Coherence effects on pion spectrum distribution

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Abstract

The effects of two kinds of coherent lengths, the wave packet length of the emitter and the radius of the coherent source, on pion spectrum distribution are studied. It is shown that both coherent lengths can cause abundant pions at low momentum, but the DCC size effects on pion spectrum distribution is more important. So observing abundant pions at low momentum may be taken as a signal of DCC effects.

PACS number(s): 25.75+r, 13.85 Hd, 24.10-i.

I. INTRODUCTION

Early in the next decade two heavy-ion accelerators, the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), will create highly excited regions, similar to heavy-ion in size and with temperatures exceeding 200\textit{MeV}. Among the most interesting speculations regarding ultra-high energy heavy ion collisions there is the idea that regions

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of misaligned vacuum might occur [1–6]. In such misaligned regions, which are analogous to misaligned domains in a ferromagnet, the chiral condensate points in a different direction from that favored in the ground state. If they were produced, misaligned vacuum regions plausibly would behave as a pion laser, relaxing to the ground state by coherent pion emission [7,8].

It is generally assumed that the fluctuation of the ratio of neutral to charged pions may be viewed as a signature of DCC phenomena. If the DCC state is formed, the product of one kind of pions should be larger comparing to other kinds of pions. Furthermore since a pion laser is formed, the mean momentum of the pions emitted from DCC regions should be much smaller. PHOBOS [9] is a compact silicon detector designed to measure particle multiplicity for all charged particles and for photons and it will be used in AGS and RHIC. The ability to measure photons will allow PHOBOS to study their parents $\pi^0$ and the fluctuation of the ratio of $\pi^0$ to $\pi^+\pi^-$. The most interesting thing of PHOBOS for us is that it can measure the low momentum particles, that is it can detect the other signature of DCC phenomena.

Two-pion Bose-Einstein correlation is widely used in high energy heavy-ion collisions to provide the information of the space-time structure, degree of coherence and dynamics of the region where the pions were produced [10–13], which is closely related to the single and double pions inclusive distribution. The coherent pion emission causes pions concentrated at low transverse momenta. This behavior should have explicit effects on the pion single particle spectrum and two-pion interferometry. Besides the large domain size of the disoriented chiral condensate (DCC) regions, there is also another kind of coherent length which corresponds to the wave packet length scale of the emitter. The wave packet length should also affect the pion spectrum distribution. Then the question arises: which one, the DCC size or the wavepacket length, affects more seriously on pion spectrum distribution. To answer this question, in this paper, we first derive the formula of single particle spectrum by taking into account both DCC size and the wave packet length in Section two. As a simple example, the DCC region size and the emitter size effects on the pion spectrum distribution are given.
in Section three. Conclusions are given in Section four.

**II. PION SPECTRUM DISTRIBUTION FOR PARTIALLY COHERENT SOURCE**

It is widely accepted that a state created by a classical pion source is described by [10,11,8,14,15]

$$|\phi> = \exp(i \int d\vec{p} \int d^4 x j(x) \exp(i p \cdot x) c^+(\vec{p})|0>,$$  \hspace{1cm} (1)

where $c^+(\vec{p})$ is the pion creation operator, $|0>$ is the pion vacuum. $j(x)$ is the current of the pion, which can be expressed as

$$j(x) = \int d^4 x' d^4 p j(x', p) \nu(x') \exp(-ip \cdot (x - x')).$$ \hspace{1cm} (2)

Here $j(x', p)$ is the probability amplitude of finding a pion with momentum $p$, emitted by the emitter at $x'$. $\nu(x')$ is a random phase factor. All emitters are uncorrelated in coordinate space when assuming:

$$<\nu^*(x')\nu(x)> = \delta^4(x' - x).$$ \hspace{1cm} (3)

