Nonfactorizable soft gluons in nonleptonic heavy meson decays

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Abstract

We include nonfactorizable soft gluon corrections into the perturbative QCD formalism for exclusive nonleptonic heavy meson decays, which combines factorization theorems and effective field theory. These corrections are classified according to their color structures, and exponentiated separately to complete the Sudakov resummation up to next-to-leading logarithms. The nonfactorizable contributions in nonleptonic decays are clearly identified in our formalism, and found to be positive for bottom decays and negative for charm decays. Our analysis confirms that the large-$N_c$ approximation is applicable to charm decays, but not to bottom decays, consistent with the phenomenological implications of experimental data. The comparison of our predictions with those from QCD sum rules is also made.
I. INTRODUCTION

Nonleptonic heavy meson decays are more difficult to analyze compared to semileptonic decays, because they involve complicated strong interactions. A naive perturbative QCD (PQCD) formalism [1], considering dynamics between the large typical scale $t$ of decay processes, which is of order of the heavy meson mass, and the hadronic scale of order $\Lambda_{\text{QCD}}$, is appropriate only for semileptonic decays. For nonleptonic decays, a more sophisticated formalism must be developed, which further includes dynamics between the $W$ boson mass $M_W$ and the typical scale $t$. Recently, such a modified PQCD framework has been proposed [2], in which nonleptonic decay rates are factorized into the convolution of a “harder” function characterized by $M_W$, a hard subamplitude by $t$, and meson wave functions by the hadronic scale. The renormalization-group (RG) method and the resummation technique [3] are then applied to organize large single logarithms $\ln(M_W/t)$ and double logarithms $\ln(t/\Lambda_{\text{QCD}})$ contained in the perturbative expansions of these convolution functions into Wilson coefficients [4] and Sudakov factors [1, 5], respectively. In the previous analysis of [6] we have considered nonfactorizable contributions to the hard subamplitude. However, nonfactorizable soft gluon effects, which cannot be absorbed into meson wave functions, were neglected because of their complicated color flows. In this paper we shall take into account these soft corrections, which produce single logarithms, systematically in the modified PQCD formalism, and improve the accuracy of the resummation up to next-to-leading logarithms.

The simplest and most widely adopted approach to exclusive nonleptonic heavy meson decays is the Bauer-Stech-Wirbel (BSW) model [7] based on the factorization hypothesis, in which decay rates are expressed in terms of various hadronic transition form factors multiplied by some coefficients. The coefficients of the form factors corresponding to external $W$ boson emission and to internal $W$ boson emission are $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$, respectively, $N_c = 3$ being the number of colors. Here $c_i$ are the Wilson coefficients appearing in the effective Hamiltonian,

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^*\left[c_1(\mu)O_1 + c_2(\mu)O_2\right], \quad (1)$$

which satisfy the matching conditions $c_1(M_W) = 1$ and $c_2(M_W) = 0$. $O_1 = (\bar{c}_L\gamma_\mu b_L)(\bar{d}_L\gamma^\mu u_L)$ and $O_2 = (\bar{d}_L\gamma_\mu b_L)(\bar{c}_L\gamma^\mu u_L)$ are the local four-fermion
operators associated with bottom decays. The form factors may be related to each other by heavy quark symmetry, and be modelled by different ansatz. The nonfactorizable contributions which cannot be expressed in terms of hadronic form factors, and the nonspectator contributions from $W$ boson exchanges are neglected. In this way the BSW method avoids complicated QCD dynamics.

Though the BSW model is simple and gives predictions in fair agreement with experimental data, it encounters several difficulties. It has been observed that the naive factorization is incompatible with experimental data for color-suppressed decay modes [7]. Therefore, the large $N_c$ limit of $a_{1,2}$, i.e. the choice $a_1 = c_1(M_c) \approx 1.26$ and $a_2 = c_2(M_c) \approx -0.52$, with $M_c$ the $c$ quark mass, must be employed in order to explain the data of charm decays [7]. However, the same limit of $a_1 = c_1(M_b) \approx 1.12$ and $a_2 = c_2(M_b) \approx -0.26$, $M_b$ being the $b$ quark mass, does not apply to the bottom case. Even after including the $c_{1,2}/N_c$ term such that $a_1 = 1.03$ and $a_2 = 0.11$, the BSW predictions are still insufficient to match the data. To overcome this difficulty, parameters $\chi$, denoting the nonfactorizable contributions which are suppressed in the large $N_c$ limit [8], have been introduced [9]. They lead to the effective coefficients

$$a_{1,2}^{\text{eff}} = c_1 + c_2 \left( \frac{1}{N_c} + \chi_1 \right), \quad a_{1,2}^{\text{eff}} = c_2 + c_1 \left( \frac{1}{N_c} + \chi_2 \right).$$  \hfill (2)

The parameters $\chi$ should be negative for charm decays, canceling the color-suppressed term $1/N_c$, and be positive for bottom decays in order to account for the observed constructive interference in $B \to D^{(*)}\pi$ decays. A phenomenological extraction [9] from the CLEO data [10] gave

$$\chi_1(D \to \bar{K}\pi) \approx \chi_2(D \to \bar{K}\pi) \approx -0.36, \quad \chi_1(B \to D\pi) \approx 0.05, \quad \chi_2(B \to D\pi) \approx 0.11.$$  \hfill (3)

We shall demonstrate that our predictions for the nonfactorizable contributions are consistent with the above values of $\chi$.

