Hadron Structure in the Non–Perturbative Regime of QCD: 
Isospin Symmetry and its Violation *

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I discuss recent progress made in calculating electromagnetic corrections in the framework of the effective field theory of QCD. In the case of elastic pion–pion scattering, strong interaction predictions have been worked out to two loop accuracy. I present first results for the electromagnetic corrections in the case of neutral pions. Here, the only sizeable effect comes from the charged to neutral pion mass difference. In the presence of nucleons, isospin violation can be measured in threshold pion photoproduction. I review the present status of the theoretical predictions and the experimental data. I argue that a deeper understanding of isospin violation based on a more precise study of such reactions can be achieved.

1. Introduction

QCD S–matrix elements and transition currents in the non–perturbative (low energy) regime can be calculated accurately by means of effective field theory methods, i.e. in chiral perturbation theory. The effective chiral Lagrangian, formulated in terms of the asymptotically observed fields (pions, nucleons) chirally coupled to external sources, admits an expansion in small external momenta and quark masses. The main physics goals of these studies are

* to pin down the value of the scalar quark condensate,
* to determine the ratios of the light quark masses,
* to test the chiral anomaly of QCD.

In this talk, I concentrate on the question of isospin symmetry violation. There are two major sources leading to a departure from this symmetry. First, there are the strong interaction effects related to the light quark mass differences. It is believed that the ratio $m_d/m_u \approx 2$ (at a canonical renormalization scale of 1 GeV)[1]. This seems to indicate that isospin should be badly violated. However, the difference $m_d - m_u \ll 1$ GeV, which is the typical strong interaction scale, and is thus effectively masked in strong interaction processes (with the exception of some reactions, which to leading order are proportional to $m_d - m_u$ like e.g. $\eta \rightarrow 3\pi$). Second, there are electromagnetic (virtual photon) effects.

To really understand the isospin violation due to the strong interactions, one has to be able to calculate these em effects precisely. In the following, I will discuss some progress made in calculating these in the framework of chiral perturbation theory.

2. Chiral perturbation theory

In the sector of the three light quarks $u$, $d$ and $s$, QCD admits a global chiral symmetry softly broken by the quark mass term,

$$
\mathcal{H}_{\text{QCD}} = \mathcal{H}_{\text{QCD}}^0 + \mathcal{H}_{\text{QCD}}^{\text{SB}} = \mathcal{H}_{\text{QCD}}^0 + m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s ,
$$

where "light" means that the current quark mass at a renormalization scale of $\mu = 1 \text{ GeV}$ can be treated as small compared to the typical scale of chiral symmetry breaking, $\Lambda_\chi \simeq 4\pi F_\pi \simeq 1.2 \text{ GeV}$, with $F_\pi \simeq 93 \text{ MeV}$ the pion decay constant. The tool to investigate these issues is chiral perturbation theory (CHPT). In CHPT, the basic degrees of freedom are the Goldstone boson fields coupled to external sources and matter fields, like e.g. the nucleons. QCD is mapped onto an effective hadronic Lagrangian formulated in terms of these asymptotically observed fields. Any matrix element involving nucleons, pions, photons and so on can be classified according to its chiral dimension, which counts the number of external momenta, quark mass insertions and inverse powers of heavy mass fields. Denoting these small parameters collectively as $p$, CHPT allows for a systematic perturbative expansion in powers of $p$, with the inclusion of loop graphs and local terms of higher dimension. The latter are accompanied by a priori unknown coupling constants, the so-called low-energy constants (LECs). This is the so-called chiral expansion, which is nothing but an energy expansion reminiscent of the ancient Euler–Heisenberg treatment of light–by–light scattering in QED at photon energies much smaller than the electron mass. In QCD, the equivalent heavy mass scale is essentially set by the first non–Goldstone resonances, i.e. the $\rho, \omega$ mesons. Symbolically, any matrix–element can be expanded as

$$
\mathcal{M} = \sum_n (p/\Lambda_\chi)^n f_n(p/\Lambda_\chi, g_i, \lambda/\Lambda_\chi) ,
$$

where $g_i$ denotes the LECs, $\lambda$ the scale of dimensional regularization and the $f_n$ are functions of order one which also contain the so-called chiral logarithms. The important observation is that chiral symmetry bounds the values of the counting index $n$ from below. This dual expansion in small momenta and quark masses can be mapped one–to–one onto an expansion in powers of Goldstone boson loops, where an $N$–loop graph is suppressed by powers of $p^{2N}$ [2]. The leading terms are in general tree graphs with lowest order insertions leading to the celebrated current algebra results. Space does not allow for a more detailed discussion of the method. Some recent developments are summarized e.g. in ref.[3] (and references therein).