This is in ideal cases. In a more realistic case, each chaotic emitter has a small coherent wave package length scale and the above equation can be replaced by

$$<\nu^*(x')\nu(x)> = \frac{1}{\delta^4} \exp\{-(\frac{x_1-x'_1}{\delta^2})^2 - (\frac{x_2-x'_2}{\delta^2})^2 - (\frac{x_3-x'_3}{\delta^2})^2 - (\frac{x_0-x'_0}{\delta^2})^2\}. \hspace{1cm} (4)$$

Here $\delta$ is a parameter which determines the coherent length (time) scale of the emitter. For simplicity, the same coherent scale is taken for both spacial and time at the moment. The above formula means that two-emitters within the range of $\delta$ can be seen as one emitter, while two-emitters out of this range are incoherent. For simplicity we also assume that

$$<\nu^*(x)> = <\nu(x)> = 0,$$ \hspace{1cm} (5)

which means that for each emitter the phases are randomly distributed in the range of 0 to $2\pi$. 

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The coherent state can be expanded in Fock-Space as

$$|\phi> = \sum_{n=0}^{\infty} \frac{(i \int j(x)e^{ip \cdot x}c^+(p)d\vec{p}d^4x)^n}{n!}|0> = \sum_{n=0}^{\infty} |n>,$$

with

$$|n> = \frac{(i \int j(x)e^{ip \cdot x}c^+(p)d\vec{p}dx)^n}{n!}|0>.$$  

Here $|n>$ is the n-pion state, using the relationship

$$[c^+(\vec{p_1}), c(\vec{p_2})] = \delta(\vec{p_1} - \vec{p_2})$$

we have

$$c(\vec{p})|n> = i \int d^4 x j(x)exp(ip \cdot x)|n - 1>,$$

then

$$I(Q, K) = <1|c^+(\vec{p_1})c(\vec{p_2})|1> = \int d^4 x_1 d^4 x_2 j^*(x_1)j(x_2)exp(-i(p_1 \cdot x_1 - p_2 \cdot x_2))$$

$$= \int d^4 x_1 d^4 x_2 j^*(x_1)j(x_2)exp(-i(K/2 + Q) \cdot x_1 - i(K/2 - Q) \cdot x_2)$$

$$= \int d^4 x_1 d^4 x_2 j^*(x_1)j(x_2)exp(-iK \cdot (x_1 - x_2)/2 - iQ \cdot (x_1 + x_2))$$

$$= \int d^4 y d^4 Y j^*(Y + y/2)j(Y - y/2)exp(-iK \cdot y/2 - i2Q \cdot Y)$$

$$= \int d^4 Y g_w(Y, k)exp(-i2Q \cdot Y).$$

Here $Y = \frac{x_1 + x_2}{2}$, $y = x_1 - x_2$ are four dimensional coordinates, while $Q = \frac{p_1 - p_2}{2}$ and $K = 2k = p_1 + p_2$ are the corresponding four dimensional momenta. The above transformation, referred as Wigner transformation, can be found in the original paper of E. Wigner [16]. $g_w(Y, K)$ is the Wigner function, which can be explained as the probability of finding a pion at $Y$ with momentum $k = K/2$ [16] and is defined as

$$g_w(Y, k) = \int d^4 y j^*(Y + y/2)j(Y - y/2)exp(-iK \cdot y/2).$$

Inserting eq.(2) into the above equation we have
\[ g_w(Y, k) = \int d^4 y e^{i k \cdot y} \]
\[ \int d^4 x' j^*(x', p_1) d\bar{p}_1 e^{i p_1 \cdot (Y + y/2 - x')} \nu^*(x') \]
\[ \int d^4 x'' j(x'', p_2) d\bar{p}_2 e^{-i p_2 \cdot (Y - y/2 - x'')} \nu(x''), \]