The rule of discarding the $1/N_c$ corrections [11], i.e. the large $N_c$ approximation, found its dynamical origin in the analyses based on QCD sum rules [12]: $\chi$ for charm decays are indeed negative, and cancel the term $1/N_c$ roughly. However, the sum rule analyses also predicted negative $\chi$ for the bottom decay $\bar{B}^0 \to D^{+}\pi^-$ in a certain kinematic limit [13], and are thus
in conflict with the phenomenological implications. Hence, the mechanism responsible for the sign change has not been understood completely, and remains a challenging subject in weak nonleptonic decays \[13\]. In our formalism with the soft gluons taken into account, the nonfactorizable contributions can be clearly identified. We shall investigate how the soft corrections modify the previous predictions for heavy meson decays \[2, 6\], and try to explore the dynamical origin for the sign change of the nonfactorizable contributions in bottom and charm decays in a unified viewpoint.

In Sect. II we briefly review the derivation of the three-scale PQCD factorization formulas for heavy meson decays, and include the nonfactorizable soft gluon effects into the formulas. The numerical results, along with a detailed comparison with those from the phenomenological and QCD sum rule analyses, are presented in Sect. III. Section IV is our conclusion.

II. SOFT GLUON CORRECTIONS

The construction of the modified PQCD formalism is as follows. Before including QCD corrections, nonleptonic heavy meson decays are described by a full Hamiltonian of four-quark current-current operators, such as

\[
H = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* (\bar{c}_L \gamma_\mu b_L)(\bar{d}_L \gamma_\mu u_L) \tag{4}
\]

for the \(B \rightarrow D^{(*)}\pi(\rho)\) decays. Consider one-loop corrections to a tree-level heavy quark decay amplitude without integrating out the \(W\) boson. The amplitude is ultraviolet finite because of the current conservation (a conserved current is not renormalized) and the presence of the \(W\) boson line. However, these corrections give rise to infrared divergences, when the radiative gluons are soft or collinear to the involved light quarks. We classify the higher-order diagrams into the reducible and irreducible types \[14\]: The former contains double logarithms from the overlap of soft and collinear divergences, while the latter contains only single soft logarithms.

We factorize the infrared sensitive contributions according to Fig. 1(a) first, which are collected by the appropriate eikonal approximation for quark propagators. The diagram in the second parentheses is absorbed into a meson wave function \(\phi(b, \mu)\), if it is two-particle reducible, or into a soft function \(U(b, \mu)\), if it is two-particle irreducible as exemplified by Fig. 1(a). Both \(\phi\)
and $U$ depend on a renormalization scale $\mu$, since the eikonal approximation brings in ultraviolet divergences. The variable $b$, conjugate to the transverse momentum $k_T$ carried by a valence quark, can be regarded as the spatial extent of the meson. $1/b$ is then the hadronic scale, which will play the role of an infrared cutoff for loop integrals below.

The diagrams in the first parentheses of Fig. 1(a) involve scales above $1/b$, and need further factorization. We express the full $O(\alpha_s)$ diagram into two terms, with the first term obtained by shrinking the $W$ boson line into a point, and the second term being the difference between the full diagram and the first term. Evidently, the former is characterized by the scale $t \ll M_W$ introduced before, and the latter by momenta of order $M_W$. We factorize the contributions with the scale $M_W$ according to Fig. 1(b), grouping it into a "harder" function $H_r(M_W, \mu)$ (not an amplitude). The remaining diagrams in the last parentheses, because of operator mixing, correspond to the local four-fermion operators $O_1$ or $O_2$ in Eq. (1). They are absorbed into a hard decay subamplitude $H(t, \mu)$. Similarly, shrinking the $W$ boson line brings in ultraviolet divergences, and thus both $H_r$ and $H$ acquire a $\mu$ dependence.

Therefore, we arrive at the factorization formula for the decay amplitude

$$M = H_r(M_W, \mu) \otimes H(t, \mu) \otimes \phi(b, \mu) \otimes U(b, \mu),$$

where $\otimes$ represents a convolution relation, since $t$ and $b$ will be integrated out at last. Assume that $\gamma_{H_r}$, $\gamma_{\phi}$, and $\gamma_U$ are the anomalous dimensions of $H_r$, $\phi$, and $U$, respectively. The anomalous dimension of $H$ is then $\gamma_H = - (\gamma_{H_r} + \gamma_{\phi} + \gamma_U)$, because a decay amplitude is ultraviolet finite as stated above. A RG treatment of Eq. (5) leads to

$$M = H_r(M_W, M_W) \otimes H(t, t) \otimes \phi(b, 1/b) \otimes U(b, 1/b)$$

$$\otimes \exp \left[ \int_t^{M_W} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{H_r}(\alpha_s(\bar{\mu})) \right]$$

$$\otimes \exp \left[ - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \left[ \gamma_{\phi}(\alpha_s(\bar{\mu})) + \gamma_U(\alpha_s(\bar{\mu})) \right] \right],$$

which becomes explicitly $\mu$-independent. The two exponentials, describing the two-stage evolutions from $M_W$ to $t$ and from $t$ to $1/b$, are the consequence of the summation of large $\ln(M_W/t)$ and $\ln(tb)$. The first exponential can be easily identified as the Wilson coefficient $c(t)$. Note that the Wilson
coefficient appears as a convolution factor here, instead of a multiplicative factor in the effective Hamiltonian, Eq. (1). In the conventional approach to exclusive nonleptonic heavy meson decays [7], which is based on Eq. (1), a value of $\mu$ must be assigned, and thus an ambiguity is introduced [15]. A merit of our formalism is then that it does not suffer the scale setting ambiguity.