3. Elastic pion–pion scattering

The purest reaction to test the spontaneous and explicit chiral symmetry breaking of QCD is elastic pion–pion scattering. In the threshold region, the scattering amplitude can be decomposed as

$$
t_I^l = q^{2l} \left[ a_I^l + b_I^l q^2 + \mathcal{O}(q^4) \right] ,
$$

(3)
where \( l \) denotes the pion angular momentum, \( I \) the total isospin of the two–pion system and \( q \) the cms momentum. Of particular interest are the S–wave scattering lengths \( a_0, a_2 \) since they vanish in the chiral limit of zero quark masses. At present, they have been worked out to two loops in the chiral expansion. Consider e.g. \( a_0 \),

\[
a_0^0 = \frac{7M_{\pi}^2}{32\pi F_{\pi}^2} \left\{ 1 + a_1 M_{\pi}^2 + a_2 M_{\pi}^4 + \mathcal{O}(M_{\pi}^6) \right\},
\]

with \( M_{\pi} \) the pion mass and the terms \( \sim a_i \) contain, of course, chiral logs. The first term in this series is the celebrated current algebra result of Weinberg[4], the second and third one are the one– and two–loop corrections given in [5] and [6], respectively. Numerically, the series converges,

\[
a_0^0 = 0.156 \cdot \left( 1 + 0.28 + 0.11 + \mathcal{O}(M_{\pi}^6) \right).
\]

Since the pion mass difference is almost entirely of em origin and the Weinberg term changes from 0.156 to 0.146 when one uses the neutral instead of the charged pion mass, one expects the em corrections to be of the same size than the strong two–loop contributions. It is thus mandatory to systematically investigate them. A first step towards this was performed in ref.[7]. There, we constructed the next–to–leading order chiral pion Lagrangian involving virtual photons (the SU(3) case was already worked out in [8]). For that, one has to assign a chiral dimension to the electric charge. Based on the observation that \( \alpha = e^2/4\pi \simeq 1/137 \simeq M_{\pi}^2/(4\pi F_{\pi})^2 \), it is natural to count \( e \) as a small momentum, \( \mathcal{O}(e) \sim \mathcal{O}(p) \). The virtual photon Lagrangian then takes the form

\[
\mathcal{L}_{\text{eff}}^\gamma = \mathcal{L}_{\text{kin}}^\gamma + \mathcal{L}_{\text{gauge}}^\gamma + \mathcal{L}_2^\gamma + \mathcal{L}_4^\gamma + \ldots
\]

where the ellipsis denotes higher order terms and the form of \( \mathcal{L}_{\text{kin}}^\gamma + \mathcal{L}_{\text{gauge}}^\gamma \) is standard. To lowest order, one can only construct a single term,

\[
\mathcal{L}_2^\gamma = C \left\langle Q_R U Q_L U^\dagger \right\rangle,
\]

where \( Q_{L,R} \) are spurions which transform linearly under left and right SU(2) transformations. At the end, one sets \( Q_L = Q_R = Q = e(3\tau_3 + 1)/6 \). The LEC \( C \) can be determined from the pion mass difference, \( (\delta M_{\pi}^2)_{\text{em}} = 2e^2C/F_{\pi}^2 \). Notice that when extended to SU(3), this term naturally leads to Dashen’s theorem, \( (\delta M_{\pi}^2)_{\text{em}} = (\delta M_{K}^2)_{\text{em}} \). An interpretation of this term and the LEC \( C \) in terms of resonance exchange can be found in [9].

