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Numerical Study for the Two-Beam Instability due to Ions in Electron-Storage Rings

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Abstract

We discuss the two-beam instability which is caused by interactions between ions and the electron beam in electron storage rings. Motion of beam and ions trapped in the beam potential was studied by using a simulation method based on a rigid Gaussian beam model. We consider two types of the two beam instability depending on the trapping time. One is the so-called ion trapping instability, in which ions are accumulated every revolution. A growth of a coupled bunch mode, which was obtained by simulations, was consistent with experiments in KEK-Photon factory. The other is the fast ion instability, in which ions are trapped during only a passage of a series of bunches (bunch train). Simulations gave a growth of a couple bunch mode for KEK-B factory.

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1 Introduction

The electron beam in a storage ring creates ions by ionizing the residual gas in a beam chamber. In a storage ring filling electron bunches with a narrow spacing, ions with a mass larger than a critical value are trapped by the attractive force of the electron beam. The trapped ions, which oscillate in the beam potential, cause the two-beam instability due to a coupling to the coherent motion of the beam. We have observed it as a coupled bunch instability.

We discuss here the two-beam instability dynamically based on a computer simulation. The simulation method, which was based on the weak strong model, was discussed concerning beam-beam interactions in Ref.[1]. This method is very useful for studying the two-beam instability due to beam-ion interactions. In this method, a weak beam, which is ions here, is expressed by macro-particles, and the barycenter motion of a strong beam is taken into consideration.

In considering the simulation, the beam-beam and beam-ion interactions are very similar. The differences between them can be summarized as follows. In the general case of beam-beam interactions, a bunch interacts with only another bunch, and both bunches move at the speed of light. However the ions move non-relativistically and interact with all of the bunches. The number of ions increases in every passing bunch. Though a space-charge force between the ions exists because of a non-relativistic effect, since the number of ion which causes the two-beam instability is generally very small as shown later, we can neglect it.

We now consider the transverse motion of electrons and ions. The magnetic field is not considered. We consider CO as being a species of ion because of dominance of the partial pressure of the residual gas and the ionization cross section. The equations of motion for electrons and ions are expressed as

$$\frac{d^2 \mathbf{x}_{e,a}}{ds^2} + K(s) \mathbf{x}_{e,a} = \frac{2r_e}{\gamma} \sum_{j=1}^{N_i} \mathbf{F}(\mathbf{x}_{e,a} - \mathbf{x}_{i,j}), \quad (1)$$

$$\frac{d^2 \mathbf{x}_{i,j}}{dt^2} = \frac{2r_e c^2}{M_i/m_e} \sum_{a=1}^{N_e} \mathbf{F}(\mathbf{x}_{i,j} - \mathbf{x}_{e,a}), \quad (2)$$

where suffices i and e denote the ion and electron, respectively. N_i and N_e are the number of each and γ is the Lorentz factor of the beam. $\mathbf{F}(\mathbf{x})$ is the Coulomb force in

two dimensional space,

$$\mathbf{F}(\mathbf{x}) = -\frac{\mathbf{x}}{|\mathbf{x}|^2} \delta(s). \quad (3)$$

These consist of differential $N_e + N_i$ equations, where that of each electron couples to the motion of all ions, and that of each ion couples to the motion of all electrons.

The two-beam instability is a collective effect obtained by solving Eqs.(1) and (2). This was originally discussed concerning the plasma physics. In accelerator physics, it was considered concerning an electron-trapping instability in a proton ring[2, 3] for the first time. Concerning ion-beam interactions, the two beam instability has been studied in most electron storage rings operated by multi-bunch mode. The instability has been discussed based on linear theory[2]. Linear theory is reviewed and commented on with an example of KEK-Photon Factory (PF) and KEK-B Factory (KEKB) in Sec.2. In Sec.3, a simulation method based on the weak-strong model is explained. This method permits one to study the two-beam instability beyond linear theory and from a dynamic aspect. The results of a simulation for PF (the ion trapping instability) and KEBB (the fast ion instability) are discussed Sec.4 and 5.