Equation (6) sums only the single logarithms. In fact, there exist the double logarithms $\ln^2(P^+b)$ in the meson wave function $\phi$, which arise from the overlap of collinear and soft divergences, $P^+$ being the largest light-cone component of the meson momentum. Hence, $\phi(b,\mu)$ in Eq. (5) should be replaced by

$$\phi(P^+,b,\mu) = \phi(b,\mu)\exp[-s(P^+,b)],$$

(7)

where the exponential $e^{-s}$, the so-called Sudakov form factor, comes from the resummation of the double logarithms. For the detailed derivation of Eq. (7) and the complete formula of $s$, refer to [1, 5]. Below we shall approximate $H_r(M_W,M_W)$ by its lowest-order expression $H_r^{(0)} = 1$, and evaluate $H(t,t)$ perturbatively, since all the large logarithms have been grouped into the exponents. For simplicity, we set the nonperturbative initial condition $U(b,1/b)$ of the RG evolution to unity due to the strong suppression at large $b$ [16], and neglect the $b$ dependence in another nonperturbative initial condition $\phi(b,1/b)$.

At the tree level, six types of diagrams contribute to the hard decay subamplitude of $B^-(P_1) \to D^0(P_2)\pi^-(P_3)$ as shown in Fig. 2, where the momenta are chosen as

$$P_1 = \frac{M_B}{\sqrt{2}}(1,1,0), \quad P_2 = \frac{M_B}{\sqrt{2}}(1,r^2,0), \quad P_3 = \frac{M_B}{\sqrt{2}}(0,1-r^2,0),$$

(8)

with $r = M_D/M_B$, $M_B$ ($M_D$) being the $B$ ($D$) meson mass. These diagrams represent external $W$ emissions, if the four quark operators are $O_1$ in Figs. 2(a) and 2(b), and $O_2$ in Figs. 2(c)-2(f). They are internal $W$ emissions, if the four quark operators are $O_2$ in Figs. 2(a) and 2(b), and $O_1$ in Figs. 2(c)-2(f). At this level, only Figs. 2(e) and 2(f) give nonfactorizable contributions. We shall argue that after including the soft gluons, Figs. 2(c) and 2(d) become nonfactorizable.

We discuss the color structure of soft gluon exchanges. If a soft gluon crosses the hard gluon vertex, the color structure is given by

$$(T^a)_{b_1b'_1}(T^H)_{b'_2b_2}(T^a)_{a_2a'_2}(T^H)_{a'_1a_1}$$
\[ = \frac{1}{2} (T^H)_{a_2b_2} (T^H)_{b_1a_1} - \frac{1}{2N_c} (T^H)_{b_1b_2} (T^H)_{a_2a_1} , \quad (9) \]

where \( T^a (T^H) \) is the color matrix associated with the soft (hard) gluon. Contracted with \( \delta_{a_1b_1} \delta_{a_2b_2} \) from the color-singlet initial- and final-state mesons, the first term, denoting an octet contribution, diminishes. The second term leads to a factor \(-1/(2N_c)\) without changing the original tree-level color flow.

If a soft gluon does not cross the hard gluon vertex, it introduces a factor \( T^a T^a = C_F = 4/3 \). It will be shown that each type of hard diagrams acquires different soft corrections, which must be organized separately.

For the external \( W \) emissions, the soft function receives two contributions as shown in Fig. 3(a):

\[ U^{(e)} = -\frac{1}{2N_c} [I(p_1, p_2) + I(k_1, k_2)] . \quad (10) \]

Employing the eikonal approximation for the quark propagators, the loop integral \( I \) is written as

\[ I(u, v) = -i g^2 \mu^\epsilon \int \frac{d^4 \epsilon}{(2\pi)^{4-\epsilon}} \frac{u^\alpha v^\beta N_{\alpha\beta}}{u \cdot l v \cdot l l^2} , \quad (11) \]

with the tensor

\[ N_{\alpha\beta} = g_{\alpha\beta} - \frac{n_{\alpha} l_{\beta} + n_{\beta} l_{\alpha}}{n \cdot l} + n^2 \frac{l_{\alpha} l_{\beta}}{(n \cdot l)^2} \quad (12) \]

from the gluon propagator in the axial gauge \( n \cdot A = 0, n = (1, -1, 0) \) being a gauge vector. The other soft contributions either vanish because of the color flow in Figs. 2(a) and 2(b), or cancel by pairs in Figs. 2(c)-2(f).

For convenience, the dimensionless vectors associated with the corresponding meson momenta are assigned as

\[ p_1 = (1, 1, 0) , \quad p_2 = (1, r^2, 0) , \quad k_1 = (0, 1, 0) , \quad k_2 = (1, 0, 0) , \quad (13) \]

for the \( b \) quark, the \( c \) quark, the light valence quark in the \( B \) meson, and the light valence quark in the \( D \) meson, respectively. The dimensionless vector associated with the pion is then \( p_3 = (0, 1, 0) \). All the above kinematic variables can be copied to charm decays directly, such as \( D \to \bar{K}\pi \), by
choosing \( r = M_K/M_D, \) \( M_K \) being the kaon mass. Varying \( r \), we can study how the soft corrections modify the predictions for bottom and charm decays.