(12)

then the single pion inclusive distribution \( P_1^{cha}(\bar{p}) \) can be expressed as (eq.10):

\[ P_1^{cha}(\bar{k}) = < 1| c^+(\bar{k}) c(\bar{k}) | 1 > = \int g_w(Y, k) d^4 Y \]
\[ = \int d^4 x' d^4 x'' d^4 p_1 d^4 p_2 j^*(x', p_1) j(x'', p_2) \]
\[ \nu^*(x') \nu(x'') \delta^4(k - \frac{p_1 + p_2}{2}) \delta^4(p_1 - p_2) e^{i p_1 \cdot (Y - x')} e^{-i p_2 \cdot (Y - x'')} \]
\[ = \int d^4 x' d^4 x'' j^*(x', k) j(x'', k) \nu^*(x') \nu(x'') e^{-i k \cdot (x' - x'')} \]

(13)

with \( k_0 \) taken to be \( k_0 = \sqrt{k^2 + m^2_\pi} \). In the above equation, we have taken into account the wave packet length \( \delta \) of the emitter which form a chaotic source.

For a system with one finite size coherent source, e.g., a finite DCC region and many other totally chaotic emitters which form a chaotic source, the state is described by [8]

\[ | \phi >_{part} = \exp(i \int d\bar{p} \int d^4 x (j(x) + j_c(x)) \exp(i p \cdot x) c^+(\bar{p}) | 0 > \]

(14)

where \( c^+(p) \) is the pion creation operator; \( j_c(x) \) is the current of pions produced by the coherent source, which can be expressed as

\[ j_c(x) = \int d^4 x' d^4 p j_c(x', p) \exp\{-i p \cdot (x - x')\}; \]

(15)

\( j(x) \) is the current of pions produced by totally chaotic source, which can be expressed as eq.(2). The difference between \( j(x) \) and \( j_c(x) \) is that: For chaotic source, each emitter have different phase (different \( \nu(x) \) in eq.(2)) while for \( j_c(x) \) each emitter have the same phase.

The state \( | \phi >_{part} \) can be expanded as

\[ | \phi > = \sum_{n=0}^{\infty} \frac{(i \int (j(x) + j_c(x)) e^{i p \cdot x} c^+(p) d\bar{p} dx)^n}{n!} | 0 > = \sum_{n=0}^{\infty} | n >_{part} \]

(16)

with

\[ | n >_{part} = \frac{(i \int (j_c(x) + j(x)) e^{i p \cdot x} c^+(p) d\bar{p} dx)^n}{n!} | 0 > . \]

(17)
Here $|n\rangle_{\text{part}}$ is the n-pion state. Then the pion’s spectrum distribution is

$$P^\text{part}_1(\vec{p}) = \langle n | c^+(\vec{p}) c(\vec{p}) | 1 \rangle_{\text{part}}$$

$$= \left( \int d^4x (j(x) + j_c(x)) e^{i p \cdot x} \right)^* \left( \int d^4x (j(x) + j_c(x)) e^{i p \cdot x} \right) \quad (18)$$

$$= \int d^4x_1 d^4x_2 (j^*(x_1) j(x_2) + j^*(x_1) j_c(x_2) + j^*_c(x_1) j(x_2) + j^*_c(x_1) j_c(x_2)) e^{-ip(\vec{x}_1 - \vec{x}_2)}$$

Taking the phase average and using the relationship of eq.(5) we have

$$< j^*(x) j_c(y) >= < j_c(x) j^*(y) >= 0 \quad (19)$$

Then the above equation can be re-expressed as

$$P_1(\vec{p}) = \int d^4x_1 d^4x_2 j^*(x_1) j(x_2) e^{-i p(\vec{x}_1 - \vec{x}_2)}$$

$$+ \int d^4x_1 d^4x_2 j^*_c(x_1) j_c(x_2) e^{-i p(\vec{x}_1 - \vec{x}_2)} \quad (20)$$

$$= \int g_w(x, \vec{p}) d^4x + |j_c(\vec{p})|^2$$

Here $j_c(\vec{p})$ can be expressed as

$$j_c(\vec{p}) = \int j_c(x) \exp(ip \cdot x) d^4x. \quad (21)$$

The above formula (eq.(20)) shows that the total pion spectrum distribution is consist of two-parts, one is spectrum distribution of the chaotic source, the other is the spectrum distribution of the coherent source. In the above derivation we have taken into account the wave packet length and the coherent pion source $j_c$, therefore, in our formulation it is possible to examine both the wave packet length of each chaotic emitter and the DCC radius effects on the pion single particle distribution when the pions emitted from the DCC region are assumed to be coherent.

