Self-consistent treatment of bubble nucleation at the electroweak phase transition

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In the standard procedure for calculating the decay rate of a metastable vacuum the solution of the classical Euclidean equation of motion of the background field is needed. On the other hand radiative corrections have to be taken into account already in the equation of motion. Hence, the latter one has to be the functional derivative of the effective action with respect to the background field. This is of crucial importance in theories in which the symmetry breaking is due to radiative corrections. Usually the effective potential is considered only, neglecting the corrections due to the derivative terms of the effective action. In this article a bounce solution from an equation of motion which takes into account the full effective action in the one-loop approximation is calculated. A computational method that yields a strict separation of the divergent contributions to the effective action from the convergent ones is obtained. This allows a wide freedom in the choice of regularization and renormalization schemes. The model under consideration is the SU(2)-Higgs model. The fluctuations of the complete bosonic sector, i.e. gauge field, Higgs and Goldstone boson contributions, are taken into account. The bounce is then self-consistent to one-loop order. The obtained results for characteristic quantities of the transition as the nucleation rate and the number of nucleated bubbles per volume are compared to other, non-self-consistent approaches.

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I. INTRODUCTION

When dealing with field theory in cosmological context, one often encounters the problem that a field is not in a state which is the absolute minimum of the potential, the true vacuum, but in a metastable local minimum, the false vacuum, which is at higher free energy. The transition from false to true vacuum proceeds by a quantum mechanical tunnelling process or by a classical transition induced by thermal fluctuations for a system at finite temperature. The transition is local in space. During the transition a bubble is formed which is a region in space with true vacuum surrounded by false vacuum. The bubble just large enough not to collapse is called critical bubble. Once nucleated, the bubble expands, converting false vacuum to true vacuum.

If the mass of the Higgs boson is not too large the electroweak phase transition is of first order and proceeds by bubble nucleation. This scenario is interesting because of the possibility to explain the baryon asymmetry within the minimal standard model [1,2].

A quantity of fundamental importance for a phase transition is the transition rate. The nucleation rate per volume, $\gamma$, can be calculated from the bounce-solution of the Euclidean field equation. This solution is also called classical solution. For a theory with only one scalar field $\Phi$ with a potential $U(\Phi)$ one has to solve the equation of motion:

$$\partial_\mu \partial_\mu \Phi - \frac{d}{d\Phi} U(\Phi) = 0$$

The boundary condition is, that the solution $\Phi = \phi$ tends to its false vacuum value if one of the $x_\mu$ tends to $\infty$. The nucleation rate can be written in the form [3,4]

$$\gamma = A \exp(-B)$$

where $B$ is the Euclidean action of the classical solution $\phi$.

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In the semiclassical approximation the factor $A$ is given by an expression involving functional determinants \([5–7]\). The negative logarithm is the one-loop contribution to the effective action.

If $U(\Phi)$ only has a single minimum the bounce-solution does not exist. Nevertheless, in some models the vacuum structure is not determined by $U(\Phi)$ alone but it is changed by radiative corrections. These can be calculated by taking into account the quantum fluctuations of the scalar field and those fields coupled to the scalar one \([8]\). Depending on the specific model, the radiative corrections yield an effective potential $V_{\text{eff}}($\Phi$)$ with different minima \([9–15]\). The bounce-solution is then calculated from a field equation that contains this effective potential. However, the effective potential is only a part of the effective action. It does not take into account the space-time dependence of the background fields, i. e. terms, that depend on derivatives of the background fields.

This raises the question, whether it is justified to neglect these terms in calculating the bounce \([16]\). In case of the electroweak phase transition the bosonic as well as the fermionic contributions to the effective action have been calculated, based on a bounce-solution, that was determined from the effective potential and not the effective action \([17,18]\). In \([14]\) an improvement to take into account non-local effects already for the calculation of the bounce was done by modifying the kinetic term of the background field. The radiative corrections were calculated using approximations as the derivative expansion \([19]\) and the heat kernel method \([14]\) or by exact numerical computation schemes \([17-20]\). The corrections due to the bosonic sector (gauge and Higgs fields) were found to be of the same order as the classical action. In such a case the non-local terms of the effective action are not negligible and should be taken into account when calculating the classical solution. As the fermionic corrections are small compared to the classical action \([18]\), their non-local contributions to the effective action can be omitted in the equation of motion.

The standard procedure to calculate the nucleation rate works in the following way: First one adds the one-loop corrections of the effective potential to the classical action $B$ and gets an approximation $\tilde{S}$ of the one-loop effective action. Based upon the functional form of $\tilde{S}$ the field equation is derived and the bounce is determined. This is used to evaluate $\tilde{S}$ and the prefactor $A$ of the transition rate which is given in more detail by

\[
\gamma = \frac{\omega_{\gamma}}{2\pi} \left( \frac{\tilde{S}}{2\pi} \right)^{\frac{3}{2}} \exp \left( -\frac{1}{2} \ln J_B \right).
\]

Here $\omega_{\gamma}$ is the modulus of the eigenvalue of the bubble’s unstable mode. $J_B$ denotes the product of all fluctuation determinants of the bosonic sector. Aside the dominant fermionic contribution due to the top-quark which is usually taken into account for the setup of $\tilde{S}$ the fluctuation determinant due to fermions is neglected here. The aim of this paper is to improve the calculation of the bounce which is then used as the numerical input for the evaluation of the nucleation rate. It is no longer determined as a saddle point of $\tilde{S}$ but as a saddle point of the full one-loop effective action. I. e., it will be obtained as a self-consistent solution of the one-loop field equation:

\[
\frac{\delta}{\delta \phi(x)} \tilde{G}^{\text{eff}} = \frac{\delta}{\delta \phi(x)} \left( \tilde{S} + \frac{1}{2} \ln J_B \right) = 0.
\]

The problem is, that no analytic representation of the effective action in the one-loop approximation is known. In \([21]\) a numerical method to calculate the functional derivative of an effective action has been presented. It is based upon a computation scheme to determine full one-loop Green’s functions \([22–24]\). In this paper this technique is applied to determine a bounce-solution from a field equation where the full space-time-dependence of the non-local terms of the quantum corrections to one-loop order is taken into account. As the numerical method to compute the functional derivative needs the background field as a numerically known input quantity the procedure we use to obtain the background field is iterative. It finally results in a self-consistent fixed point of the iteration condition, i. e. a solution of the one-loop field equation \((5)\).

A one-loop Green’s function is a divergent quantity that has to be renormalized. An important issue of the presented method is the strict separation of the divergent parts, which have to be handled analytically, and the convergent part, which may be computed numerically. As no expansion in any quantity is needed a conceptual advantage of this method in contrast to approximation schemes like the derivative or the heat kernel expansion is that the unphysical infrared divergencies these approximations suffer from do not appear.

Among the various models for which self-consistent solutions are interesting this paper deals with the electroweak phase transition. Recent lattice calculations show that the perturbation theory is not reliable for Higgs boson masses above 60\,GeV. The phase transition seems to disappear at a critical Higgs boson mass which is in the range between 66\,GeV and 80\,GeV \([25]\) and the experimental lower limit of the Higgs boson mass already is beyond 66\,GeV. Hence,
the generation of the baryon asymmetry at the phase transition of the standard model is ruled out already, and
the interest has shifted to extensions of the standard model. Nevertheless, the investigation of the first order phase
transition at small Higgs boson masses is still of interest and is at present actively studied on the lattice [26]. Some
aspects of the transition, like the determination of the surface tension and the latent heat, are still in a state of
development. Hence, we have the possibility to develop new calculational methods in a well known model that are
straightforward to extend to more complicated models.

The plan of this paper is as follows: In the next section the model will be specified and the relevant fluctuation
operators are given. The next section deals with the renormalization of the effective action that is improved by
resummation. In section IV the formulas for the numerical procedure how to calculate the functional derivative of an
effective action and the effective action itself are derived. Finally the results are presented and discussed.

II. BASIC EQUATIONS

The Lagrangian of the SU(2)-Higgs model with gauge fields and fermions in the limit of vanishing electroweak
mixing angle is given by

\[ L = L_{\text{gauge}} + L_{\text{Higgs}} + L_{\text{ferm}}, \]

where

\[ L_{\text{gauge}} = -\frac{1}{4} W_{\mu
u}^a W^{\mu\nu a}, \]
\[ L_{\text{Higgs}} = \frac{1}{2} (D_\mu \Phi)\,^\dagger (D^\mu \Phi) - U(\Phi, \Phi), \]
\[ L_{\text{ferm}} = \sum_f \bar{\Psi}_f \gamma_\mu \partial_\mu \Psi_f - \sum_{ff'} g_{ff'}^Y \bar{\Psi}_f \gamma^5 \Psi_{f'}, \]
\[ W_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon^{abc} A_\mu^b A_\nu^c, \]
\[ D_\mu = \partial_\mu - ig^2 \tau^a A_\mu^a, \]
\[ U(\Phi) = \frac{\lambda}{4} (\Phi\Phi - v_0^2)^2. \]

Here \( W_{\mu\nu}^a \) denotes the field strength tensor of the gauge fields, \( \Phi \) the Higgs-doublet, \( \tau^a \) the Pauli matrices, \( \Psi_f \) the
fermion fields of the quarks, \( v_0 \) the vacuum expectation value of the Higgs field at temperature \( T = 0 \) and \( g_{ff'}^Y \) the
Yukawa-couplings of the fermions to the Higgs field. The sum \( f \) runs over the different flavors and colors of the
quarks.