Concentrating only on the pole terms, we obtain

\[
I(p_1, p_2) = -\frac{\alpha_s}{\pi} \frac{1}{\epsilon} \left[ f(p_1, p_2) - f(p_1, n) - f(p_2, n) + 1 \right],
\]

\[
I(k_1, k_2) = -\frac{\alpha_s}{\pi} \frac{1}{\epsilon} \left[ \ln \frac{k_1 \cdot k_2}{2} - \frac{1}{2} \ln \left( \frac{(k_1 \cdot n)^2}{n^2} \right) - \frac{1}{2} \ln \left( \frac{(k_2 \cdot n)^2}{n^2} \right) + 1 \right],
\]

with

\[
f(u, v) = u \cdot v \int_0^1 dx \left[ x^2 u^2 + 2x(1-x)u \cdot v + (1-x)^2 v^2 \right]^{-1}.
\]

A straightforward calculation gives

\[
I(p_1, p_2) = -\frac{\alpha_s}{\pi} \frac{1}{\epsilon} \left( \frac{4r^2}{r^4 - 1} \ln r + 1 \right),
\]

\[
I(k_1, k_2) = -\frac{\alpha_s}{\pi} \frac{1}{\epsilon},
\]

from which we extract the anomalous dimension

\[
\gamma^{(e)}_{U} = -\frac{\alpha_s}{\pi} \frac{1}{N_c} \left( \frac{2r^2}{r^4 - 1} \ln r + 1 \right).
\]

For the internal \( W \) emissions, the analysis is more complicated. Soft corrections to Fig. 2(a) and 2(b) are similar to Eq. (10) but with \( p_2 \) and \( k_2 \) replaced by \( p_3 \), that is,

\[
U^{(i)1} = -\frac{1}{2N_c} [I(p_1, p_3) + I(k_1, p_3)].
\]

Here \( k_1 \) should be chosen as \( k_1 = (1, 0, 0) \) to render the second term meaningful. Other soft corrections, again, vanish because of the color flow. To Figs. 2(c) and 2(e), six soft gluon exchange diagrams contribute as shown in Fig. 3(b). The soft function is given by

\[
U^{(i)2} = -\frac{1}{2N_c} [I(p_1, p_2) - I(p_1, k_2) + I(p_1, p_3) + I(k_1, p_3)]
\]

\[+C_F[I(k_2, p_3) - I(p_2, p_3)].\]
Notice the minus signs in front of $I(p_1, k_2)$ and $I(p_2, p_3)$. A minus sign appears, when the soft gluon attaches a quark and an antiquark [16]. If the $D$ meson is massless, pair cancellations occur between $I(p_1, p_2)$ and $I(p_1, k_2)$ in the first brackets, and between the two terms in the second brackets. $U^{(i2)}$ then reduces to $U^{(i1)}$. The soft function associated with Figs. 2(d) and 2(f) is

$$U^{(i3)} = C_F[I(p_1, p_2) - I(p_1, k_2)] - \frac{1}{2N_c}[I(p_1, p_3) + I(k_1, p_3) + I(k_2, p_3) - I(p_2, p_3)], \quad (22)$$

according to Fig. 3(b). We present only the explicit expression of $I(p_1, p_3)$,

$$I(p_1, p_3) = -\frac{\alpha_s}{\pi} \frac{1}{c} \left[ \frac{1}{2} \ln \left( \frac{p_1 \cdot p_3}{p_1^2} \right) - \frac{1}{2} \ln \left( \frac{p_3 \cdot n}{n^2} \right) - f(p_1, n) + 1 \right]. \quad (23)$$

All other $I$’s can be derived simply from Eqs. (14), (15) and (23) with appropriate replacement of the kinematic variables. Following the similar procedures, we have

$$\gamma_{U}^{(i1)} = -\frac{\alpha_s}{\pi} \frac{1}{N_c},$$

$$\gamma_{U}^{(i2)} = -\frac{\alpha_s}{\pi} \left[ \frac{1}{N_c} \left( \frac{2r^2}{r^4 - 1} \right) \ln r + 1 \right] - C_F \frac{2r^2}{r^2 + 1} \ln r \right],$$

$$\gamma_{U}^{(i3)} = -\frac{\alpha_s}{\pi} \left[ \frac{1}{N_c} \left( \frac{r^2}{r^2 + 1} \ln r + 1 \right) \right] - C_F \frac{4r^2}{r^4 - 1} \ln r \right]. \quad (24)$$

### III. FACTORIZATION FORMULAS

The decay rate of $B^- \rightarrow D^0 \pi^-$ is [2]

$$\Gamma = \frac{1}{128\pi} G_F^2 |V_{cb}|^2 |V_{ud}|^2 M_B^3 \left( \frac{1 - r^2}{r} \right)^3 |\mathcal{M}|^2, \quad (25)$$

with the decay amplitude

$$\mathcal{M} = f_{\pi} [(1 + r) \xi_+ - (1 - r) \xi_-] + f_D \xi_i + \mathcal{M}_e + \mathcal{M}_i, \quad (26)$$

9
$f_D$ and $f_\pi$ being the $D$ meson and pion decay constant, respectively. The form factors $\xi_{\pm}$ associated with the external $W$ emissions from Figs. 2(a)-2(d) are given by

$$\xi_+ = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_D(x_2) \alpha_s(t_e) a_1(t_e)$$

$$\times [(1 + \zeta_+ x_2 r) h_e(x_1, x_2, b_1, b_2, m_e) + (r + \zeta_+ x_1) h_e(x_2, x_1, b_2, b_1, m_e)]$$

$$\times \exp[-S_B(t_e) - S_D(t_e) - S_U^{(c)}(t_e)],$$

$$\xi_- = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_D(x_2) \alpha_s(t_e) a_1(t_e)$$

$$\times \zeta_- [x_2 r h_e(x_1, x_2, b_1, b_2, m_e) - x_1 h_e(x_2, x_1, b_2, b_1, m_e)]$$

$$\times \exp[-S_B(t_e) - S_D(t_e) - S_U^{(c)}(t_e)],$$

with the constants [5]

$$\zeta_+ = \frac{1}{2} \left[ \eta - \frac{3}{2} + \sqrt{\frac{\eta - 1}{\eta + 1}} \left( \eta - \frac{1}{2} \right) \right],$$

$$\zeta_- = -\frac{1}{2} \left[ \eta - \frac{1}{2} + \sqrt{\frac{\eta + 1}{\eta - 1}} \left( \eta - \frac{3}{2} \right) \right].$$

