A unified BFKL/DGLAP description of Deep Inelastic Scattering

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Abstract. We introduce a coupled pair of evolution equations for the unintegrated gluon distribution and the sea quark distribution which incorporate both the resummed leading \( \ln(1/x) \) BFKL contributions and the resummed leading \( \ln(Q^2) \) DGLAP contributions. We solve these unified equations in the perturbative QCD domain. With only two physically motivated parameters we obtain an excellent description of the HERA \( F_2 \) data.

1. INTRODUCTION

One of the striking features of the measurements of deep inelastic scattering at HERA is the strong rise of the proton structure function \( F_2 \) as \( x \) decreases from \( 10^{-2} \) to below \( 10^{-4} \) [1]. At first sight it appeared that the rise was due to the (BFKL) resummation of leading \( \ln(1/x) \) contributions. In this approach the basic dynamical quantity at small \( x \) is the gluon distribution \( f(x, k_T^2) \) unintegrated over its transverse momentum \( k_T \). Observables are computed in terms of \( f \) via the \( k_T \) factorization theorem. For example

\[
F_2 = F_2^{\gamma g} \otimes f \quad \text{where} \quad F_2^{\gamma g} = \sum_q e_q^2 B_q
\]

and where \( \otimes \) denotes a convolution in transverse \( (k_T) \), as well as longitudinal, momentum, see Fig. 1(a). \( F_2^{\gamma g} \) is the off-shell gluon structure function which at lowest order is given by the “quark box and crossed-box” contributions, \( \gamma g \rightarrow q\bar{q} \). On the other hand the conventional DGLAP approach is based on collinear factorization

\[
\frac{\partial F_2}{x \partial \ln(Q^2/\Lambda^2)} = \sum_i e_i^2 P_{qg} \bar{g} + \sum_i e_i^2 P_{qg} \bar{q} (q_i + \bar{q}_i)
\]
where $\otimes$ is simply a convolution over longitudinal momentum. DGLAP evolution effectively sums up the leading $\ln Q^2$ contributions, and is able to describe the $F_2$ data at the smallest $x$ values observed (even for $Q^2 \sim 1\text{GeV}^2$) with an appropriate choice of input parton distributions. These non-perturbative input shapes in $x$ mean that there is more freedom in the pure DGLAP description than in the BFKL approach. In the latter approach, the shape in $x$ emerges, in principle, from the perturbative $\ln(1/x)$ resummation. In practice it is not so clear cut since non-perturbative, and also subleading $\ln(1/x)$, effects can modify the BFKL prediction. Here we should attempt to find a unified description of $F_2$ which incorporates both (DGLAP and BFKL) of these perturbative effects.

2. UNIFIED BFKL/DGLAP FORMALISM

We argue that the unintegrated gluon distribution $f(x, k_T^2)$ and the $k_T$ factorization theorem provides the natural framework for describing observables at small $x$. To determine $f$ we arrange the BFKL equation so that we only need to solve it in the perturbative domain $k_T^2 > k_0^2$ [2]. We also include the residual DGLAP contributions. To be precise we have

\[
f(x, k_T^2) = \frac{\alpha_s}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z} g(z, k_0^2) + \]

\[
+ \frac{3\alpha_s}{\pi} k_T^2 \int_x^1 \frac{dz}{z} \int_{k_0^2}^{k_T^2} \frac{dk_1^2}{k_1^2} \left[ f\left(\frac{z}{x}, k_1^2\right) \theta\left(\frac{k_T^2}{z} - k_1^2\right) - f\left(\frac{z}{x}, k_T^2\right) \right] + \frac{f\left(\frac{x}{z}, k_T^2\right)}{(4k_T^2 + k_0^2)^{1/2}} \]

\[
\left[ \frac{k_T^2}{k_1^2} - \frac{k_0^2}{k_1^2} \right] \left[ \frac{4k_T^2}{k_1^2} + k_0^2 \right] \right]\]
\[
+ \frac{3\alpha_s}{\pi} \int_x^1 dz \left( \frac{P_{gg}(z)}{6} - \frac{1}{z} \right) \int_{k_0^2}^{k_T^2} \frac{dk_T^2}{k_T^2} f \left( \frac{x}{z}, k_T^2 \right) + \frac{\alpha_s}{2\pi} \int_x^1 dz P_{gq} \sum \left( \frac{x}{z}, k_T^2 \right)
\]