The critical electroweak bubble is a pure real Higgs field configuration. On the classical level it is

\[ \Phi(x)_{\text{cl}} = \phi(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad A_\mu^a(x)_{\text{cl}} = 0, \quad \Psi(x)_{\text{cl}} = 0. \]

The configuration is taken to be stationary, i.e. time independent. As explained in the introduction the bubble profile
calculated from the functional derivative of the classical action (3) does not exist because the tree-level potential \( U(\phi) \)
only has a single minimum. Hence, instead of the classical action one has to use the effective action. For the model
under consideration this effective action is given by:

\[ \Gamma[\phi, T] = S_{\text{cl}}[\phi] + S_{\text{eff, gauge}}[\phi, T] + S_{\text{eff, ferm}}[\phi, T] + S_{\text{eff, Higgs}}[\phi, T]. \]

This functional cannot be computed exactly. Usually it is calculated in the semiclassical approximation to one-loop
order. In order to calculate the one-loop approximation one needs to know the fluctuation operators. They are found
by expanding the Lagrangian (6) around the classical field configuration (13) to second order in the small fluctuations:

\[ \Phi(x) = [\phi(x) + h(x) + i\tau^a \varphi^a(x)] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \]
\[ A_\mu^a(x) = a_\mu^a(x), \]
\[ \Psi(x) = \psi(x). \]
Here \( h(x), \varphi^a(x) \), \( a^a_\mu(x) \) and \( \psi(x) \) denote the fluctuations of the isoscalar– and isovector–part of the Higgs field, the gauge field and the fermions respectively. The second order contribution to the Lagrangian is given by:

\[
L^{(2)} = -\frac{1}{2} \partial_\mu a^a_\nu \partial^\mu a^{\nu a} + \frac{1}{2} \partial_\mu a^a_\nu \partial^\nu a^{\mu a} + \frac{g^2}{8} \varphi^2 a^a_\mu a^{\mu a} + \sum_f \bar{\psi}^f i\gamma^\mu \partial_\mu \psi^f - \sum_{f'} g^{f f'} \phi \bar{\psi}^f \psi^{f'} + \frac{1}{2} \partial_\mu \partial^\mu h + \frac{1}{2} \partial_\mu \varphi^a \partial^\mu \varphi^a + \frac{g}{2} (\partial_\mu \phi) a^{a a} \varphi^a - \frac{g}{2} \phi a^a_\mu \partial^\mu \varphi^a - \frac{1}{2} (3\lambda \phi^2 - \lambda \nu^2_0) \varphi^a \varphi^a ,
\]

(16)

One has to add a gauge-fixing term and the contributions of the Faddeev-Popov-ghosts to the Lagrangian. Here the ‘t Hooft-Feynman-background gauge is used:

\[
L_{\text{GF}} := -\frac{1}{2} \mathcal{F}^a \mathcal{F}^a \quad \text{with} \quad \mathcal{F}^a := \partial_\mu a^{a a} + \frac{g}{2} \varphi^a ,
\]

(17)

\[
L_{\text{FP}} = \eta^a \left( \partial_\mu \partial^\mu + \frac{g^2}{4} \varphi^2 \right) \eta^a .
\]

(18)

From \( L^{(2)}_{\text{tot}} = (L + L_{\text{GF}} + L_{\text{FP}})^{(2)} \) the fluctuation operators are read of. They are given in Euclidean metric as the bounce solution we want to obtain is a solution of an Euclidean field equation. There are three contributions of the bosonic sector to the effective action. The first is due to the isoscalar fluctuations of the Higgs field, the second is due to the coupled channel of the gauge-fields and the isovector components of the Higgs field (would-be-Goldstone bosons) and the last is due to the time component of the gauge fields and the Faddeev-Popov-ghosts. Each of them can be written as:

\[
S_i[\phi] = c_i \ln \mathcal{J}_i = c_i \ln \det \left( \frac{-\partial^2 + U_i(\phi)}{-\partial^2 + U_i(0)} \right) .
\]

(19)

In order to shorten the notation the quantities

\[
c_{aa} = -\frac{3}{2} , \quad c_b = \frac{1}{2} , \quad c_{a\varphi} = \frac{3}{2} , \quad m^a_\varphi(\phi) = \frac{g^2}{4} \phi^2 , \quad m^b_\varphi(\phi) = 3\lambda \phi^2 - \lambda \nu^2_0 , \quad m^c_\varphi(\phi) = \frac{g^2}{4} \phi^2 + \lambda (\phi^2 - \nu^2_0) ,
\]

(20)

\[
U_{a a a}(\phi) = m^a_\varphi(\phi) , \quad U_b(\phi) = m^b_\varphi(\phi) , \quad U_{a a \varphi}(\phi) = \begin{pmatrix}
m^a_\varphi(\phi) & 0 & 0 & 0 \\
0 & m^b_\varphi(\phi) & 0 & 0 \\
0 & 0 & m^c_\varphi(\phi) & 0 \\
0 & 0 & 0 & m^a_\varphi(\phi)
\end{pmatrix} + g \zeta \nabla \phi ,
\]

(21)

\[
\zeta := (\zeta^1 \quad \zeta^2 \quad \zeta^3)^T ,
\]

(22)

\[
\zeta^1 := \begin{pmatrix} 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix} , \quad \zeta^2 := \begin{pmatrix} 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} , \quad \zeta^3 := \begin{pmatrix} 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} .
\]

(23)

are introduced and one uses that the background field is time independent.

The contribution of the top-quark is the dominant one in the fermionic sector. Therefore, nothing but its contribution is taken into account. It is given by

\[
S_t[\phi] = -3 \ln \det \left( \begin{pmatrix} \gamma_\mu \partial_\mu - g_t \phi \\
\gamma_\mu \partial_\mu \end{pmatrix} \right) ,
\]

(24)

where the factor three is due to the quark color and \( g_t \) is the strength of the Yukawa-coupling of the top-quark. As explained in the introduction only the leading temperature dependence of the top-quark contribution, which is part of the finite temperature effective potential, will be taken into account. Hence, this calculation is not self-consistent with respect to the fermionic sector.
The first step is to extract the contributions of the different fluctuating fields to the high temperature approximation of the one-loop effective potential \([9,13,27,28]\). Based upon the fluctuation operators given above the sum of the tree level potential \(U(\phi)\) and these one-loop corrections can be written as:

\[
V_{\text{ht}}(\phi, T) = \frac{1}{2}m^2_H \phi^2 - ET\phi^3 + \frac{\lambda_T}{4} \phi^4
\]

with \(m^2_H := 2D(T^2 - T_0^2)\)

\[
D := \frac{1}{8v_0^2} \left(3m_W^2 + m_H^2 + 2m^2_T\right)
\]

\[
B := \frac{3}{64\pi^2v_0^2} \left(2m_W^4 + \left(\frac{m_H^2}{2} + m_W^2\right)m_H^2 + \frac{7}{8}\right)m_H^4 - 4m_T^4 \right)
\]

\[
T_0^2 := \frac{m_H^4 - 8v_0^2B}{4D}
\]

\[
E := \frac{3g^3}{32\pi}
\]

\[
\lambda_T := \lambda - \frac{3}{16\pi^2v_0^2} \left[2m_W^4 \ln \frac{m_W^2}{a_B T^2} + \left(\frac{m_H^2}{2} + m_W^2\right)^2 \ln \frac{m_W^2}{a_B T^2}
\]

\[
+ \frac{3}{4}m_H^2 \ln \frac{m_W^2}{a_B T^2} - 4m_T^2 \ln \frac{m_T^2}{a_B T^2} \right]
\]

where \(\ln a_B = 2 \ln 4\pi - 2\gamma \simeq 3.91, \ln a_F = 2 \ln \tau - 2\gamma \simeq 1.14, v_0 = 246\text{GeV}, m_W = \frac{1}{2}gv_0\) and \(\Theta_W = 0\). The form of the constants \(B, D, E\) and \(\lambda_T\) differs from those in \([9]\), because of the contributions of the Higgs boson and would-be-Goldstone boson fluctuations taken into account here. The renormalization has been performed in the minimum of the classical potential at \(T = 0\).

The given representations of the effective action are quite formal. They are divergent and have to be renormalized. Furthermore, they have to be evaluated for a system at finite temperature. This is done in the next section.

**III. RENORMALIZATION OF THE RESUMMED EFFECTIVE ACTION**

The vacuum structure is changed due to the contributions of the different fluctuating fields to the one-loop effective potential. The tree-level mass and the tree-level self-coupling of the Higgs field are modified due to effects of finite temperature. They will be modified again by effects of higher order. Dependent on the temperature these corrections are not negligible and a pure one-loop calculation is not reliable \([29,30]\).

In order to take into account at least the dominant contributions of the higher order effects one has to consider the sum of one-loop-Daisy-graphs, replacing the expressions due to the inserted tadpole-graphs by their leading temperature dependence which is proportional to \(T^2\). Also the vertices are replaced by those which take into account the temperature dependence of the coupling.