**III. COHERENT LENGTH EFFECTS ON PION SPECTRUM**

In this section, we will give an example to investigate the wave packet length and the coherence source radius effects on single pion distributions. We assume that the chaotic emitter amplitude distribution is
\[ j(x, k) = \exp\left(-\frac{x_1^2 - x_2^2 - x_3^2}{2R_0^2}\right) \delta(x_0) \exp\left(-\frac{k_1^2 + k_2^2 + k_3^2}{2\Delta^2}\right). \] (22)

Where \( R_0 \) and \( \Delta \) are parameters which represent the radius of the chaotic source size and the momentum range of pions respectively. Here \( x = (x_0, x_1, x_2, x_3) \) and \( k = (k_0, k_1, k_2, k_3) \) is pion’s coordinate and momentum respectively. Bringing eq.(4) and eq.(22) into eq.(13), we can easily get the pion single particle spectrum distribution

\[ P_{cha}^1(\vec{p}) = \left(\frac{1}{\Delta^2} + \frac{R_0^2 \delta^2}{\pi^2 + 4R_0^2}\right) \frac{1}{\pi} \exp\left\{-\frac{\vec{p}^2}{\Delta^2} \cdot \left(1 + \frac{R_0^2 \delta^2}{\pi^2 + 4R_0^2}\right)\right\}. \] (23)

From the above expressions, we can see that the wave packet length of each chaotic emitter has great influence on the pion single particle inclusive distribution. The single particle momentum distribution is shown in fig.1. The input value of \( R_0 \) and \( \Delta \) is 5\,fm and 0.3\,GeV, respectively. The solid, dashed and dot-dashed lines correspond to \( \delta = 0\,fm, 0.5\,fm \) and \( 1\,fm \), respectively. It is clearly shown that as the wave packet length \( \delta \) increases, the mean momentum of pions gets smaller, which means that the wave packet length of the chaotic emitter can cause abundant pions at low momentum.

Now we consider the finite size coherence source effects on the pion single particle inclusive distribution. We assume that the emitting amplitude of coherent pions is

\[ j_c(x, k) = \exp\left(-\frac{x_1^2 - x_2^2 - x_3^2}{2R_c^2}\right) \delta(x_0) \exp\left(-\frac{k_1^2 + k_2^2 + k_3^2}{2\Delta_c^2}\right). \] (24)

Here \( R_c \) and \( \Delta_c \) are parameters which represent the radius of the coherent source, e.g. the DCC region, and the mean momentum of coherent pions. \( x = (x_0, x_1, x_2, x_3) \) and \( k = (k_0, k_1, k_2, k_3) \) is pion’s coordinate and momentum respectively. The normalized pion single particle distribution can be expressed as

\[ P_{nor}^{part}(\vec{p}) = A \cdot P_{nor}^{cha}(\vec{p}) + (1 - A)P_{nor}^c(\vec{p}), \] (25)

where \( A \) is a parameter which determines the incoherent degree of the source and is defined by

\[ A = \frac{\int d\vec{p} P_{nor}^{cha}(\vec{p})}{\int d\vec{p} (P_{nor}^{cha}(\vec{p}) + P_{nor}^c(\vec{p}))}. \] (26)
For $A = 1$ the source is totally chaotic, for $A = 0$ the source is totally coherent, otherwise the source is partially coherent. Here $P_{\text{nor}}^{\text{cha}}(\vec{p})$ can be expressed as