At next–to–leading order, one has in total 13 terms (including the ones which are only needed for renormalization) accompanied by scale–dependent LECs called \( k_i(\lambda) \). These terms and the corresponding \( \beta \)–functions for the LECs are enumerated in [7]. As it is the case for the hadronic LECs in the two–flavor case [5], one can introduce scale–independent couplings, called \( \bar{k}_i \), via

\[
\bar{k}_i(\lambda) = \frac{\kappa_i}{32\pi^2} \left[ \bar{k}_i + \ln \frac{M_{\pi}^2}{\lambda^2} \right].
\]

Notice that one chooses the neutral pion mass as the reference scale. This is natural since the neutral pion mass is almost entirely a hadronic effect, in contrast to the charged one.

In [7], the numerical consequences for the process \( \pi^0 \pi^0 \rightarrow \pi^0 \pi^0 \) were worked out. This involves the diagrams c), d) and j) shown in Fig. 1. In the \( \sigma \)–model gauge, the insertion
of the dimension two operator Eq.(7) leads only to terms quadratic in the pion fields and thus can be entirely absorbed in a redefinition of the pion propagator

$$\Delta_{\pi}^{ab}(\ell) = \frac{i\delta^{ab}}{\left[\ell^2 - M_{\pi^0}^2 - \delta M^2 (1 - \delta^{3a})\right]}, \quad (9)$$

with \(\ell\) the pion four–momentum and \('a,b' isospin indices. It is then straightforward to work out the em corrections to the S–wave scattering length \(a_0\) and effective range \(b_0\). While \(a_0\) only changes by 5% (which is still smaller than the hadronic two–loop correction), \(b_0\) increases by 36%. This can be understood by looking at the partial wave amplitude \(t_0 = t_{00}^0/3 + 2t_{20}^2/3\) divided by the scattering length shown in Fig. 1. Above the \(\pi^0\pi^0\) threshold at \(W_0 = 2M_{\pi^0}\) the charged pion threshold opens at \(W_c = 2M_{\pi^+}\). This leads to a pronounced cusp effect which is expected to scale as \(\sqrt{M_{\pi^+}^2 - M_{\pi^0}^2}/M_{\pi^+} \simeq 26\%\). Also shown in [7] is that the effect of the terms \(\sim \bar{k}_i\) (graph j) is completely absent in this channel. The dominant isospin–violating effect appears to be given by the charged to neutral pion mass difference.

![Graphs and plot](image)

**Fig. 1:** Left panel: Tree and one–loop graphs contributing to the em corrections for \(\pi\pi\) scattering. Solid and wiggly lines denote pions and photons, in order. The em contact insertions of dimension two and four are depicted by filled circles and boxes, respectively. Right panel: The normalized S–wave scattering amplitude for the process \(\pi^0\pi^0 \rightarrow \pi^0\pi^0\) versus the pion cm energy \(W = \sqrt{s}\). The solid and dashed lines give the results for \(e \neq 0\) and \(e = 0\), respectively.

For reactions involving charged pions, matters are more complicated. One finds e.g. that graph e) is infrared divergent, and this IR divergence is not cancelled by the divergent parts of the LECs. Indeed, one also has to account for the soft photon emission of the external legs. Furthermore, to define the scattering lengths, one has to properly subtract the long–range em interaction. A systematic analysis of these effects is under way. It is important to note that only in the effective field theory framework one can hope to develop a consistent scheme to treat these various complications induced by the em interactions.
4. The nucleon sector: General aspects

As stressed first by Weinberg [10], reactions involving nucleons and neutral pions are particularly suited to assess the quark mass difference $m_d - m_u$. In the pion sector, G-parity allows to leading order only a term which is sensitive to the sum of the quark masses, i.e. the terms $\sim m_d - m_u$ appear at next-to-leading order. Furthermore, as discussed above, virtual photons start to contribute already at lowest order. In the presence of nucleons, the effective Lagrangian takes the following form

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \ldots .$$

Since the charge matrix always appears quadratically in physical processes, the leading term $\mathcal{L}_{\pi N}^{(1)}$ cannot be directly affected by the inclusion of virtual photons. The only effect at this order comes from the covariant derivative, $D_\mu = \partial_\mu + i A_\mu Q$ which generates vertices to be used in the calculation of photon loops. At next order (dimension two), one has exactly two new terms that influence the pion–nucleon couplings. Furthermore, at this order exactly one term $\sim m_d - m_u$ appears [11]. The corresponding LECs are finite. The dimension three and four Lagrangians have not yet been worked out in full detail. I will therefore take a more pedestrian approach here and try to estimate what size of isospin violation apart from the one induced by the pion mass difference we can expect. For that, I will review the theoretical and experimental status of threshold pion photoproduction and draw some lessons from that. For other discussions of isospin violation in the pion–nucleon system, see e.g. van Kolck [12] and Bernstein [13].