In practice the summation of Daisy-graphs is performed by replacing the tree-level-potential \(U(\phi)\) in the Lagrangian by the high temperature potential \(V_{\text{ht}}\) given in the preceding section \([31–34]\). Then the fluctuation operators and the contributions to the effective action are determined. The effective action based upon these fluctuation operators is improved by resummation. It can be written as

\[
S_R[\phi, T] = \sum_i S_{i,R}[\phi, T] + \sum_i \sum_{n=0}^{+\infty} c_i \ln \left[\frac{-\Delta + \nu_n^2 + U_i(\phi, T)}{-\Delta + \nu_n^2 + U_i(0, T)}\right],
\]

where \(U_i(\phi, T)\) is given by \((21)\)ff. with \(m^2_i(\phi)\) and \(m^2_i(\phi)\) replaced by

\[
m^2_i(\phi, T) = 3\lambda_T \phi^2 - 6ET\phi + m_H^2, \quad m^2_i(\phi, T) = \frac{g^2}{4} \phi^2 + \lambda_T \phi^2 - 3ET\phi + m_H^2.
\]

Expanding the effective action in a series of one-loop Feynman-graphs it can be rewritten as

\[
S_{i,R} \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}c_i}{k} \sum_{n=-\infty}^{+\infty} \text{Tr} \left\{\left[-\Delta + \nu_n^2 + U_i(0, T)\right]^{-1} \left[U_i(\phi, T) - U_i(0, T)\right]\right\}^k
\]
\[ \sum_{k=1}^{+\infty} S_{i,R}^{(k)} . \]  

Here \([-\Delta + \nu_n^2 + U_i(0,T)]^{-1}\) is a formal representation of the Green’s function associated to that operator. The superscript \((k)\) denotes the order of the corresponding graph (see fig. 1).

In analogy also the improved one-loop effective potential \(V_{i,R}(\phi, T)\) exists. It has the following relation to the usual one-loop effective action \(V_i(\phi, T)\):

\[
V_{i,R}(\phi, T) = c_i T \operatorname{tr} \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \ln \left[ p^2 + \nu_n^2 + U_i(\phi, T) \right] 
= c_i T \operatorname{tr} \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \ln \left[ p^2 + \nu_n^2 + U_i(\phi) \right] 
+ c_i T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \ln \left[ \frac{p^2 + \nu_n^2 + U_i(\phi, T)}{p^2 + \nu_n^2 + U_i(\phi)} \right] 
= V_i(\phi, T) + V_{i,\text{ring}}(\phi, T) .
\]  

(38)

The divergencies in \(V_i\) are temperature independent. They are cancelled by the familiar \(T = 0\)-counter terms. However, also \(V_{i,\text{ring}}\) contains divergencies which furthermore are temperature dependent. Hence, the substitution of \(U_i(\phi)\) by \(V_{i,\text{ct}}(\phi)\) accounts for finite and infinite contributions of higher orders. The latter are removed by the use of thermal counter terms \([34,35]\). They are not fixed by renormalization conditions but are chosen in such a way that they cancel the two divergent orders of \(V_{i,\text{ring}}(\phi, T)\) exactly. These two orders \((k = 1 \text{ and } k = 2)\) can be read of from:

\[
V_{i,\text{ring}}(\phi, T) = \sum_{k=1}^{+\infty} V_{i,\text{ring}}^{(k)} = c_i T \operatorname{tr} \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k} \times \left\{ \left[ U_i(\phi, T) - U_i(0, T) \right] \right\}^k 
- \left[ U_i(\phi) - U_i(0, T) \right] \}
\]  

(39)

However, the high temperature contribution of \(V_{i,\text{ring}}^{(2)}\), i.e. the contribution of the static Matsubara mode, is finite and is not subtracted.

For the numerical application the resummed one-loop effective action is separated into a finite part that is evaluated numerically and an infinite contribution that has to be renormalized. The latter is given by the first two orders of the series \((34)\). In performing the renormalization by adding the counter terms one finds contributions already taken into account in the high temperature potential \(V_{i,\text{ht}}(\phi, T)\); we denote these terms as \(V_{i,\text{ht}}\). These terms must be subtracted in order to avoid double counting; we call the resulting difference \(\Delta S_{i,R,\text{ren}}^{(1+2)}\):

\[
\Delta S_{i,R,\text{ren}}^{(1+2)} = S_{i,R}^{(1)} + S_{i,R}^{(2)} - S_{i,R,\text{ht}} - \beta \int d^3x \left[ V_{i,\text{ring}}^{(1)} + V_{i,\text{ring}}^{(2)} - V_{i,\text{ring,ht}}^{(2)} + V_{i,\text{ht}} \right] + S_{i,\text{ct}} .
\]  

(40)

Here we make use of that the high temperature approximation of the second order contribution, \(S_{i,R,\text{ht}}^{(2)}\), is finite. Therefore, it is treated together with the finite part, \(S_{i,R}^{(2)}\), and is evaluated numerically:

\[
S_{i,R,\text{num}} = S_{i,R,\text{ht}}^{(2)} + S_{i,R,\text{ht}}^{(3)} + S_{i,R,\text{num}}^{(3)} = S_{i,R,\text{ht}}^{(2)} + S_{i,R,\text{num}}^{(3)} .
\]  

(41)

Here a superscript \((k)\) denotes the sum of contributions beginning with the \(k\)-th order. The index ‘ns’ denotes the contributions of the non-static Matsubara modes. It is known to be approximated sufficiently by a gradient expansion \([18]\). It is found to be negligible small compared to the contribution due to the static mode. Therefore it is neglected and only \(S_{i,R,\text{ht}}^{(2)}\) is evaluated numerically. This is postponed to the next section.

Using

\[
T \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + \nu_n^2 + m^2} = \int \frac{d^4p}{(2\pi)^3} \frac{1}{p^2 + m^2}
\]  

(42)
\[
T \sum_{n=-\infty}^{+\infty} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{(p^2 + \nu_n^2 + m^2)^2} = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{(p^2 + m^2)^2}
+ \frac{T}{8\pi m} \ln \frac{m^2}{a_B T^2} + \delta_1(T, m^2),
\]
and neglecting the terms with \(\delta_1\) and \(\delta_2\), which are small at the temperature we are working at, the analytic part \(\Delta S_{R, \text{ren}}^{(1+2)}\) is found to be:

\[
\Delta S_{R, \text{ren}}^{(1+2)} = \beta \int d^3 x \left\{ \frac{3\lambda}{32\pi^2} \left( \frac{m_H^2}{a_B T^2} + 2 \right) \ln \frac{m_H^2}{a_B T^2} + 2 - \frac{3\lambda m_H T}{8\pi} \right\} \phi^2 \\
+ \frac{g^2 T}{32\pi} \phi^3 + \left( g^2 + \lambda \right) \left[ -\frac{3m_H^2 T}{8\pi} - \frac{3m_W^2}{32\pi^2} + \frac{3m_H^2}{32\pi^2} + \frac{3m_W^2}{64\pi^2} \ln \frac{m_W^2}{a_B T^2} \right] \phi^2 \\
+ \frac{g^2 T}{16\pi} \phi^3 - \frac{3g^2}{32\pi^2} \left( \nabla \phi \right)^2 \ln \frac{m_W^2}{a_B T^2} \right\}.
\]

IV. DESCRIPTION OF THE NUMERICAL METHOD

A. The functional derivative of the effective action

In the previous section the calculation of the finite contribution to the resummed effective action due to the different fluctuating fields was postponed. Combining them to \(S_{R, \text{fin}}^{(2)}\), its functional derivative with respect to the background field \(\phi\) can be represented as

\[
\frac{\delta S_{R, \text{fin}}}{\delta \phi(x)} = \text{tr} \left[ c \frac{\delta U(\phi, T)}{\delta \phi(x)} G(x, x) \right] \\
= \text{tr} \left[ c \frac{\delta \mathcal{M}^2(\phi, T)}{\delta \phi(x)} G(x, x) \right] - g \text{ tr} [c \xi \nabla G(x, x)].
\]

where \(U, \mathcal{M}^2(\phi, T)\) and \(\xi\) are the \((6 \times 6)\) generalisations of the matrices defined in \((21)\) ff. at finite temperature. They are given explicitly in appendix A. \(c\) contains the factors of degeneracy for the different fluctuations. Its non-zero components are given by

\[
c_{11} = c_{22} = c_{33} = c_{44} = -c_{66} = \frac{3}{2}, \text{ and } c_{55} = \frac{1}{2}.
\]

The Green’s function \(G(x, x')\) is the solution of

\[
\left[ -\Delta + U(\phi, T) \right] G(x, x') = 1 \cdot \delta^{(3)}(x - x')
\]