$\eta = (1 + r^2)/(2r)$ is the maximal velocity transfer involved in the decay process. The form factors $\xi_i = \xi_{i1} + \xi_{i23}$ associated with the internal $W$ emissions are

$$\xi_{i1} = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \phi_\pi(x_3) \alpha_s(t_i) c_2(t_i)$$

$$\times [(1 + x_3(1 - r^2)) h_i(x_1, x_3, b_1, b_3, m_i) + x_1 r^2 h_i(x_3, x_1, b_3, b_1, m_i)]$$

$$\times \exp[-S_B(t_i) - S_\pi(t_i) - S_U^{(1)}(t_i)],$$

from Figs. 2(a) and 2(b), and

$$\xi_{i23} = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \phi_\pi(x_3) \alpha_s(t_i) \frac{c_1(t_i)}{N}$$

$$\times \{ [1 + x_3(1 - r^2)] h_i(x_1, x_3, b_1, b_3, m_i) \exp[-S_U^{(2)}(t_i)]$$

$$+ x_1 r^2 h_i(x_3, x_1, b_3, b_1, m_i) \exp[-S_U^{(3)}(t_i)] \}$$

$$\times \exp[-S_B(t_i) - S_\pi(t_i)],$$

$$10$$
from Figs. 2(c) and 2(d). Strickly speaking, ξ_i should not be classified as a factorizable contribution because of the irreducible soft gluons that attach the B and D mesons and the pion and D meson. Only at the lowest order of soft corrections, i.e. S_U = 0, is it a factorizable contribution. We shall consider this point, when identifying the nonfactorizable contributions below.

The exponents

\[
S_B(\mu) = s(x_1 P_1^-, b_1) + 2 \int_{1/b_1}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) ,
\]

\[
S_D(\mu) = s(x_2 P_2^+, b_2) + s((1 - x_2) P_2^+, b_2) + 2 \int_{1/b_2}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) ,
\]

\[
S_\pi(\mu) = s(x_3 P_3^-, b_3) + s((1 - x_3) P_3^-, b_3) + 2 \int_{1/b_3}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) ,
\]

(32)
correspond to the summation of the reducible radiative corrections grouped into the B meson wave function φ_B, the D meson wave function φ_D, and the pion wave function φ_π, respectively. The quark anomalous dimension γ = −α_s/π, is related to γ_φ = 2γ introduced before. The exponents

\[
S_U^{(j)}(\mu) = \int_w^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{U^{(j)}}(\alpha_s(\bar{\mu})) ,
\]

(33)
for j = e, i1, i2, and i3, correspond to the summation of the irreducible corrections grouped into the soft function U, where the lower bound w = min(1/b_i) is chosen to collect the largest soft logarithms. The wave functions satisfy the normalization

\[
\int_0^1 \phi_{B,D,\pi}(x) dx = \frac{f_{B,D,\pi}}{2\sqrt{6}} ,
\]

(34)
f_B being the B meson decay constant.

In Eqs. (27), (28), (30) and (31) the functions h’s, the Fourier transform of the lowest-order \( \hat{H} \) from Figs. 2(a)-2(d), are given by

\[
h_e(x_1, x_2, b_1, b_2, m_e) = K_0(\sqrt{x_1 x_2 m_e} b_1) \\
\times [\theta(b_1 - b_2) K_0(\sqrt{x_2 m_e} b_1) I_0(\sqrt{x_2 m_e} b_2) \\
+ \theta(b_2 - b_1) K_0(\sqrt{x_2 m_e} b_2) I_0(\sqrt{x_2 m_e} b_1)] ,
\]

(35)
\[
h_i(x_1, x_3, b_1, b_3, m_i) = h_e(x_1, x_3, b_1, b_3, m_i) ,
\]

(36)
with $m_e = M_B^2$ and $m_i = (1 - r^2)M_B^2$. The hard scales $t$'s take the maximum of all energies involved in $H$:

$$
t_e = \max(\sqrt{x_1 m_e}, \sqrt{x_2 m_e}, 1/b_1, 1/b_2)
$$
$$
t_i = \max(\sqrt{x_1 m_i}, \sqrt{x_3 m_i}, 1/b_1, 1/b_3)
$$

(37)

It is an important feature that the Sudakov form factor $e^{-s}$ exhibits a strong suppression in the large $b$ region. Hence, Sudakov suppression guarantees that the main contributions arise from large $t$ region, where the running coupling constant $\alpha_s(t)$ is small, and perturbation theory is relatively reliable.