2 Linear theory

The linear theory presented by E.Keil et.al. is reviewed here. The following approximations are assumed in the theory:

- The transverse motions of beam and ions can be expressed by their barycenters.
- The coasting-beam approximation is used; that is, the beam has no longitudinal structure, and is specified by its transverse barycenter at longitudinal position z from a reference bunch and time $s = ct + z$.
- The beam size can be treated as a constant along the ring; that is, the beta function is assumed to be a constant.
- The beam-ion force is assumed to be linear for the distance between each barycenter. The strength of the linear force is determined by assuming that both of the beam and ions distribute according to a Gaussian with the same size in the transverse plane.

Now, the equations of motion for the beam and ion are expressed as follows:

$$\frac{D^2 \bar{\mathbf{x}}_e}{Ds^2} + (2\pi\nu_\beta)^2 \bar{\mathbf{x}}_e = \frac{2n_i r_e}{\gamma} \mathbf{F}_L(\bar{\mathbf{x}}_e - \bar{\mathbf{x}}_i), \quad (4)$$

$$\frac{d^2 \bar{\mathbf{x}}_i}{dt^2} = \frac{2n_e r_e c^2}{M_i/m_e} \mathbf{F}_L(\bar{\mathbf{x}}_i - \bar{\mathbf{x}}_e), \quad (5)$$

where

$$F_{L,y}(\mathbf{x}) + iF_{L,x}(\mathbf{x}) = -\frac{1}{\sigma_x + \sigma_y} \left(\frac{y}{\sigma_y} + i \frac{x}{\sigma_x} \right). \quad (6)$$

Here, n_e and n_i are the number of electrons and ions in a unit length. The differentiation for s is expressed by

$$\frac{D}{Ds} = \frac{\partial}{\partial s} + \frac{\partial}{\partial z}. \quad (7)$$

Here, we try to obtain a solution with a following form:

$$\mathbf{x}(s, z) = \boldsymbol{\xi} \exp i \left(\frac{2\pi m z}{L} - \frac{\Omega s}{c} \right), \quad (8)$$

where m is an integer. We obtain a forth-order equation,

$$(\Omega^2 - \omega_i^2) [(\Omega - m\omega_0)^2 - \omega_\beta^2 - \omega_e^2] = \omega_e^2 \omega_i^2. \quad (9)$$

Here ω_i and ω_e are the frequency of the ion and betatron frequency shift ($= 2\omega_\beta \delta\omega_\beta$) of the beam, respectively, and are expressed as

$$\omega_i = \frac{2n_e r_e c^2}{M_i/m_e} \frac{1}{\sigma_y(\sigma_x + \sigma_y)}, \quad \omega_e = \frac{2n_i r_e c^2}{\gamma} \frac{1}{\sigma_y(\sigma_x + \sigma_y)}. \quad (10)$$

The two-beam instability occurs when the frequency (Ω) has an imaginary part. We investigate the case where the frequencies of the ions (ω_i) and beam ($\omega_\beta - m\omega_0$) in the experimental frame are close each other. Because the two-beam instability is interpreted as being a coupling resonance of the two oscillators, which originate due to the collective motions of the beam and ions. We obtain the threshold and growth rate as

$$\omega_{e,th} = \frac{(m\omega_0 - \omega_i)^2 - \omega_\beta^2}{2\sqrt{\omega_i(m\omega_0 - \omega_i)}} \quad \text{Im} \left(\frac{\Omega}{\omega_0} \right) = \frac{\omega_e}{2\omega_0} \sqrt{\frac{\omega_i}{m\omega_0 - \omega_i}}. \quad (11)$$

In linear accelerators and the bunch train of circular accelerators, since m becomes continuous, a threshold does not exist in the ion-beam system. It is determined by a comparison with other damping effects.

It is well-known that these equation shows a very low threshold and a very high growth rate in recently developed accelerators. We discuss them in the cases of KEK-PF and KEKB. The parameters are given in Table 1.

Table 1: Parameters of PF and KEKB

	PF	KEKB
Circumference(m)	186	3016
Revolution frequency($\omega_0/2\pi$)	1.6×10^6	10^5
Energy(GeV)	2.5	8
Current (A)	0.4	1.1
Vertical tune (ν_y)	3.305	43.08
Harmonic number	312	5120
Emittances ($\varepsilon_x(nm)/\varepsilon_y(nm)$)	130/ 1.5	18 / 0.36
Transverse radiation damping rate	8×10^{-5}	1.2×10^{-4}

Table 2: Results obtained by linear theory

	I(A)	ω_i/ω_0	η_{th}	Im(Ω/ω_0)
PF	0.04	0.740	0.0015	0.022
	0.4	2.339	0.003	0.169
KEKB	1.1	139.535	8.1×10^{-7}	0.192

Table 2 gives examples of the results obtained by linear theory. These high growth rates surprises us. However, since we do not consider the ion increase due to production, we should not immediately believe the growth rate.