A method for the numerical computation of such a one-loop Green’s function was derived in \([22-24]\). In order to determine \(G(x, x)\) it is decomposed into its partial wave contributions. The details of this calculation are given in appendix A. It turns out that by the decomposition one channel of the coupled system can be combined with the Faddeev-Popov channel so that the system reduces to a \((5 \times 5)\)-system. Extracting the order \((2)\) from \((45)\) it takes the form:

\[
\frac{\delta S_{R, \text{fin}}^{(2)}}{\delta \phi(x)} = \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} \left\{ \frac{3g^2}{4} \phi \left( \frac{g_{11,l}}{g_{11,l}} + g_{22,l} \right) + \frac{3}{2} \left( \frac{g^2}{2} \phi + 2\lambda \phi - 3\pi T \right) g_{33,l} \right\} \\
+ 3(\lambda \phi - ET) g_{44,l} \left\{ \frac{3g^2}{16\pi} \phi g_{55,0} \right\} \\
- \frac{3g^2}{2} \sum_{l} \frac{2l + 1}{2\pi} \left\{ -c_1 \left( \frac{g_{11,l}}{g_{11,l}} + \frac{2}{r} g_{22,l} \right) + c_0 \left( \frac{g_{23,l}}{g_{23,l}} + \frac{2}{r} g_{23,l} \right) \right\}.
\]
This coincides with [17] where it was derived by reduction of the sphaleron system. The radial Green’s function is

\[ \kappa_p \]

powers of \( b \) where \( V \)
The non-zero components of the potential \( V(\phi) \) are

\[ V_{11} = V_{22} = V_{55} = \frac{g^2}{4} \phi^2(r), \quad V_{33} = \left( \frac{g^2}{4} + \lambda_T \right) \phi^2(r) - 3ET\phi(r), \]

\[ V_{44} = 3\lambda_T \phi^2(r) - 6ET\phi(r), \quad V_{13} = V_{31} = -\sqrt{\frac{l+1}{2l+1}} g\phi'(r), \]

and \( V_{23} = V_{32} = \sqrt{\frac{l}{2l+1}} g\phi'(r). \)

This coincides with [17] where it was derived by reduction of the sphaleron system. The radial Green’s function is given by [23]

\[ g_{km,l}(r, r') = \kappa \left[ f^{\alpha^+}_{k,l}(r)f^{\alpha^-}_{m,l}(r')\Theta(r-r') + f^{\alpha^-}_{k,l}(r)f^{\alpha^+}_{m,l}(r')\Theta(r'-r) \right], \]

where \( f^{\alpha^+}_{k,l} \) and \( f^{\alpha^-}_{k,l} \) are the linear independent solutions of the homogeneous equation

\[ \left[ \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l_n(l_n+1)}{r^2} - m_n^2 \right) \delta_{nk} - V_{nk}(\phi, T) \right] f^{\alpha^\pm}_{k,l}(r) = 0. \]

The mode functions \( f^{\alpha^+}_{k,l} \) \( (f^{\alpha^-}_{k,l}) \) are regular (singular) at infinity. In order to calculate the mode functions numerically their asymptotic behaviour at infinity is separated from the rest by the following ansatz [24]:

\[ f^{\alpha^\pm}_{k,l}(r) = b^\pm_{k,l}(z) \left( \delta^\alpha_k + b^\pm_{k,l}(r) \right) \quad \text{with} \quad z = kr. \]

The functions \( b^\pm_{k,l}(z) \) satisfy the equation

\[ \left( \frac{\partial^2}{\partial z^2} + \frac{2}{z} \frac{\partial}{\partial z} - \frac{l_n(l_n+1)}{z^2} - 1 \right) b^\pm_{k,l}(z) = 0, \]

One obtains

\[ b^+_1(z) = z^{-(l+2)!}(2l+1)!! \quad b^+_1(z) = \frac{z^{l+1}}{(2l+3)!!}, \]

\[ b^-_1(z) = z^{-l}(2l+1)!! \quad b^-_1(z) = \frac{z^{l-1}}{(2l+1)!!}, \]

\[ b^+_1(z) = b^+_1(z) = k_l(z) \quad b^+_1(z) = b^+_1(z) = i_l(z), \]

\[ b^-_1(z) = 1 \quad b^-_1(z) = 1. \]

In the massive channels we obtain the modified Bessel functions, while in the massless ones the solutions reduce to powers of \( z \). The fifth channel only contributes in the s-wave. Therefore, its Bessel function is given only for \( l = 0 \). The normalization constants and \( \kappa = m_H \) are chosen in such a way that the Wronskians of the different channels become equal:

\[ W\left[b^+_1(z), b^-_1(z)\right] = \frac{1}{z^2}. \]
The differential equation of the rest mode functions $h_{n,l}^{\alpha \pm}$ reduces to:

$$
\left[ \frac{\partial^2}{\partial r^2} + 2 \left( \frac{1}{r} + \kappa \frac{b_{l,m}^+(z)}{b_{l,m}^-(z)} \right) \frac{\partial}{\partial r} \right] h_{n,l}^{\alpha \pm}(r) = V_{nk}(\delta_{k,l}^\alpha + h_{k,l}^{\alpha \pm}(r)) \frac{b_{l,m}^+(z)}{b_{l,m}^-(z)} .
$$

(57)

This equation can be used for generating the functions $h_{n,l}^{\alpha \pm}$ order by order in the potential $V$. Introducing the contribution of order $k$ in the potential as $h_{n,l}^{\alpha(k)}$ and defining

$$
h_{n,l}^{\alpha(k)} := \sum_{j=k}^{+\infty} h_{n,l}^{\alpha(j)}
$$

as in [19], the order $(1)$ of the Green’s function is given by:

$$
g_{km,l}(r,r) = \frac{\kappa}{2} \left[ b_{l,m}^-(z)b_{l,m}^+(z) \left( h_{m,l}^{-(1)}(r) + h_{k,l}^{m+(1)}(r) + h_{k,l}^{n+(1)}(r)h_{m,l}^{n-(1)}(r) \right) \\
+ b_{l,m}^-(z)b_{l,m}^+(z) \left( h_{k,l}^{m-(1)}(r) + h_{m,l}^{k+(1)}(r) + h_{m,l}^{n+(1)}(r)h_{k,l}^{n-(1)}(r) \right) \right]
$$

(58)

**B. Evaluation of the effective action**

In order to calculate the nucleation rate $\gamma$ the effective action in the one-loop approximation is needed. A method to calculate its functional derivative with respect to the background field has been given in the previous subsection. For the purpose of renormalization the divergent parts have been separated from the rest and have been handled analytically. The same separation can be used here in order to evaluate the effective action itself. The analytic part is given by (44)ff. The numerical contributions $S_{1,1u}$ are determined in the following.

For the numerical computation of the effective action the fluctuation determinant $J$ is decomposed into partial waves

$$
\ln J = \sum_l (2l + 1) \ln J_l,
$$

(59)

with $J_l = \det \underline{M}/\det \underline{M}^0$.

Here $\underline{M}$ and $\underline{M}^0$ are the fluctuation operators defined in (49). We need to know two $(n \times n)$ matrices $\tilde{f}(\nu, r)$ and $\tilde{f}^0(\nu, r)$ which contain the $n$ linear independent solutions $f_{k,l}^{\nu}(\nu, r)$ and $f_{k,l}^{0\nu}(\nu, r)$ of the system of coupled differential equations

$$
(\underline{M} + \nu^2)_{nk} \tilde{f}_{k,l}^{\nu}(\nu, r) = 0 \quad \text{and} \quad (\underline{M}^0 + \nu^2)_{nk} \tilde{f}_{k,l}^{0\nu}(\nu, r) = 0 .
$$

(60)

The boundary conditions of these functions are chosen to be regular at $r = 0$. The lower index denotes the $n$ components while the different solutions are labelled by the Greek upper index. These solutions are normalized such that

$$
\lim_{r \to 0} \tilde{f}_l(\nu, r) \left( \tilde{f}_l^{0}(\nu, r) \right)^{-1} = 1 .
$$

(61)

On these conditions the statement of the theorem is [19]:

$$
J_{l,1}(\nu) := \frac{\det \underline{M} + \nu^2}{\det \underline{M}^0 + \nu^2} = \lim_{r \to \infty} \frac{\det f(\nu, r)}{\det f^0(\nu, r)} .
$$

(62)

The theorem has been applied for computing the one-loop effective action of a single scalar field on a bubble background [20], and of a fermion system on a similar background [19] previously. Furthermore it has been used to calculate the bosonic fluctuation determinant of the critical bubble [17]. It was found to yield very precise results, in addition to providing a very fast computational method.
If the theorem is applied at \( \nu = 0 \) the fluctuation determinant \( \mathcal{J} \equiv \mathcal{J}(0) \) is obtained. The consideration of finite values of \( \nu \) is necessary in the discussion of zero modes. For simplicity the following formulas are given only for \( \nu = 0 \).

In analogy to the mode functions \( f_{k}^{0}(r) \) also the functions \( \tilde{f}_{l}(r) \) can be decomposed as:

\[
\tilde{f}_{l}(r) = h_{l}(k\nu) \left( \delta_{k}^{0} + \tilde{h}_{k,l}^{0}(r) \right),
\]

with \( h_{l}(k\nu) \) given by (55). The consequence is that the separated functions \( \tilde{h}_{k,l}^{0}(r) \) satisfy the same type of differential equation as the rest mode functions \( h_{k,l}^{0}(r) \). Only the boundary conditions differ, namely \( \tilde{h}_{k,l}^{0}(0) = 0 \) and \( \tilde{h}_{k,l}^{0}(\infty) = \text{const} \). The two functions are related by:

\[
h_{l}^{0}(r) = \left[ 1 + \tilde{h}_{l}(r) \right] \left[ 1 + \tilde{h}_{l}(\infty) \right]^{-1} - 1.
\]

After determining \( \tilde{f}_{l}(r) \) one obtains the desired order of the fluctuation determinant.

\[
\ln \mathcal{J}_{l}^{(0)} = \lim_{r \to \infty} \left\{ \ln \det[1 + \tilde{h}_{l}(r)] - \ln \tilde{h}_{l}^{(1)} \right\}.
\]

As the determinant \( \mathcal{J} \) is a product of three individual determinants due to the coupled and uncoupled channels one has to take into account the different factors of degeneracy for each of them (see (45)).