5. Threshold pion photoproduction

In the physical basis, we have four reactions, two involving neutral and two involving charged pions, i.e. $\gamma p \to \pi^+ n$, $\gamma n \to \pi^- p$, $\gamma p \to \pi^0 p$ and $\gamma n \to \pi^0 n$. Working to first order in the em coupling, the corresponding amplitudes can be expressed in terms of three isospin amplitudes, commonly denoted as $A^{(0,\pm)}$, via

$$A(\gamma p \to \pi^+ n) = \sqrt{2} (A^{(0)} + A^{(-)}) , \quad A(\gamma n \to \pi^- p) = \sqrt{2} (A^{(0)} - A^{(-)}) ,$$

$$A(\gamma p \to \pi^0 p) = A^{(0)} + A^{(+)} , \quad A(\gamma n \to \pi^0 n) = -A^{(0)} + A^{(+)} .$$

If isospin were to be an exact symmetry, one would not need to measure all four amplitudes but rather could deduce the fourth one from the “triangle relation”, e.g.

$$A(\pi^0 n) = A(\pi^0 p) - \frac{1}{\sqrt{2}} \left( A(\pi^+ n) + A(\pi^- p) \right) .$$

Any deviation from this is a measure of isospin violation either due to the light quark mass difference or from the virtual photons. In the threshold region, where the produced pion has a very small three momentum, it is advantageous to decompose the amplitudes into S- and P-wave multipoles. I will now review the status of our knowledge about the (S-wave) electric dipole amplitude $E_{0+}$ for the four reaction channels. Space forbids to discuss in detail the interesting physics related to the P-wave multipoles, in particular the novel low-energy theorems for the multipole combinations $P_1$ and $P_2$ which have recently been derived [14].
5.1. Charged pion photoproduction
Charged pion photoproduction at threshold is well described in terms of the Kroll–Ruderman contact term, which is non–vanishing in the chiral limit,

\[ E_{0^+}^{\text{thr}}(\pi^+ n) = \frac{e g_{\pi N}}{4\pi \sqrt{2m (1 + \mu)^{3/2}}} = 27.6 \cdot 10^{-3}/M_\pi, \]

\[ E_{0^+}^{\text{thr}}(\pi^- p) = -\frac{e g_{\pi N}}{4\pi \sqrt{2m (1 + \mu)^{1/2}}} = -31.7 \cdot 10^{-3}/M_\pi, \quad (13) \]

with \( \mu = M_\pi/m \) and using \( g_{\pi N}^2/4\pi = 14.28, \ e^2/4\pi = 1/137.036, \ m = 928.27 \text{MeV} \) and \( M_\pi = 139.57 \text{MeV} \). By now, all chiral corrections including the third order in the pion mass have been calculated [15]. The chiral series is quickly converging and the theoretical error on the CHPT predictions is rather small, see table 1. Notice that these uncertainties do not account for the variations in pion–nucleon coupling constant, about which no consensus has been reached yet. Also given in that table are recent results from the dispersion theoretical (DR) analysis of the Mainz group [16]. A theoretical uncertainty has not yet been determined within that framework. The available threshold data are quite old, with the exception of the recent TRIUMF experiment on the inverse reaction \( \pi^- p \rightarrow \gamma n \). While the overall agreement is quite good for the \( \pi^+ n \) channel, in the \( \pi^- p \) channel the CHPT prediction is on the large side of the data. Clearly, we need more precise data to draw a final conclusion. It is, however, remarkable to have predictions with an error of only 2%.

Table 1
Predictions and data for the charged pion electric dipole amplitudes.