The factorization formulas for the nonfactorizable external and internal $W$-emission amplitudes $\mathcal{M}_e$ and $\mathcal{M}_i$, respectively, contain the kinematic variables of all the three mesons. The integration over $b_3$ can be performed trivially, leading to $b_3 = b_1$ or $b_3 = b_2$. Their expressions are

$$
\mathcal{M}_e = 32\pi \sqrt{2N C_F} \sqrt{r M_B^2} \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_D(x_2) \phi_n(x_3)
$$
$$
\times \left\{ \alpha_s(t_e^{(1)}) c_2(t_e^{(1)}) \frac{N}{N} \exp[-S(t_e^{(1)})|_{b_3 = b_2} - S_U^{(e)}(t_e^{(1)})] \right\}
$$
$$
\times \left\{ [(1 - r^2)(1 - x_3) - x_1 + (r - r^2)(x_1 - x_2)] h_e^{(1)}(x_i, b_i)
$$
$$
- \alpha_s(t_e^{(2)}) c_2(t_e^{(2)}) \frac{N}{N} \exp[-S(t_e^{(2)})|_{b_3 = b_2} - S_U^{(e)}(t_e^{(2)})] \right\}
$$
$$
\times \left\{ [(2 - r)x_1 - (1 - r)x_2 - (1 - r^2)x_3] h_e^{(2)}(x_i, b_i) \right\},
$$

(38)

$$
\mathcal{M}_i = 32\pi \sqrt{2N C_F} \sqrt{r M_B^2} \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_D(x_2) \phi_n(x_3)
$$
$$
\times \left\{ \alpha_s(t_i^{(1)}) c_1(t_i^{(1)}) \frac{N}{N} \exp[-S(t_i^{(1)})|_{b_3 = b_1} - S_U^{(i)}(t_i^{(1)})] \right\}
$$
$$
\times \left\{ [x_1 - x_2 - x_3(1 - r^2)] h_i^{(1)}(x_i, b_i)
$$
$$
+ \alpha_s(t_i^{(2)}) c_1(t_i^{(2)}) \frac{N}{N} \exp[-S(t_i^{(2)})|_{b_3 = b_1} - S_U^{(i)}(t_i^{(2)})] \right\}
$$
$$
\times \left\{ [(x_1 + x_2)(1 + r^2) - 1] h_i^{(2)}(x_i, b_i) \right\},
$$

(39)

with $[dx] \equiv dx_1 dx_2 dx_3$ and $S = S_B + S_D + S_n$. The functions $h^{(j)}$, $j = 1$ and 2, appearing in Eqs. (38) and (39), are derived from Figs. 2(e) and 2(f):

$$
h_e^{(j)} = [\theta(b_1 - b_2)K_0(BM_B b_1) I_0(BM_B b_2)
$$
\[ h^{(j)}_i = \begin{cases} 
\theta(b_1 - b_2)K_0(B_Mb_1)I_0(B_Mb_2) & \text{for } B_j \geq 0 \\
\theta(b_2 - b_1)K_0(D_Mb_2)I_0(D_Mb_1) & \text{for } B_j \leq 0 
\end{cases} \times \left( \frac{i\pi}{2}H_0^{(1)}(|D_Mb_2|) \right) \]

with the variables

\[ \begin{align*}
B^2 &= x_1x_2(1 - r^2), \\
B_1^2 &= (x_1 - x_2)x_3(1 - r^2) + x_1x_2(1 + r^2), \\
B_2^2 &= x_1x_2(1 + r^2) - (x_1 - x_2)(1 - x_3)(1 - r^2), \\
D^2 &= x_1x_3(1 - r^2), \\
D_1^2 &= (x_1 - x_2)x_3(1 - r^2), \\
D_2^2 &= (x_1 + x_2)r^2 - (1 - x_1 - x_2)x_3(1 - r^2).
\end{align*} \]

Similarly, the scales \( t^{(j)}_i \) are chosen as

\[ \begin{align*}
t^{(j)}_e &= \max(B_M, |B_j|M_B, 1/b_1, 1/b_2), \\
t^{(j)}_i &= \max(D_M, |D_j|M_B, 1/b_1, 1/b_2).
\end{align*} \]

**IV. RESULTS AND DISCUSSIONS**

In the evaluation of the various form factors and amplitudes, we adopt 
\[ G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \] the decay constants 
\[ f_B = 200 \text{ MeV}, f_D = 220 \text{ MeV}, \] and 
\[ f_a = 132 \text{ MeV} \] [10], the CKM matrix elements 
\[ |V_{cb}| = 0.040 \] and 
\[ |V_{ud}| = 0.974, \] the masses 
\[ M_B = 5.28 \text{ GeV} \] and 
\[ M_D = 1.87 \text{ GeV} \] [17], the \( B^- \) meson lifetime 
\[ \tau_{B^-} = 1.68 \text{ ps} \] [18], the \( B \) and \( D \) meson wave functions [19],

\[ \phi_{B,D}(x) = \frac{N_{B,D}}{16\pi^2} \frac{x(1-x)^2}{M_{B,D}^2 + C_{B,D}(1-x)}, \]
and the Chernyak-Zhitnitsky pion wave function \[20\],

\[
\phi_\pi(x) = \frac{5\sqrt{6}}{2} f_\pi x(1-x)(1-2x)^2. \tag{45}
\]

The normalization constants \(N_B = 590.8 \text{ GeV}^3\) and \(N_D = 92.85 \text{ GeV}^3\), and the shape parameters \(C_B = -27.6 \text{ GeV}^2\) and \(C_D = -3.372 \text{ GeV}^2\) are determined by fitting the predictions from Eq. (25) to the data of the decay rates of \(B \to D^{(*)}\pi\) \[10\].