**C. Self-consistent determination of the bubble profile**

1. Computing the start profile for the iteration

In order to solve the one-loop field equation iteratively we have to choose a suitable separation of the equation into a left-hand and right-hand side. Then setting the right-hand side to zero results in a first approximate solution of the equation. This is taken as the start profile for the iteration.

In particular we choose the following form of the field equation

\[
-\Delta \phi + \frac{\partial}{\partial \phi} \phi V_{R}(\phi, T) = \mathcal{I}(\phi),
\]

with

\[
V_{R}(\phi, T) := V_{m}(\phi, T) - \frac{T}{12\pi} \left( 3\lambda_{T} \phi^{2} + \tilde{m}_{H}^{2} \right)^{\frac{3}{2}} + \frac{3\lambda_{T} \tilde{m}_{H} T}{8\pi} \frac{\phi^{2}}{\tilde{m}_{H}^{2}} + \frac{\tilde{m}_{H}^{2}}{12\pi} \frac{3\lambda_{T} \tilde{m}_{H} T}{8\pi} \left( \frac{\phi^{2}}{4} + \lambda_{T} \right) \phi^{2} - 3\lambda_{T} \tilde{m}_{H} T \phi^{2} - 3\lambda_{T} \tilde{m}_{H} T \phi^{2}
\]

\[
+ \frac{3\lambda_{T} \tilde{m}_{H}^{2}}{8\pi} \left( \frac{3\lambda_{T} \tilde{m}_{H} T}{32\pi} + \frac{3\lambda_{T} \tilde{m}_{H}^{2}}{32\pi} + \frac{3\lambda_{T} \tilde{m}_{H} T}{32\pi} \left( \frac{\phi^{2}}{4} + \lambda_{T} \right) \phi^{2} - 3\lambda_{T} \tilde{m}_{H} T \phi^{2} - 3\lambda_{T} \tilde{m}_{H} T \phi^{2} \right)
\]

\[
+ \left( \frac{3\lambda_{T} \tilde{m}_{H}^{2}}{32\pi} + \frac{3\lambda_{T} \tilde{m}_{H} T}{32\pi} \left( \frac{\phi^{2}}{4} + \lambda_{T} \right) \phi^{2} - 3\lambda_{T} \tilde{m}_{H} T \phi^{2} - 3\lambda_{T} \tilde{m}_{H} T \phi^{2} \right)
\]

I. e., in addition to \( V_{m} \) the contributions of the isoscalar and isovector part of the Higgs field to the resummed effective potential are shifted to the left-hand side. For a suitable chosen temperature the potential \( V_{R} \) has a secondary minimum at a nonzero value of \( \phi \). We denote this minimum as \( \bar{v}(T) \). It is used to scale the dimensional quantities as follows:

\[
\phi = \bar{v}(T) \bar{\phi}, \quad x_{\mu} = \frac{1}{\bar{g}(T)} x_{\mu}, \quad \text{and} \quad T = g(T) \bar{T}.
\]

All masses are scaled with \( g(T) \). The inhomogeneity \( \mathcal{I}(\phi) \) consists of (44) and (48) reduced by those terms that were shifted to the left-hand side.
\[ I(\phi) = -\frac{3T}{2} \left\{ \frac{g^2}{2} \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} \left[ g_{11,l} + g_{22,l} \right] + \left( \frac{g^2}{2} \phi + 2\lambda_T \phi - 3ET \right) \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} \frac{g_{11,l}}{g_{33,l}} \right\} \\
+ 3gT \sum_{l=0}^{\infty} \frac{2l + 1}{2\pi} \left[ -c_1 \left( \frac{g_{11,l}}{g_{33,l}} + \frac{2}{r} g_{13,l} \right) + c_0 \left( \frac{g_{23,l}}{g_{33,l}} + \frac{2}{r} g_{23,l} \right) \right] \\
- 3T \left( \lambda_T \phi - ET \right) \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} g_{h,l} + \frac{3g^2T}{16\pi} \phi \phi_{a_0} \right\} - 3\lambda_T \phi \left[ \frac{3\lambda_T \phi^2 + \tilde{m}_H^2}{4} - \bar{m}_H \right] - \frac{3g^2T}{16\pi} \phi^2 - \frac{3g^2}{32\pi^2} \Delta \phi \ln \frac{m_W^2}{a_B T^2} \\
- \frac{3T}{8\pi} \left[ \left( \frac{g^2}{4} + \lambda_T \right) \phi - 3ET \right] \left[ \left( \frac{g^2}{4} + \lambda_T \right) \phi^2 - 3ET \phi + \tilde{m}_H^2 \right]^\frac{1}{2} - \bar{m}_H \right]. \]

For the purpose of comparison the solution \( \phi_{cl} \) of the equation

\[ -\Delta \phi_{cl} + \frac{\partial}{\partial \phi} V_{\text{int}}(\phi, T) \bigg|_{\phi=\phi_{cl}} = 0 \]  

is also calculated. It is denoted as the classical profile.

In order to avoid large changes between two subsequent profiles especially at the beginning of the iteration we use a relaxation, i.e. the field equation (66) is written as

\[ -\Delta \phi^{(n+1)} + \frac{\partial}{\partial \phi} V_R(\phi, T) \bigg|_{\phi=\phi^{(n+1)}} = (1 - \varepsilon) I(\phi^{(n-1)}) + \varepsilon I(\phi^{(n)}). \]

By a suitable chosen \( \varepsilon \) the convergence of the iterations of the profile can be improved.

\section*{2. Treatment of zero modes and unstable mode}

For the isoscalar fluctuations there are two contributions to the effective action and its functional derivative that have to be treated individually: namely the unstable mode in the partial wave \( l = 0 \) and the translation zero mode in the partial wave \( l = 1 \).

A bound state occurs in the s-wave, i.e. a state with a negative eigenvalue. As this is the only negative eigenvalue the total determinant is also negative and the one-loop contribution to the effective action is complex. This corresponds to the fact that the state under consideration, the critical bubble, is unstable.

In the p-wave there exists a mode with the eigenvalue zero. It corresponds to the fact that the critical bubble breaks translational invariance. Because of this mode the determinant itself is zero and the effective action diverges.

For the calculation of the nucleation rate instead of the fluctuation determinant with these two properties we need one where the negative eigenvalue has been replaced by its modulus and where the three eigenvalues zero have been removed. These changes to the determinant are indicated by two primes:

\[ J_h = \prod_{n=\infty}^{+\infty} \frac{\det \left[ -\Delta + \nu_n^2 + m_n^2(\phi, T) \right]}{\det \left[ -\Delta + \nu_n^2 + m_n(0, T) \right]} \]

As three eigenvalues zero have been removed from the determinant of the isoscalar fluctuations the rate gets its dimension (energy)$^3$, i.e. (volume)$^{-1}$. The further dimension (time)$^{-1}$ is due to the eigenvalue \( \omega^- \) of the unstable mode.

The numerical method presented in the previous sections does not explicitly make use of the eigenvalues. Therefore, we have to show how these two contributions are removed from the effective action – this has also been discussed in [17] – and its functional derivative.

In case of the unstable mode this is performed easily. One computes the particular partial wave contributions according to (65). In the partial wave \( l = 0 \) only the sign of the determinant has to be inverted before taking the logarithm. For the functional derivative the sign is irrelevant because due to

\[ \frac{\delta}{\delta \phi(x)} \ln J = \frac{\delta}{\delta \phi(x)} \ln \prod_{\alpha} \omega_{\alpha}^2 = \sum_{\alpha} \frac{1}{\omega_{\alpha}^2} \frac{\delta \omega_{\alpha}^2}{\delta \phi(x)} \]

\[ \frac{\delta}{\delta \phi(x)} \ln J = \frac{\delta}{\delta \phi(x)} \ln \prod_{\alpha} \omega_{\alpha}^2 = \sum_{\alpha} \frac{1}{\omega_{\alpha}^2} \frac{\delta \omega_{\alpha}^2}{\delta \phi(x)} \]

\[ \frac{\delta}{\delta \phi(x)} \ln J = \frac{\delta}{\delta \phi(x)} \ln \prod_{\alpha} \omega_{\alpha}^2 = \sum_{\alpha} \frac{1}{\omega_{\alpha}^2} \frac{\delta \omega_{\alpha}^2}{\delta \phi(x)} \]

\[ \frac{\delta}{\delta \phi(x)} \ln J = \frac{\delta}{\delta \phi(x)} \ln \prod_{\alpha} \omega_{\alpha}^2 = \sum_{\alpha} \frac{1}{\omega_{\alpha}^2} \frac{\delta \omega_{\alpha}^2}{\delta \phi(x)} \]

\[ \frac{\delta}{\delta \phi(x)} \ln J = \frac{\delta}{\delta \phi(x)} \ln \prod_{\alpha} \omega_{\alpha}^2 = \sum_{\alpha} \frac{1}{\omega_{\alpha}^2} \frac{\delta \omega_{\alpha}^2}{\delta \phi(x)} \]

\[ \frac{\delta}{\delta \phi(x)} \ln J = \frac{\delta}{\delta \phi(x)} \ln \prod_{\alpha} \omega_{\alpha}^2 = \sum_{\alpha} \frac{1}{\omega_{\alpha}^2} \frac{\delta \omega_{\alpha}^2}{\delta \phi(x)} \]
the sign of each eigenvalue enters the equation twice.