When the threshold is extremely low, as shown in the KEKB case, we should consider the two-beam instability due to transient ions, the so-called fast ion instability. If we use a production cross-section of $\sigma_{CO} = 2 \times 10^{-22} m^2$ and a partial pressure of $P_{CO} = 10^{-9} Torr$, a bunch including $N_e = 1.4 \times 10^{10}$ ($n_e = 2.1 \times 10^{10} m^{-1}$) electrons produces about 100 ions in a meter. If 1000 bunches pass, the neutralization factor becomes $\eta = 5 \times 10^{-6}$, which already exceeds the threshold. This suggests the possibility of the fast-ion instability.

Eq.(9) has a singular property at $\omega_i = \pm(m\omega_0 - \omega_\beta)$. In practical cases, the ion frequency (ω_i) has a spread due to the nonlinearity of the beam-ion force and the beam size from the variation of the $\beta(s)$ function. The linear theory has been extended to include the frequency spread by Ref.[4, 5, 6]. A linear theory including an ion increase for linear accelerators is given in Ref.[5].

3 Beyond linear theory

We now restart our discussion from Eqs.(1) and (2). The differential equations can be solved with macro-particles for both electrons and ions. It is called a strong-strong simulation similar to the phrase of beam-beam interactions. Strong-strong simulations have been recently performed for the fast-ion instability[5, 7]. In the strong-strong simulations, the motion of many macro electrons and ions are tracked by using many computer resources. Now, the weak-strong model is being proposed to study the two-beam instability. In this model, the transverse distribution of the beam is assumed to be a rigid Gaussian, where the deviation is determined by the emittance and beta function; that is, deformations of the bunch due to ions are neglected. The effects of the bunch length and the synchrotron motion are not considered. The beam is not characterized by each electron, but by a series of bunches with a rigid Gaussian distribution. By averaging Eq.(1), we take into account only the first-order moment. Ions are treated exactly, except for neglecting their longitudinal motion. The equation of motion for a bunch and ions are expressed as

$$\frac{d^2 \bar{\mathbf{x}}_e}{ds^2} + K(s) \bar{\mathbf{x}}_e = \frac{2r_e}{\gamma} \sum_{j=1}^{N_i} \mathbf{F}_G(\bar{\mathbf{x}}_e - \mathbf{x}_{i,j}; \boldsymbol{\sigma}(s)), \quad (12)$$

$$\frac{d^2 \mathbf{x}_{i,j}}{dt^2} = \frac{2N_e r_e c^2}{M_i/m_e} \mathbf{F}_G(\mathbf{x}_{i,j} - \bar{\mathbf{x}}_e; \boldsymbol{\sigma}(s)), \quad (13)$$

where $\mathbf{F}_G$ is expressed by Bassetti-Erskine formula[8],

$$F_{G,y}(\mathbf{x}) + iF_{G,x}(\mathbf{x}) = \sqrt{\frac{\pi}{2(\sigma_x^2 - \sigma_y^2)}} \quad (14)$$

$$\times \left[w \left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - \exp \left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) w \left(\frac{\frac{\sigma_y}{\sigma_x} x + i \frac{\sigma_x}{\sigma_y} y}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right] \delta(s). \quad (15)$$

Here N_e is the number of particles in a bunch. These equations of motion, Eq.(12) and (13), can be solved by an approximation in which ions are represented by macro particles, and the barycenter motion of every bunch is obtained. If a coherent mode, in which bunches have a correlation with each other, grows, this means that a coupled-bunch instability is caused. In this case, it is guessed that ions also move coherently. It can be made certain to calculate the moments of macro-ions.