Our formalism can be applied to the decay \(D^+ \to \bar{K}^0\pi^+\) directly, which occurs through a similar effective Hamiltonian but with the four-fermion operators \(O_1 = (\bar{s}_L\gamma_\mu c_L)(\bar{d}_L\gamma_\mu u_L)\) and \(O_2 = (\bar{d}_L\gamma_\mu c_L)(\bar{s}_L\gamma_\mu u_L)\). The expression of the decay rate \(\Gamma\) is also similar to Eq. (25), but with the CKM matrix element \(|V_{cb}| = 1.0\) substituted for \(|V_{cb}|\), and the meson masses \(M_D\) and \(M_K = 0.497 \text{ GeV}\) \[21\] for \(M_B\) and \(M_D\), respectively. In all the form factors and amplitudes the kinematic variables of the \(B\) (\(D\)) meson are replaced by those of the \(D\) (\(K\)) meson. The \(D^+\) meson lifetime is \(\tau_{D^+} = 1.05 \text{ ps}\) \[21\], and the \(D\) meson wave function has been defined above. For the kaon, we adopt the decay constant \(f_K = 160 \text{ MeV}\), and the wave function \[20\],

\[
\phi_K(x) = \frac{\sqrt{6}}{2} f_K x(1-x)[3.0(1-2x)^2 + 0.4]. \tag{46}
\]

Because of the smaller \(D\) meson mass, the transverse degrees of freedom are more important in the definitions of the hard scales \(t\). Hence, we choose the maximum of the scales \(1/b_i\) for the arguments \(t\) of the Wilson coefficients. In this case Sudakov suppression is weaker, and insufficient to diminish the contributions from the region with \(t\) close to \(\Lambda_{\text{QCD}}\), where \(c_{1,2}(t)\) diverge. To have meaningful predictions, we choose \(t = \max(1/b_i, t_c)\) in the numerical analysis, \(t_c = (1 + \epsilon)\Lambda_{\text{QCD}}\), such that the Wilson coefficients are frozen at \(c_{1,2}(t_c)\) as \(\max(1/b_i) \leq t_c\). The parameter \(\epsilon = 0.0000227\) is determined from the data of the decays \(D \to K^{(*)}\pi\) \[21\].

After including the soft gluons, the factorizable external \(W\)-emission contribution \(\chi^{(f)}_e\) and the factorizable internal \(W\)-emission contribution \(\chi^{(f)}_i\) to the decay amplitude \(\mathcal{M}\) in Eq. (26) should be identified as

\[
\chi^{(f)}_e = f_\pi[(1 + r)\xi_+ - (1 - r)\xi_-],
\]
\[
\chi^{(f)}_i = f_{D(K)}\xi_i|_{SU=0}, \tag{47}
\]

14
where \(\xi_i|_{SU=0}\) denotes the lowest order internal \(W\)-emission contributions obtained from Figs. 2(a)-2(d) by turning off the soft corrections. The nonfactorizable external \(W\)-emission contribution \(\chi^{(nf)}(nf)\) and the nonfactorizable internal \(W\)-emission contribution \(\chi^{(nf)}_i\) are then

\[
\begin{align*}
\chi^{(nf)}_e &= M_e, \\
\chi^{(nf)}_i &= f_{D(K)}(\xi_i - \xi_i|_{SU=0}) + M_i.
\end{align*}
\]  
(48)

We shall show that our predictions of \(\chi_{1,2}\) are positive and negative for the \(B\) and \(D\) meson decays, respectively, consistent with the phenomenological arguments in Eq. (3) [9, 22]. However, the positive nonfactorizable contribution \(\chi_1\) for bottom decays is contrary to the QCD sum rule predictions [13, 23, 24], which we shall comment on later.

For \(r = M_D/M_B = 0.354\), we have the anomalous dimensions \(\gamma^{(e)}_U = -0.421\alpha_s/\pi, \gamma^{(i)}_U = -0.333\alpha_s/\pi, \gamma^{(i2)}_U = -0.730\alpha_s/\pi, \) and \(\gamma^{(i3)}_U = 0.413\alpha_s/\pi\). For \(r = M_K/M_D = 0.265\), we have \(\gamma^{(e)}_U = -0.396\alpha_s/\pi, \gamma^{(i1)}_U = -0.333\alpha_s/\pi, \gamma^{(i2)}_U = -0.628\alpha_s/\pi, \) and \(\gamma^{(i3)}_U = 0.196\alpha_s/\pi\). The results of the various form factors and amplitudes for the decays \(B^- \to D^0\pi^-\) and \(D^+ \to \bar{K}^0\pi^+\) are exhibited in Table I. The rows entitled by \(SU \neq 0\), whose values match the data, are derived with the soft corrections taken into account. Those by \(SU = 0\) only help to extract the nonfactorizable contributions \(\chi^{(nf)}\), and to investigate the importance of the soft corrections. Because the \(D\) meson is lighter, the scales \(t\) can run to a lower value as indicated by Eqs. (37) and (43). The exponentials \(e^{-SU}\), which basically act as enhancing factors, then amplify the contributions from this region with smaller \(t\), where the Wilson coefficients are larger. Therefore, the soft gluon effects are more important in charm decays as shown in Table I.

The factorizable external \(W\)-emission contributions are positive in both the \(B\) and \(D\) meson decays, and their magnitudes increase, after including the soft corrections with \(\gamma^{(e)}_U < 0\). \(\xi_i\) changes sign, since the Wilson coefficient \(c_2\) in \(\xi_{i1}\) becomes so negative, when evolving from the characteristic scale of the \(B\) meson decay to that of the \(D\) meson decay, that it overcomes the positive \(c_1/N_c\) in \(\xi_{i23}\). Their magnitudes also increase because of \(\gamma^{(i1)}_U, \gamma^{(i2)}_U < 0\). The real parts of the nonfactorizable amplitudes \(M_e (M_i)\) are always negative (positive) due to the negative \(c_2\) (positive \(c_1/N_c\)) in the \(B\) and \(D\) meson decays. At first sight, these observations differ from the phenomenological
extractions in Eq. (3) [9, 22]. As argued before, the nonfactorizable contribu-
tions should be appropriately identified according to Eq. (48), whose results
are listed in Table I. We have $\text{Re}(\chi^{(nf)}_i) = +0.0189$ GeV for the $B$
meson decay and $\text{Re}(\chi^{(nf)}_i) = -0.4715$ GeV for the $D$ meson decay. It is easy to
attribute the sign change of $\chi^{(nf)}_i$ to the stronger enhancement of $\xi_i$ by the
soft corrections in charm decays.