The removal of the translational zero mode is more complicated. An alternative representation for the Green’s function $g_{44,1}(r, r')$ compared to (51) can be deduced from the eigenvalues and eigenfunctions of

$$
\left[ -\frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + \frac{l(l+1)}{r^2} + m^2_H + 3\lambda T \phi^2 - 6ET \phi \right] \psi_\alpha(r) = \omega_\alpha^2 \psi_\alpha(r). \tag{74}
$$

It is given by:

$$
g_{44,1}(r, r') = \sum_\alpha \frac{\psi_\alpha(r)\psi_\alpha^*(r')}{\omega_\alpha^2}. \tag{75}
$$

Obviously this Green’s function is divergent when the spectrum of the operator contains the eigenvalue zero. Defining a new Green’s function as

$$
g_{44,1}(r, r', \nu) = \sum_\alpha \frac{\psi_\alpha(r)\psi_\alpha^*(r')}{\nu^2 + \omega_\alpha^2}, \tag{76}
$$

the divergence manifests itself in the limit $\nu \to 0$ in the form of a $1/\nu^2$-pole. As the zero mode is the only mode which contributes to this pole its contribution can be removed by defining the Green’s function at $\nu = 0$ without the zero mode in the following way:

$$
g_{44,1}(r, r', 0) := \lim_{\nu \to 0} \left[ g_{44,1}(r, r', \nu) - \frac{\psi_0(r)\psi_0^*(r')}{\nu^2} \right]. \tag{77}
$$

In the special case under consideration, where the Green’s function only has to be calculated at $r = r'$, the practical way to subtract the zero mode in the $p$-wave is to compute the function $g_{44,1}(r, r, \nu)$ for sufficient many different values of $\nu$. Then in the function

$$
\tilde{g}_{44,1}(r, r, \nu) := \frac{A(r)}{\nu^2} + B(r) + C(r)\nu^2 \tag{78}
$$

the parameters $A$, $B$ and $C$ are adjusted by a fit with respect to $\nu$ for each value of $r$. The subtraction of the zero mode is then done by setting $g_{44,1}(r, r, 0) := B(r)$.

The computation of the function $g_{44,1}(r, r, \nu)$ is done in the same way as described in section IV. The only difference is, in the definition of the radial part of the Green’s function (49) and in all consecutive equations a $\nu^2$ has to be added to the partial wave operator $M^0$. Hence, the ‘mass’ $\kappa$ of the isoscalar channel changes to $\kappa = \sqrt{m^2_H + \nu^2}$.

The one-loop contribution of the isoscalar fluctuation to the fluctuation determinant is determined according to (65) in each partial wave. In the $p$-wave the following expression has to be evaluated:

$$
\lim_{r \to \infty} \ln \left[ 1 + \tilde{h}_{4,1}^4(r) \right]. \tag{79}
$$

The numerical investigation shows that $\tilde{h}_{4,1}^4(\infty) = -1$ and the logarithm diverges due to the zero mode in this partial wave. In order to subtract the mode the computational method to determine the fluctuation determinant is extended in the same way as before in case of the Green’s function. I. e., the theorem mentioned above is applied at finite $\nu$. In order to subtract the zero mode the determinant $\tilde{J}_0,1(\nu)$ has to be replaced by [17,20]

$$
\lim_{\nu \to 0} \lim_{r \to \infty} \frac{d}{d(\nu^2)} \tilde{h}_{4,1}^4(\nu, r). \tag{80}
$$

In practice $\tilde{h}_{4,1}^4(\nu, \infty)$ is computed for some sufficient small values of $\nu$ and is used to fit the constants $A$, $B$ and $C$ in

$$
1 + \tilde{h}_{4,1}^4(\nu, \infty) = A\nu^2 + B\nu^4 + C\nu^6. \tag{81}
$$

With these techniques the unstable mode and the translation mode are removed from the effective action as well as from its functional derivative. Concerning the translation mode a further remark is necessary. An exact zero mode only exists when the profile $\phi$ is determined from the classical field equation (70). The one-loop equation differs from the classical one due to the non-vanishing right-hand side of the equation. As a consequence one finds a bound state in the spectrum in the $p$-wave below the continuum threshold but its eigenvalue only is near to zero but it is not exact zero. Nevertheless this mode is subtracted.

In the coupled system of gauge fields and would-be-Goldstone bosons a zero mode exists in the $s$-wave. For the treatment of this mode we adopt the prescription given in [17].
In order to determine the profile for one step of the iteration a boundary problem has to be solved. At $r = 0$ the slope of the profile has to vanish and at $r = \infty$ the function has to tend to zero exponentially. The profile is obtained using a shooting method: The value of the profile at $r = 0$ is varied until the profile gets the correct behavior at infinity. We typically use 1000 steps on a scale from $r = 0$ to $r_{\text{max}} \approx 1/\tilde{m}_H$ where the latter typically is below 200 in the units $\tilde{g} \tilde{v}$ we are working in. The numerical technique used to integrate the differential equation is the Nyström method.

For small values of $r$ the profile can be represented as $\phi(r) \approx a_0 + a_2 r^2 + a_4 r^4$. Hence, for a given profile the parameters can be obtained using a simple fit. Then for the rest mode functions $\tilde{h}^\alpha(r)$ which are calculated from the differential equations (57) the behavior at small $r$ can be evaluated analytically using a power series expansion. Using this expansion the functions have the correct starting properties for the following numerical integration. Hence, we can avoid numerical problems due to the divergent behavior of the Bessel functions in this region.

The radial Green’s functions are composed from the rest mode functions according to (58) and then the partial wave summation (69) is performed. This sum is executed explicitly up to $l_{\text{max}} = 15$. At large $l$ the product $(2l + 1)g_{l,i}(r, r)$ behaves as $a(r)/l^2 + b(r)/l^3 + c(r)/l^4$, no matter which channel $i$ one considers. Using a fit the coefficients $a$, $b$ and $c$ can be obtained for each value of $r$ and then the value of the sum from $l_{\text{max}}$ to infinity can be estimated. As a cross-check we start this estimate at $l = 10$ and it is tested whether it approaches a constant when $l$ reaches $l_{\text{max}}$.

In order to check the precision various cross-checks have been implemented. The equality

$$\omega^\alpha := \tilde{m}_H r^2 W \left[ f_{n,l}^\alpha (r), f_{n,l}^\beta (r) \right] = \delta^\beta_\alpha$$

is satisfied numerically with a relative deviation of the order $O(10^{-6})$. Especially the Wronskian between $f^+(r)$ and $\tilde{f}(r)$ satisfies:

$$\kappa a^2 W \left[ f^\alpha (r), \tilde{f}^\beta (r) \right] = \delta^\beta_\alpha + \tilde{h}^\beta_\alpha (\infty)$$

$$= \delta^\beta_\alpha + \tilde{h}^\alpha_\beta (0).$$

The relative deviations from these identities are of the order $O(10^{-4})$ for (83) and $O(10^{-6})$ for (84). The former being not as precise as the latter is due to the bad convergence of the functions $\tilde{h}^\alpha_\beta$ at large $r$. In the massless channels they only achieve their asymptotic values as $1/r^4$. Hence, their value at $r = \infty$ only can be estimated from a fit. On the other hand the constancy of the product of $r^2$ with the Wronskian is satisfied excellently. Therefore we do not make use of this fitted quantity in the further process.

V. RESULTS AND DISCUSSION

The numerical results are given in table 1 and in the figures 2 to 7. The Higgs boson masses used here are 30GeV and 40GeV. The mass of the top-quark is taken to be $m_t = 170$GeV and for the gauge boson mass we use a value of $m_W = 80.2$GeV. The gauge coupling $g$ is taken as $g = 0.651616$. The vacuum expectation value of the Higgs field is then fixed to be $v_0 = 246$GeV. The relevant temperature interval is determined from the approximation of the resummed effective potential $V_{\text{eff}}$. It is found to be smaller than the interval determined from the high temperature approximation $V_{\text{ht}}$ of the effective potential. The values are given in table 1. Within this interval we choose equidistant steps for the temperature. We leave out the range of temperatures close to the roll-over temperature $T_\beta$, because these correspond to small thick wall bubbles which are of minor cosmological interest.

We give the results for the logarithm of the nucleation rate $\gamma$, i.e. the rate including the one-loop corrections and prefactors as defined in (4). It is evaluated using two different bubble profiles. First we use the self-consistent profile, i.e. the solution of (66). The results have to be compared to that ones we obtain from using the profile $\phi_{\text{cl}}$, which is the solution of (70). The results are displayed in the figures 2 and 3 where we give the logarithm of the nucleation rate versus the temperature. For the results based on the self-consistent profile we also present a function that fits the explicitly calculated points very well. The function has the form

$$F(T) = -a \left( \frac{(T_{\text{cl}}^H)^2}{(T_c^H)^2 - T^2} \right)^b.$$  

The results for $a$ and $b$ depend on the specific parameter set. We find $a = 3.48 \cdot 10^{-2}$ and $b = 1.350$ for $m_H = 30$GeV and $a = 2.38 \cdot 10^{-2}$ and $b = 1.362$ for $m_H = 40$GeV.
At $m_H = 30\text{GeV}$ the self-consistently calculated transition rate is smaller than the rate obtained from the classical profile, while at $m_H = 40\text{GeV}$ it is just the other way around. This is due to the relation between the temperature intervals. While in both cases the interval suitable for finding the self-consistent solution is located within the interval derived from the high temperature potential it is shifted towards higher temperatures with increasing Higgs boson mass (see table 1). For comparison we map the temperature intervals $[T_0^H, T_0^{cH}]$ and $[T_0^B, T_0^{cB}]$ onto the interval $[0,1]$ by introducing the dimensionless quantity $\zeta = (T - T_0)/(T_c - T_0)$. The results of both parameter sets are given in figure 4. For both parameter sets the self-consistently obtained rate is increased compared to the ordinary one. Hence, the effect of using the self-consistent profile is an enhancement of bubble nucleation.

