca + Ca$ reaction at three different energy beams.

Table Captions

- Table 1: Comparison of the Binding energies and Mean Square Radius obtained for Calcium and Lead nuclei, with a DYWAN and a Vlasov calculation.

BE (MeV)	HF	Exp	DYWAN
Ca	-8.55	-8.55	-7.1
Pb	-7.86	-7.86	-7.5
RMS (fm)	HF	Exp	DYWAN
Ca	3.52	3.49	3.61
Pb	5.51	5.50	5.9

Table 1

Figure 1

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Figure 20