The incoherent feature of ions is considered by a simulation, while that of a beam, such as emittance growth, can not be treated. If the tune spread due to the ion distribution

is large, the coherent motion may be smeared and enlarge the beam size; i.e. Landau damping occurs. The smear is expressed as

$$\delta\omega_\beta = \frac{\omega_c^2}{2\omega_\beta} = \frac{n_i r_e c^2}{\gamma \omega_\beta} \frac{1}{\sigma_y(\sigma_x + \sigma_y)}. \quad (16)$$

The problem is which is the larger, growth rate of the two-beam instability or the damping rate due to the smear. In our examples, linear theory shows that the growth rate is larger. Using PF, I=40mA and $\eta = 0.01$, as an example, the growth rate is 0.045, while the damping rate (tune spread) is 0.009. If the Landau damping rate becomes important, this simulation will overestimate the growth of the instability. In the strong-strong simulation[5], the smear effect is included, though the calculation requires much CPU time. The overestimate of the growth is a trade off the shorter CPU time in this simulation method. The simulations should be performed while paying attention to the Landau damping rate. We discuss this problem again concerning the applications.

We also comment on the smear of the ion motion. The smear of ions, which is just ω_i , is much faster than that of the beam in the usual case. In terms of a beam-beam interaction, it says that the tune shift of ions is larger than that of the beam in the ion-beam interactions. The smear of the beam may be reduced by the faster smear of ions. This weak-strong model completely involves the smear of ions.

The ion trapping and the fast ion instability are discussed by using this simulation in Sec.4 and in Sec.5, respectively. Both instabilities are caused by the same mechanism. The difference is how long trapped in the beam potential. The former is caused by ions which are accumulated turn by turn, and the later is by which by bunches in a train.

4 Application to the ion trapping

We discuss the two-beam instability due to the ion-trapping using PF as an example. To survey features of the instability, some (fixed) amount of macro ions were put along an unperturbed beam orbit, and motion of the ions and bunches were investigated. The ring was represented by four ionization points with different values of β functions. It was checked that the results did not depend on the number of ionization points. A thousand macro-ions were put at each point. All of the bunches were set with zero displacement. We calculated the kick which ions felt from the beam using Eq.(13). The force which the beam feels was obtained by summing the force of the ions. The beam felt a kick due

Figure 1: Coupled-bunch pattern due to ion-trapping in KEK-PF. The neutralization factor used to be 1%.

to the macro-ions at each ionization point, and was transferred to the next point. The statistical noise of the ions is a seed of the instability in the simulations. We discuss the vertical motion of ions and beam, because simulations show that the coherent mode of the horizontal motion is much less than the vertical one.

The time evolution of the amplitude of the bunches which pass through a ionization point in a ring is shown in Fig.1. The amplitude was obtained at a position with $\beta_y = 28.3m$: $\sigma_y = 0.19mm$. We found that the phase advance of the coupled-bunch motion was about $m - \nu_y \sim 4 - 3.3 = 0.7$ in each revolution, which means an excitation of the lowest mode. The barycenter and size of ions of the ionization point are also plotted in Fig.1. The barycenter of the ions also oscillate with the same frequency as the bunches. The coupled-bunch mode means that the two-beam instability is caused by a resonance between the frequencies of the beam coherent motion ($(1 - \Delta\nu_y)\omega_0$) and the ions (ω_i). It is interesting that the ion size also becomes larger along with the growth of the dipole amplitude. The coherent motion of the ions is accompanied by their blow-up coming from the rapid smear due to the beam force.

The same calculations were then performed for various values of the neutralization factor. Fig.2 shows the time evolution of the maximum amplitudes of the bunches and ion sizes for each revolution, respectively. The figures shows that both of the ion size and

the beam amplitude grow at $\eta \geq 0.005$. We may be able to say that the threshold for the neutralization factor is guessed to be ≤ 0.005 .

In an actual accelerator, since ions are continuously supplied, the time evolution of the instability depends on the creation rate of ions, which is related to the vacuum pressure. We took into consideration the ion increase in the simulation: macro ions are produced by bunches in proportion to the vacuum pressure, and the bunches feel a force due to the ions, which increase in every bunch passage.