Based on the results for the nucleation rate we can investigate some aspects of the cosmological electroweak phase transition. As the universe expands the temperature $T$ decreases. Once the critical temperature $T_c$ is reached critical bubbles can start to nucleate and grow. The corresponding time is obtained using the relation between time and temperature $[36]$: 

$$t \approx \frac{0.03m_{PL}}{T^2} \quad \text{with } m_{PL} = 1.2 \cdot 10^{19}\text{GeV}. \quad (86)$$

Hence, the nucleation rate $\gamma$ can be considered as a function of time as well. For a given velocity $v$ of the bubble wall the number density of bubbles of size $r$ at the time $t$ is given by

$$n(t, r) = \gamma \left( t - \frac{r}{v} \right). \quad (87)$$

Then the fraction of space that is still in the symmetric phase is obtained as $[37]$:

$$\rho_s(t) = \exp \left[ -\frac{4\pi}{3} \int_{t_c}^{t} \! \! v^3(t - t_1)^3 \gamma(t_1) \right]. \quad (88)$$

This quantity is used to define implicitly the time $t_e$ when the nucleation is completed by $\rho_s(t_e) = 1/e$. As it marks the end of the phase transition it is a quantity that is easier accessible in lattice simulations than other characteristics of the transition. The fraction of space that is already in the asymmetric phase is simply $\rho_a(t) = 1 - \rho_s(t)$. The total number of droplets per unit volume, $N(t)$, also depends on the fraction $\rho_s(t)$ because new bubbles can only nucleate in a region that is still in the symmetric phase:

$$N(t) = \int_{t_e}^{t} \! \! \gamma(t_1)\rho_s(t_1). \quad (89)$$

This quantity is important for the discussion of structure formation. The average bubble radius, $R(t)$, is obtained as:

$$R(t) = \frac{1}{N(t)} \int_{t_e}^{t} \! \! v(t - t_1)\gamma(t_1)\rho_s(t_1). \quad (90)$$

The quantities $\rho_s(t)$, $N(t)$ and $R(t)$ are evaluated for the parameter set $m_H = 30\text{GeV}$, $m_t = 170\text{GeV}$ for different values of the bubble wall velocity $v$. The results are displayed in figure 5 to 7. The completion time $t_e$ depends only weakly on the bubble wall velocity and is roughly $t_e - t_c \approx 0.15 \cdot 10^{-12}\text{sec}$ (see figure 5). This can be compared to a previous calculation of this quantity $[38]$, where a value of roughly $3 \cdot 10^{-12}\text{sec}$ is given. One sees the effect of enhancing the rate. The time scale on which the nucleation proceeds is decreased by an order of magnitude. Also for the number of bubbles per volume after the completion time the results can be compared. For three values of the bubble wall velocity, namely $c$, $c/10$ and $c/1000$ the corresponding values for $\ln(N/\text{cm}^3)$ can be read off from figure 6: they are 28.1, 34.7 and 47.8. For the same velocities in $[38]$ the values 9.3, 12.3 and 18.0 are given; hence, the number of bubbles per volume is increased substantially. The average bubble radius $R(t)$ for the three velocities under consideration comes out to be the same as in $[38]$ and is plotted in figure 7. As the initial radius of the critical bubble is of the order of 1fm it is neglected throughout this calculation.

The presented method to calculate self-consistently the solution of a one-loop field equation has the following essential features:

- The complete one-loop determinant and its functional derivative is evaluated. No expansion in any quantity is needed. In view of the fact that the derivative and heat kernel expansion generate unphysical infrared divergencies order by order this is an important conceptual advantage of the method presented here.
• Coupled with an iterative procedure the numerical computation of the functional derivative of an effective action allows the determination of a self-consistent solution of a full one-loop field equation.

• The divergent and convergent contributions to the effective action can be treated separately. While the convergent part is calculated numerically the divergent part is evaluated analytically. So the choice of the regularization and renormalization prescription is free.

• The numerical computation of the functional derivative of an effective action only requires numerical integration of a few differential equations per partial wave. This can be done with high precision and is not computer time consuming.

In conclusion we find that taking into account the radiative corrections already in the determination of the bounce solution has the following effects:

• The width of the temperature interval within that a first order phase transition is possible is decreased and with increasing the Higgs boson mass it is shifted towards higher temperatures.

• The transition rate is increased substantially relative to the rate obtained from the standard calculation.

• The time scale within that the transition is completed is reduced. The number of bubbles per volume is increased.

ACKNOWLEDGEMENTS

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APPENDIX A: PARTIAL WAVE DECOMPOSITION OF THE GREEN’S FUNCTION

In [17] the partial wave decomposition of the Green’s function was done by reduction of the sphaleron system. An alternative derivation will be given here. The Green’s function \( G(x, x') \) introduced in (45) satisfies the matrix equation

\[
\begin{pmatrix}
- \frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \partial^2 + M^2(\phi, T) + g \partial G
\end{pmatrix} G(x, x') = \mathbb{I} \cdot \delta^{(3)}(x - x') .
\]

(A1)

The non-zero components of \( M^2(\phi, T) \) and \( \zeta \) are given by

\[
M_{11}^2 = M_{22}^2 = M_{33}^2 = M_{66}^2 = m_6^2(\phi) , \quad M_{44}^2 = m_4^2(\phi, T) , \quad M_{55}^2 = m_5^2(\phi, T) , \quad M_{22} = m_2^2(\phi, T) , \quad \zeta_{14} = 1 , \quad \zeta_{24} = 1 , \quad \zeta_{43} = 1 .
\]

(A2)

For the partial wave expansion of the solution \( \Psi(x) \) of the corresponding homogenous problem we use the following ansatz:

\[
\begin{align*}
\Psi(x) &= \sum_{l=0}^{+\infty} \psi_l(x) \\
\psi_l(x) &= \sum_{m=-l}^{+l} \begin{pmatrix}
\varphi_1^l(\hat{r}) & \varphi_2^l(\hat{r}) & \varphi_3^l(\hat{r}) & 0 & 0 & 0 \\
0 & 0 & 0 & \varphi_4^l(\hat{r}) & 0 & 0 \\
0 & 0 & 0 & 0 & \varphi_5^l(\hat{r}) & 0 \\
0 & 0 & 0 & 0 & 0 & \varphi_6^l(\hat{r})
\end{pmatrix} \begin{pmatrix}
f_1(r) \\
f_2(r) \\
f_3(r) \\
f_4(r) \\
f_5(r) \\
f_6(r)
\end{pmatrix}
\end{align*}
\]

(A3)

\[
\begin{align*}
\varphi_1^l(\hat{r}) &= [Y_{l+t} m(\hat{r})]_j , & \varphi_2^l(\hat{r}) &= [Y_{l} m(\hat{r})]_j , & \varphi_3^l(\hat{r}) &= [Y_{l} m(\hat{r})]_j \\
\varphi_4^l(\hat{r}) &= \varphi_5^l(\hat{r}) = \varphi_6^l(\hat{r}) = Y_{lm}(\hat{r}) .
\end{align*}
\]

(A4)

Here \( \hat{r} := x/r \) means the unit vector in \( x \)-direction. To shorten the notation the \( l- \) and \( m- \)dependence of the functions \( \varphi_j^l \) and the \( l- \)dependence of the functions \( f_j \) is not shown explicitly in the following. The vector spherical harmonics \( Y \) (see e. g. [39]) are related to the common surface spherical harmonics:
\[ Y_{l+1 \ell m} = \frac{1}{c_1} \frac{1}{2l+1} \left( -(l+1)\hat{\mathbf{a}}Y_{l \ell m} + r\nabla Y_{l \ell m} \right), \tag{A7} \]
\[ Y_{l-1 \ell m} = \frac{1}{c_0} \frac{1}{2l+1} \left( l\hat{\mathbf{a}}Y_{l \ell m} + r\nabla Y_{l \ell m} \right), \tag{A8} \]
\[ Y_{l \ell m} = \frac{1}{\sqrt{l(l+1)}} \mathbf{IL}_{l \ell m}, \tag{A9} \]
\[ c_0 := \sqrt{\frac{l}{2l+1}}, \quad c_1 := \sqrt{\frac{l+1}{2l+1}}. \tag{A10} \]

For the partial wave \( l = 0 \) one has to consider the following peculiarity: neither the function \( Y_0 = 0(\hat{\mathbf{a}}) \) nor \( Y_0 = 0(\hat{\mathbf{a}}) \)
exists, because the normalizing prefactor diverges. Scaling \( f_2(r) \) with \( c_0 \) and \( f_3(r) \) with \( \sqrt{l(l+1)} \), the products \( Y_{l-1 \ell m} f_2(r) \) and \( Y_{l \ell m} f_3(r) \) are well defined also at \( l = 0 \); they yield zero. I. e., at \( l = 0 \) the components \( f_2(r) \) and \( f_3(r) \) vanish identically. Hence, the coupled \((4 \times 4)\)-system reduces to a coupled \((2 \times 2)\)-system.