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<td>( E_{0^+}^{\text{thr}}(\pi^+ n) )</td>
<td>28.2 ± 0.6</td>
<td>28.0</td>
<td>27.9 ± 0.5[17], 28.8 ± 0.7[18]</td>
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<tr>
<td>( E_{0^+}^{\text{thr}}(\pi^- p) )</td>
<td>-32.7 ± 0.6</td>
<td>-31.7</td>
<td>-31.4 ± 1.3[17], -32.2 ± 1.2[19], -31.5 ± 0.8[20]</td>
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5.2. Neutral pion photoproduction off nucleons
The threshold production of neutral pions is much more subtle since the corresponding electric dipole amplitudes vanish in the chiral limit. Space does not allow to tell the tale of the experimental and theoretical developments concerning the electric dipole amplitude for neutral pion production off protons, for details see [21]. Even so the convergence for this particular observable is slow, a CHPT calculation to order \( p^4 \) does allow to understand the energy dependence of \( E_{0^+} \) in the threshold region once three LECs are fitted to the total and differential cross section data [22] as shown in fig. 2. The threshold value agrees with the data and the dispersion theoretical determination, see table 2. More interesting is the case of the neutron. Here, CHPT predicts a sizeably larger \( E_{0^+} \) than for the proton (in magnitude), whereas the dispersion relations tend to give values of the same size (note however that the DR treatment for the neutral channels is less stable than for the charged ones). The CHPT prediction for \( E_{0^+}(\pi^0 n) \) in the threshold region is shown in fig. 3. Both amplitudes clearly exhibit the unitary cusp due to the opening of the secondary threshold,
Figure 2. CHPT prediction for the $\pi^0 p$ electric dipole amplitude compared to the MAMI [23] and SAL [24] data.

Figure 3. CHPT prediction for the $\pi^0 n$ electric dipole amplitude.

$\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ and $\gamma n \rightarrow \pi^- p \rightarrow \pi^0 n$, respectively. Note, however, that while $E_{0^+}(\pi^0 p)$ is almost vanishing after the secondary threshold, the neutron electric dipole amplitude is sizeable ($-0.4$ compared to $2.8$ in units of $10^{-3}/M_{\pi^+}$).

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<tr>
<td>$E_{0^+}^{\text{thr}}(\pi^0 p)$</td>
<td>$-1.16$</td>
<td>$-1.22$</td>
<td>$-1.31 \pm 0.08[23]$, $-1.32 \pm 0.11[24]$</td>
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<tr>
<td>$E_{0^+}^{\text{thr}}(\pi^0 n)$</td>
<td>$2.13$</td>
<td>$1.19$</td>
<td>$1.9 \pm 0.3[25]$</td>
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5.3. Neutral pion photoproduction off the deuteron

The question arises how to measure the neutron amplitude? The natural neutron target is the deuteron. The transition matrix for $\pi^0$ production off the deuteron (d) takes the form

$$\mathcal{T} = 2i \, E_d \, \vec{J} \cdot \vec{\epsilon} + \mathcal{O}(q^2)$$

with $\vec{J}$ the total angular momentum of the d and $\vec{\epsilon}$ the polarization vector of the photon. Although the deuteron electric dipole amplitude could be calculated entirely within CHPT, a more precise calculation is based on the approach suggested by Weinberg [26], i.e. to calculate matrix elements of the type $\langle \Psi_d | K | \Psi_d \rangle$ by using deuteron wave functions $\Psi_d$ obtained from accurate phenomenological NN potentials and to chirally expand the kernel $K$. Diagrammatically, one has the single scattering (ss) terms which contain the desired $\pi^0 n$ amplitude. In addition, there are the so-called three–body (th) contributions (meson exchange currents). To leading order $p^3$, one only has the photon coupling to the
pion in flight and the seagull term [27]. The latter involves the charge exchange amplitude and is thus expected to dominate the single scattering contribution. However, to obtain the same accuracy as for the ss terms, one has to calculate also the corrections at order $p^4$. This has been done in [28]. It was shown that the next-to-leading order three-body corrections and the possible four-fermion contact terms do not induce any new unknown LEC and one therefore can calculate $E_d$ in parameter-free manner. One finds

$$E_d = E_{d}^{\text{ss}} + E_{d}^{\text{tb},3} + E_{d}^{\text{tb},4}$$

$$= 0.36 - 1.90 - 0.25 = (-1.8 \pm 0.2) \cdot 10^{-3}/M_{\pi^+}.$$ (15)