$$P_{\text{nor}}^{\text{cha}}(\vec{p}) = \left(\frac{1}{\Delta^2} + \frac{R_0^2 \delta^2}{\delta^2 + 4 R_0^2}\right)^{3/2} \exp\left\{ -\vec{p}^2 \left( \frac{1}{\Delta^2} + \frac{R_0^2 \delta^2}{\delta^2 + 4 R_0^2} \right) \right\} \tag{27}$$

and $P_{\text{nor}}^{c}(\vec{p})$ can be expressed as

$$P_{\text{nor}}^{c}(\vec{p}) = \left(\frac{1}{\Delta_c^2} + \frac{R_c^2}{\pi}\right)^{3/2} \exp\left\{ -\vec{p}^2 \left( \frac{1}{\Delta_c^2} + R_c^2 \right) \right\} \tag{28}$$

Then the single particle inclusive distribution for partially coherent source is shown in fig.2, where the input values of $A$, $R_0$, $\Delta$, $\Delta_c$ and $\delta$ is 0.5, 5 fm, 0.3 GeV, 0.15 GeV and 0.3 fm, respectively. The solid, dashed and dot-dashed lines correspond to $R_c = 1$ fm, 2 fm and 3 fm, respectively. It is clear that as $R_c$ becomes larger, that is DCC region becomes larger, the pion’s mean momentum becomes smaller, this condition is consistent with the nature of the coherent property of the source. It can be seen from fig.2 that DCC effects on pion spectrum distribution is more important than emitter size effects. So observing abundant pions at low momentum can be taken as a signature of DCC effect.

**IV. CONCLUSIONS**

It has been suggested that a large DCC region may be formed in relativistic heavy-ion collisions. If the DCC region is formed, a large number of lower momentum pions should be produced, which is one of the signature of DCC phenomena. There is also another kind of coherent length which corresponds to the size of the wave packet length of each chaotic emitter and which also affects the pion momentum distribution. Therefore it is very interesting to analyze the effects of the two kind of lengths, namely the DCC size and wave packet length, on pion single particle inclusive distributions and to find out whose effect is more imporatnt.

In this paper, as a simple example, we have derived the formula of the pion spectrum distribution by taking into account both the wavepacket length and DCC size and analyzed
the effect of the two coherent lengths on pion inclusive distributions. It has been shown that both coherent lengths can cause the abundance of pions at low momentum. Among the two, the DCC size effects on the pion spectrum distribution is more important. Therefore observing abundant pions at low momentum may provide a signal of the DCC effects. Such a signal can be detected by PHOBOS at RHIC.

**ACKNOWLEDGEMENT**

The authors would like to express their gratitude to the referees for their helpful suggestion and comments. One of the authors (Q.H.Z.) would like to express his thanks to Dr. Pang Yang for helpful discussions. This study was partially supported by the National Natural Science Foundation of China, Post-doctoral Science Foundation of China and the Alexander von Humboldt Foundation in Germany.
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Fig.1 The pion inclusive distribution. The input values of $R_0$ and $\Delta$ are 5fm and 0.3GeV. The solid, dashed and dot-dashed lines corresponds to $\delta = 0 fm$, 0.5fm and 1fm, respectively.

Fig.2 The pion inclusive distribution for partial coherent source. The input values of $R_0$, $\Delta$, $\Delta_c$, $\delta$ and $A$ are 5fm, 0.3GeV, 0.15GeV, 0.3fm and 0.5. The solid, dashed and dot-dashed lines corresponds to $R_c = 1fm$, 2fm and 3fm, respectively.
Fig. 1   Q.H. Zhang et al
Fig. 2  Q.H. Zhang et al.

$p (\text{GeV/c})$

$P(p)$

$R_c = 1 \text{ fm}$

$R_c = 2 \text{ fm}$

$R_c = 3 \text{ fm}$