In the following the index \( l \) of the functions \( \psi \) is suppressed. Using the index notation the ansatz (A4) can be formulated as
\[ \psi_n = \varphi_n^i f_i, \tag{A12} \]
and with
\[ \mathcal{D}_{nk} := \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \mathbf{IL}^2 \right) \delta_{nk} - \mathcal{M}^2_{nk} \tag{A13} \]
the homogeneous equation for \( \Psi \) takes the form:
\[ \mathcal{D}_{nk} \psi_k = g\phi^i \zeta_{nk}^i \frac{x_j}{r} \psi_k. \tag{A14} \]
Using
\[ \mathbf{IL}^2 \varphi_n^i = l_i(l_i + 1)\varphi_n^i \quad \text{with} \quad l_i = (l+1, l-1, l), \tag{A15} \]
we get
\[ \mathcal{D}_{nk} \psi_k = \varphi_k \left[ \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l_i(l_i + 1)}{r^2} \right) \delta_{nk} - \mathcal{M}^2_{nk} \right] f_i = g\phi^i \zeta_{nk}^i \frac{x_j}{r} \psi_k. \tag{A16} \]
Let \( j \) be an index that takes values from 1 to 3. Then we have
\[ \mathcal{D}_{nj} \psi_j + \mathcal{D}_{n4} \psi_4 + \mathcal{D}_{n5} \psi_5 + \mathcal{D}_{n6} \psi_6 = g\phi^i \delta_{n4} \frac{x_j}{r} \psi_j + g\phi^j \delta^i_{n} \frac{x_j}{r} \psi_4. \tag{A17} \]
With the useful relations [39, S. 103f.]
\[ \frac{x_j}{r} \varphi_4^i = -c_1 \varphi_4^i + c_0 \varphi_7^i, \tag{A18} \]
\[ \frac{x_j}{r} \psi_j = \frac{x_j}{r} \varphi_j^i f_i = \mathbf{\hat{x}} \left( f_1 Y_{l+1 \ell m} + f_2 Y_{l-1 \ell m} + f_3 Y_{l \ell m} \right) \]
\[ = -f_1 c_1 \varphi_4^i + f_2 c_0 \varphi_7^i, \tag{A19} \]
the right-hand side of (A17) can be written as:
\[ g\phi^i \delta_{n4} \left( -f_1 c_1 \varphi_4^i + f_2 c_0 \varphi_7^i \right) + g\phi^j \delta^i_{n} \left( -c_1 \varphi_4^i + c_0 \varphi_7^i \right) f_4. \tag{A20} \]
Therefore, as the vector spherical harmonics are linear independent, (A16) only can be satisfied for \( f_k \) being a solution of
\[ \left[ \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l_n(l_n + 1)}{r^2} \right) \delta_{nk} - \mathcal{V}(\phi) \right] f_k(r) = 0, \tag{A21} \]
with \( \mathcal{V}(\phi) = \) \[ \begin{pmatrix}
  m_2^2(\phi) & 0 & 0 & -c_1 g\phi^i & 0 & 0 \\
  0 & m_2^2(\phi) & 0 & c_0 g\phi^i & 0 & 0 \\
  -c_1 g\phi^i & c_0 g\phi^i & m_2^2(\phi, T) & 0 & 0 & 0 \\
  0 & 0 & 0 & m_2^2(\phi, T) & 0 & 0 \\
  0 & 0 & 0 & 0 & m_2^2(\phi) & 0 \\
  0 & 0 & 0 & 0 & 0 & m_2^2(\phi) \\
\end{pmatrix}. \tag{A22} \]
These considerations motivate the following ansatz for the Green’s function $G(x, x')$:

$$\quad G_{km}(x, x') = \sum_{l,m} g_{lr}(r, r') \varphi_k^r(\hat{x}) \varphi_m^{*r}(\hat{x}') \; ,$$

(A23)

where the radial Green’s functions – their $l$–dependence is not displayed explicitly – satisfy

$$\frac{\partial^2}{\partial r^2} + 2 \frac{\partial}{\partial r} \frac{l}{r} \delta_{nk} - \mathcal{V}_{nk}^r(\phi) \right] g_{km}(r, r') = - \frac{1}{r^2} \delta(r - r') \delta_{nm} \; .$$

(A24)

In (45) one needs to know the following expressions:

\[
\begin{align*}
\text{tr} \left[ e^M g \right] &= \sum_{i=0}^{+\infty} \frac{2l+1}{4\pi} \left\{ \frac{3g^2}{4} \phi (g_{11} + g_{22} + g_{33}) + \frac{3}{2} \left( \frac{g^2}{2} \phi + 2\lambda_T \phi - 3ET \right) g_{44} \\
&+ 3(\lambda_T \phi - ET)g_{55} - \frac{3g^2}{4} \phi g_{66} \right\} , \\
\text{tr} \left[ e^G \right] &= \frac{3}{2} \sum_{l,m} \sum_{i,j} \nabla_n g_{ij} \varphi_m^a \varphi_n^b \nabla_r g_{ij} \varphi_m^a \varphi_n^b \\
&= \frac{3}{2} \sum_{l,m} \sum_{i,j} \nabla_n g_{ij} \varphi_m^a \varphi_n^b \nabla_r g_{ij} \varphi_m^a \varphi_n^b .
\end{align*}
\]

(A25)

(A26)

Obviously the components $g_{33}$ and $g_{66}$ satisfy the same differential equation (see (A24)). Hence, they can be combined. Due to the different sign of their factors of degeneracy they cancel exactly, except for the s–wave. As mentioned above at $l = 0$ the amplitudes $f_2(r)$ and $f_3(r)$ vanish and therefore $g_{33} \equiv 0$ in this particular partial wave. Hence, the $(6 \times 6)$– reduces to a $(5 \times 5)$–system, where the fifth channel only contributes at $l = 0$. The final result is given in (48).


APPENDIX: TABLES

<table>
<thead>
<tr>
<th>$m_H$</th>
<th>$T_0$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
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<td>75.464</td>
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<td>75.433</td>
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Table 1: The values of the roll-over temperature $T_0$ and the critical temperature $T_c$ obtained from the high temperature potential $V_{ht}$ and the approximation of the one-loop resummed effective potential $V_R$. The top-quark mass is chosen as $m_t = 170 GeV$. The Higgs boson mass $m_H$ and the temperatures are given in GeV.

APPENDIX: FIGURE CAPTIONS

Figure 1: The expansion of the one-loop correction $S_{1,R}$ to the effective action. The lines represent the propagators and the dots the vertex factors given by $U_i(\phi, T) - U_i(0, T)$.

Figure 2: Comparison of the logarithm of the nucleation rate $\gamma$ (see (4)), i.e. including the one-loop corrections and prefactors, versus the temperature $T$. $\gamma$ has dimension GeV$^4$ and is calculated in units of $(g^2(T))$. The squares represent the results we obtain using the profile $\phi_{cl}$, which is a solution of (70). They are connected by straight dashed lines. The dots are the results we obtain using the the self-consistent solution of the full one-loop field equation (66). The solid line is the function $F(T)$ as defined in (85), with $a = 3.48 \cdot 10^{-2}$ and $b = 1.350$. The parameter set used here is $m_H = 30 GeV$ and $m_t = 170 GeV$.

Figure 3: The same as in figure 2 for the parameter set $m_H = 40 GeV$ and $m_t = 170 GeV$. Here the solid line is the function $F(T)$ as defined in (85), with $a = 2.38 \cdot 10^{-2}$ and $b = 1.362$.

Figure 4: Comparison of the logarithm of the nucleation rate after mapping of the temperature intervals $[T_0, T_c]$ and $[T_1, T_2]$ to $[0, 1]$. For the parameter set $m_H = 30 GeV$ ($m_H = 40 GeV$) and $m_t = 170 GeV$ the squares (triangles) are the results obtained when using the profile $\phi_{cl}$ and the dots (diamonds) are due the self-consistent profile.

Figure 5: The logarithm of the fraction $\rho_0(t)$ of space that has been converted to the asymmetric phase in dependence of the time $t$. The three lines are due to three different values of the bubble wall velocity $v$. The solid line is obtained for $v = c$, the short-dashed line for $v = c/10$ and the long-dashed one for $v = c/1000$. The parameter set used here is $m_H = 30 GeV$ and $m_t = 170 GeV$.

Figure 6: The total number of bubbles per unit volume $N(t)$ in dependence of the time $t$. The three lines are due to three different values of the bubble wall velocity $v$. The description is the same as in figure 5.

Figure 7: The average bubble radius $\bar{R}(t)$ in dependence of the time $t$. The three lines are due to three different values of the bubble wall velocity $v$. The description is the same as in figure 5.
$$S_{i,R} = +c_i - \frac{c_i}{2} + \frac{c_i}{3} + \ldots$$
Figure 4

\[ \ln \gamma \]

Figure 5

\[ \ln \rho_i(t) \]