As previously done, the ring was represented by 4 ionization points. In every passing of the bunches through an ionization point, new macro-ions were generated at the transverse position of the bunch with a Gaussian distribution. Though 1 or 10 macro-ions were generated for every revolution time, results were not affected by the number. Fig.3 shows the growth of the dipole amplitude and the ion size for $P_{CO} = 10^{-7}$, 4×10^{-8} , 10^{-8} , 4×10^{-9} and $10^{-9}Torr$. In these parameters, the neutralization factor increase by 10^{-4} for $P_{CO} = 10^{-7}Torr$ every revolution time. The maximum amplitude exceeds $\sigma_y \equiv 0.19mm$ at a neutralization factor of ~ 0.01 . Fig.3(b) shows that the ion size increases rapidly in spite of the small beam amplitude ($\ll \sigma_y$). Though the figure shows a saturation at around a few cm, it results from the boundary (5cm) of the vacuum chamber. Fig.4 shows the number of accumulated ions in the vacuum chamber. The ions are thrown away in spite of the relatively small amplitude of the beam ($\sim \sigma_y$). This explains the pulsation phenomena[9]. Number of trapped ions, which measured with bremsstrahlung, correlated to the pulsation of the instability in the experiment at PF.

Fig.5 shows the growth rate for various stored currents and vacuum pressures. The same calculations were performed for a stored current (I) of 400mA; the results are plotted in the figure. PF has been operated with $P_{CO} = 10^{-9}Torr$ or less. The figure shows that the growth rate is ~ 1200 and ~ 200 turns for stored currents of 40mA and 400mA, respectively, at the pressure. The two-beam instability due to ion-trapping has been observed in PF[9]; its threshold was about 30-50mA[10]. The instability has been controlled by exciting octapole magnets, which caused a Landau damping of several hundreds to ~ 1000 turn near to the threshold. These results seem to be consistent with the experiments.

We now consider the smear of the beam to be due to the distribution of ions. This simulation did not include the smear directory because we used the rigid bunch model. However, we obtain the smear from the ion distribution by Eq.(16), where $\sigma_{x,y}$ was calculated by averaging the macro ions. Fig.6 shows the tune spread which leads the smear.

Figure 2: Maximum vertical amplitude of the bunch coherent motion and vertical size of the ions. The amplitude and size are plotted in every revolution for various neutralization factors as written in the figure.

Figure 3: Maximum amplitude of the bunches and size of ions. The amplitude and size are plotted in every revolution for various vacuum pressure as written in the figure.

Figure 4: Number of ions in a vacuum chamber. The vertical axis is normalized by the number of ions which are created in one revolution.

Figure 5: Relation between growth rate and vacuum pressure.

Figure 6: Spread of the vertical tune due to the trapped-ion distribution.

Since the smear is proportional to the square of the beam amplitude, the coherent motion is not smeared at small amplitude. On the other hand, for a larger amplitude than σ_y , since the ion distribution becomes broad, the smear become weak. In this regime, ions are diffused by the beam, and are eventually thrown out of the chamber as shown in Figs.3 and 4.

5 Application to the fast ion instability

In low-emittance high-intensity storage rings, the growth rate of the two-beam instability is very high. It is possible to cause an instability by ions which are produced and trapped in only one revolution, as mentioned in Sec.2. It is called the first ion instability[5]. We can simulate it in the same way.

We consider bunch trains in a ring. By putting a sufficient gap between the trains, the ions which are trapped during a passing bunch train are cleared after its passage. Thus, the first bunch of a train does not face ions, while following bunches feel ions produced by the previous bunches in the train. The ions and bunches of the tail of the train cause the two-beam instability.

The simulation results of KEKB are presented here. The train length is assumed to be 500. Four ionization points were chosen in the ring, as was done in the case of trapping.

Figure 7: Coupled-bunch pattern due to the fast ion instability. The vacuum pressure is assumed to be 1 nTorr. Bunch patterns at the 200, 400, 600, 800, and 1000-th revolution are drawn.

Macro-ions were created with a Gaussian distribution at the transverse position of the passing bunch at an ionization point. The number of macro ions was 10 for passing a bunch at a point. Results were not affected for the number of half. Fig.7 shows the bunch pattern which is observed at the position of the ring. The figure shows the coherent-mode oscillation, whose frequency is equal to ion's one, is induced.