To compare our predictions with Eq. (3), we present in Table I the BSW
parameters $a_1$, $a_2$, $c_2\chi_1$, and $c_1\chi_2$, corresponding to $\chi^{(f)}_e$, $\chi^{(f)}_i$, $\chi^{(nf)}_e$, and $\chi^{(nf)}_i$, respectively. The signs, except for those of $\text{Re}(\chi^{(nf)}_e)$ and $c_2\chi_1$ associated with the $D$ meson decay, are consistent. The discrepancy between the signs
of $\text{Re}(\chi^{(nf)}_e)$ and $c_2\chi_1$ for the $D$ meson decay will be resolved, if two-loop soft
functions $U$ are considered. At this order, two soft gluons attach the valence
quark lines of, for example, the $B$ meson and the pion in Figs. 2(a) and 2(b),
for which the associated color traces do not vanish. Then the form factors
$\xi_\pm$ should be classified as being nonfactorizable, and $\chi^{(nf)}_e$ be redefined by a
similar expression to Eq. (48). It is expected based on Table I that $\text{Re}(\chi^{(nf)}_e)$
for the $B$ meson decay remains negative, while that for the $D$ meson decay changes sign.

At last, we come to the comparision of our predictions with those derived
from QCD sum rules [13]. The conclusion that the nonfactorizable contribu-
tion $\chi_1$ in bottom decays is negative holds only in the kinematic limit $M_B,$
$M_D \to \infty$ but with $\Delta M = M_B - M_D$ fixed [13]. In this limit both the $B$
and $D$ mesons are at rest, and soft corrections should be more important.
The reason is similar to that for the larger soft effects in charm decays given
before. For extremely heavy mesons, the wave functions peak at $x \to 0$ [5],
and the characteristic scales $t$ in Eq. (37) becomes smaller. Then the form
factors $\xi$ may increase by a significant factor, since the contributions from
the region with large Wilson coefficients are enhanced by the exponentials
e$^{-S_U}$. Following Eq. (48), it is possible that the sign of $\chi^{(nf)}$ for bottom
decays is reversed. However, we argue that the above kinematic condition is
inappropriate for realistic $B$ meson decays, and thus the conclusion in [13] is
in doubt.

In this paper we have not only completed the Sudakov resummation up
to next-to-leading logarithms, but also investigated the soft gluon effects in
heavy meson decays based on the three-scale PQCD factorization theorem.
By identifying the nonfactorizable contributions carefully, we are able to
explain the variation and especially the sign changes of the amplitudes in bottom and charm decays. Certainly, these subjects still need more thorough studies in order to have a full understanding of their dynamical origin.

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References


TABLE I. The various form factors and amplitudes for the decays $B^{-} \to D^{0}\pi^{-}$ and $D^{+} \to \bar{K}^{0}\pi^{+}$. The unit is $10^{-3}$ GeV.

<table>
<thead>
<tr>
<th>Decay</th>
<th>External W</th>
<th>$f_{D}\xi_{i}$</th>
<th>$M_{e}$</th>
<th>$M_{i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D\pi$</td>
<td>$S_{U} = 0$</td>
<td>106.5</td>
<td>2.5</td>
<td>$-5.8 + 19.8i$</td>
</tr>
<tr>
<td></td>
<td>$S_{U} \neq 0$</td>
<td>108.5</td>
<td>2.6</td>
<td>$-5.8 + 20.0i$</td>
</tr>
<tr>
<td>$D \to \bar{K}\pi$</td>
<td>$S_{U} = 0$</td>
<td>267.0</td>
<td>$-21.3$</td>
<td>$-18.8 + 19.0i$</td>
</tr>
<tr>
<td></td>
<td>$S_{U} \neq 0$</td>
<td>1075.0</td>
<td>$-529.3$</td>
<td>$-18.5 + 13.6i$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>$\chi_{(a)1}^{(f)}$</th>
<th>$\chi_{(a)2}^{(f)}$</th>
<th>$\chi_{(c)1}^{(nf)}$</th>
<th>$\chi_{(c)2}^{(nf)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D\pi$</td>
<td>108.5</td>
<td>2.5</td>
<td>$-5.8 + 20.0i$</td>
<td>$18.9 - 11.0i$</td>
</tr>
<tr>
<td></td>
<td>(1.03)</td>
<td>(0.11)</td>
<td>(-0.01)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$D \to \bar{K}\pi$</td>
<td>1075.0</td>
<td>$-21.3$</td>
<td>$-18.5 + 13.6i$</td>
<td>$-471.5 - 30.3i$</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(-0.1)</td>
<td>(0.19)</td>
<td>(-0.45)</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1. (a) $O(\alpha_s)$ factorization of infrared and hard contributions. (b) $O(\alpha_s)$ factorization into a “harder” function, a soft function and a hard decay subamplitude.

Fig. 2. Lowest-order diagrams for the hard subamplitude.

Fig. 3. Soft gluon corrections to Fig. 2.