Some remarks concerning this result are in order. First, one finds indeed that the tb contribution is bigger than the single scattering one. However, the former can be calculated precisely, i.e. the first corrections amount to a meager 13%. This signals good convergence. I remark that a recent claim about large higher order (unitarity) corrections [29] needs to be quantified in a consistent CHPT calculation. Second, the resulting $E_d$ is very sensitive to $E_{0+}(\pi^0 n)$. If one were to set $E_{0+}(\pi^0 n) = 0$, $E_d$ changes to $-2.6 \cdot 10^{-3}/M_{\pi^+}$, i.e. the threshold cross sections would change by a factor of two. Note that the theoretical error given in Eq.(15) is an educated guess, see [28]. Third, the CHPT prediction nicely agrees with the empirical value of $E_d^{\exp} = (-1.7 \pm 0.2) \cdot 10^{-3}/M_{\pi^+}$ [25]. This agreement might, however, be fortuitous since the extraction of the empirical number relies on the input from the elementary proton amplitude to fix a normalization constant. The TAPS collaboration intends to redo this measurement at MAMI.

5.4. The size of isospin violation

I can now give a very rough estimate for the effects of isospin violation beyond the one from the charged to neutral pion mass difference. For that, let us compare the neutron amplitude as predicted by CHPT or DR (labelled ”pre”) and compare with the result of the ”triangle relation”, Eq.(12), labelled ”tri”. This gives

CHPT : $E_{0+}^{\pi^0 n,\text{pre}} = 2.13$, $E_{0+}^{\pi^0 n,\text{tri}} = 2.06$

DR : $E_{0+}^{\pi^0 n,\text{pre}} = 1.19$, $E_{0+}^{\pi^0 n,\text{tri}} = 1.38$. (16)

This indicates that threshold pion photoproduction is sensitive to isospin violation induced by the light quark mass difference $m_d - m_u$ and virtual photon effects (besides the ones leading to $M_{\pi^0} \neq M_{\pi^\pm}$) of the order of a few up to 15%. Clearly, such an estimate should only be considered indicative since it is not based on a fully self-consistent calculation. Also, the rather large discrepancy one obtains in the DR approach might to some extent reflect the uncertainty of the method used in the analysis. This remains to be clarified. However, it is rather obvious that combining precise calculations with very accurate measurements, one can obtain significant information on the origin of isospin violation in the pion–nucleon system. This is a rather novel situation in nuclear physics, namely that such seemingly old-fashioned reactions can and will be used to test in a quantitative manner our understanding of certain aspects of the QCD in the low energy domain, i.e. where the strong coupling constant is really large.
6. Summary and outlook

As outlined before, there are many problems still to be tackled to gain a deeper insight into the violation of isospin in low energy hadronic reactions. Let me mention here only a few of these:

⋆ The effects of virtual photons on $\pi\pi$ scattering involving charged pions are in the process of being worked out to one loop order. These effects are complementary to the strong one and two loop corrections calculated so far but are needed for a precise comparison with the $K_{\ell4}$ data expected from DAΦNE or the pionium ones from CERN.

⋆ The construction of the effective chiral pion–nucleon Lagrangian to one loop is underway,

$$\mathcal{L}_{\text{eff}}[U, N, A_\mu] = \mathcal{L}^{(2)}[\ldots] + \mathcal{L}^{(3)}[\ldots] + \mathcal{L}^{(4)}[\ldots],$$  \hspace{1cm} (17)

with $U$, $N$ and $A_\mu$ parametrizing the pions, nucleons and virtual photons, in order. As explained before, the effective Lagrangian includes also the strong operators $\sim m_d - m_u$.

⋆ The next step will then be to apply this machinery to pion photoproduction and pion–nucleon scattering. Over the last few years, there have been various claims of large isospin–breaking effects in low–energy $\pi N$–scattering. Only with the consistent machinery of the effective chiral Lagrangian involving virtual photons, we can hope to put such claims on firm grounds (or to dismiss them).

I finally remark that 20 years have passed since Weinberg’s seminal paper [10] and that only now the theoretical machinery as well as the experimental methods allow us to address these questions in a truly quantitative manner. I am hopeful that within a few years from now further considerable progress will have been made.

Acknowledgements

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