Fig.8 shows the time evolution of the growth of the dipole amplitude of the bunches, where the amplitude is half the Courant-Snyder invariant ($J_y \equiv (\gamma_y y^2 + 2\alpha_y y y' + \beta_y y'^2)/2$). The growth rates were 10 and 100 turns for vacuum pressures of 10^{-8} and 10^{-9} Torr, respectively, at a vertical amplitude of $0.5\sigma_y$. The growth becomes slow at an amplitude of about σ_y . The growth reduction seems to result from ion size which is about $1/\sqrt{2}$ of beam's one. These were consistent with those obtained by the strong-strong simulation[7]. We have considered a feedback system to cure the instability in KEKB. The damping rate of the feedback has been designed to be 100 turns. Length of the bunch train for which the feedback can cure was 500 for $P_{CO} = 10^{-9}$ Torr and was 100 for $P_{CO} = 10^{-8}$ Torr.

We investigated an effect of residual ions which were not cleared by a gap between bunch trains. If 100 empty bucket (200ns) was taken, the effect from the residual ions could be neglected. The luminosity reductions by the clearing gap are 20% and 100% for $P_{CO} = 10^{-9}$ and 10^{-8} Torr, respectively. Good vacuum is important to keeping the design

luminosity.

Fig.9 shows the tune spread which leads the smear due to the ion distribution. The smear does not work for a small amplitude of less than σ_y . When the amplitude reaches $\sim \sigma_y$, the coherent motion may be smeared, since the growth is reduced, as shown in Fig.8. This leads to a pulsation of the amplitude growth.

In a worse vacuum, for example $P_{CO} = 10^{-7}$ Torr, the coherent amplitude of the beam was not saturated by $\sim \sigma_y$. Since the ion distribution blew-up at the tail of the train, the beam amplitude grew over σ_y in the same way as in the trapping case. This was also clarified by a strong-strong simulation[11].

We considered the case of mixture of more species of ion briefly. Each species of ion has a proper frequency (ω_i) given by Eq.(10) in the beam potential. In a scope of linear theory, the superposition rule will be satisfied. Simulations including multi-species of ions showed coupled-bunch patterns so as to suggest the superposition.

6 Conclusions

Simulations based on a weak strong model were performed for studying two types of two-beam instabilities due to ions. Concerning the ion trapping instability, growth rates for some vacuum pressure and stored current were obtained by using parameters of PF. The growth rate was about 1200 turn at $P_{CO} = 10^{-9}$ Torr and $I = 40$ mA. It was consistent with experiments at PF. In these condition, the two beam instability has been observed and been cured by exciting octapole magnets with a Landau damping of ~ 1000 turn. Blow-up and diffusion of ions were obtained at the vertical beam amplitude of $\sim \sigma_y$ by the simulations. It can explain the pulsation of the instability.

The other is the fast ion instability. In the example of KEKB, the growth rate is obtained to be 100turn at $P_{CO} = 10^{-9}$ Torr for 500 bunch train. Since the rate is near the limit for curing, the vacuum pressure of $P_{CO} = 10^{-9}$ Torr is required for KEKB. It was consistent with a result from the strong-strong model[7], and was performed using a much smaller computer resource.

The instability has usually been cured by a feedback system or Landau damping. In a recent low-emittance ring, a bunch by bunch feedback system was used for such purposes, because rapid Landau damping can not be expected according to the small beam size. Since the dynamics of the ion-beam system is affected by the feedback, it

Figure 8: Time evolution of the amplitude of each bunch amplitude. Amplitudes of 100, 200, 300, 400, and 500-th bunches are plotted for $P = 10^{-8} \text{ Torr}$ (a) and $P = 10^{-9} \text{ Torr}$ (b).

Figure 9: Spread of the vertical tune due to the ion distribution. Tune spreads of 300, 400, and 500-th bunches are plotted for $P = 10^{-8} \text{ Torr}$ (a) and $P = 10^{-9} \text{ Torr}$ (b).

is not clear whether the comparison between the growth and damping rates predicts an actual phenomenon. It may be necessary to perform simulations which include the feedback system.

This method is also available for beam-photoelectron interactions[12]. Regarding the beam-photoelectrons issue, the linearity of the wake is satisfied as in the usual case. However, the linearity is broken in some cases[13]. This method, which does not require linearity of the wake, can be used to solve such nonlinear wake effects.

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