(3)

where \(-1/z\) is taken from DGLAP because it is already included in BFKL. The input term comes from two sources: the \(k_T^2 < k_0^2\) parts of BFKL and DGLAP terms. We specify the input in terms of a simple two parameter form \(g(x, k_0^2) = N(1 - x)^\beta\). In addition to restricting the solution of the BFKL equation to the perturbative region \(k_T^2 > k_0^2\) and to including the DGLAP terms, we have also introduced a \(\theta\) function which imposes the constraint \(k_T^2 < k_0^2/z\) on the real gluon emissions. The origin of this constraint is the requirement that the virtuality of the exchanged gluon is dominated by its transverse momentum \(|k'|^2 \simeq k_T^2\). We take a running coupling \(\alpha_s(k_T^2)\), which is supported by the results of the next-to-leading order \(\ln(1/x)\) analyses of Fadin, Lipatov, Camici and Ciafaloni. The final term in (3) depends on the quark singlet momentum distribution \(\Sigma\). At small \(x\) the sea quark components \(S_q\) of \(\Sigma\) dominate. They are driven by the gluon via the \(g \rightarrow q\bar{q}\) transitions, that is \(S_q = B_q \otimes f\) where at lowest order \(B_q\) is the box (and crossed box) contribution indicated in Fig. 1(a). Besides the \(z\) and \(k_T^2\) integrations symbolically denoted by \(\otimes\) the box contribution implicitly includes an integration over the transverse momentum \(\kappa_T\) of the exchanged quark. The evolution equation for \(\Sigma\) may be written in the form

\[
\Sigma = S^{(0)} + \sum_q B_q(k_T^2 = 0) \otimes z g(z, k_0^2) + \sum_q B_q \otimes f + P_{qq} \otimes S_q + V
\]

(4)

where the first three terms on the right hand side are the “\(B_q \otimes f\)” contributions coming from the regions I, II, III of the \(k_T^2\) and \(\kappa_T^2\) integrations that are shown in Fig. 1(b). First, in the non-perturbative domain, region I, the \(u, d, s\) sea quark contribution is parametrized in the form \(S^{(0)} = \mathcal{C}_P x^{-0.08}(1 - x)^8\) consistent with soft pomeron and counting rule expectations, where \(\mathcal{C}_P\) is independent of \(Q^2\). The constant \(\mathcal{C}_P\) is fixed in terms of the two parameters, \(N\) and \(\beta\), by the momentum sum rule. In region II we apply the strong \(k_T\) ordering approximation with \(B_q \approx B_q(k_T^2 = 0)\) so that the \(k_T^2\) integration can be carried out to give a contribution proportional to \(g(x/z, k_0^2)\). Finally in region III we evaluate the full box contribution; this gives the main contribution and is responsible for the rise of \(F_2\) with decreasing \(x\). The last two terms in (4) give the sea \(\rightarrow\) sea evolution contribution, and the valence contribution \(V(x, Q^2)\) which is taken directly from a recent parton set. The charm quark

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1) A more general treatment of the gluon ladder which incorporates both the BFKL equation and DGLAP evolution is given by the CCFM equation [3], which is based on angular ordering of the gluon emissions. The angular ordered and kinematic constraints lead to similar subleading \(\ln(1/x)\) effects, but the kinematic constraint overrides the angular ordered constraint, except when \(Q^2 < k_T^2\) in the large \(x\) domain [4].
component of the sea is given totally by perturbative QCD, since for $k_T^2 < k_0^2$ the box $B(k_T^2 = 0)$ is finite as $\kappa_T^2 \to 0$ due to $m_c \neq 0$.

3. DESCRIPTION OF $F_2$ DATA AND DISCUSSION

We solve the coupled integral equations (3) and (4) for the gluon $f$ and the quark singlet $\Sigma$ in the perturbative domain, $k_T^2 > k_0^2$. The only input is the gluon $g(x, k_0^2)$. We take $k_0^2 = 1\text{GeV}^2$. We determine the values of the two input parameters by fitting to the available data [1] for $F_2$ with $x < 0.05$ and $Q^2 > 1.5\text{GeV}^2$. The continuous curves in Fig. 2 show the description of a sample of the data. Overall the fit is excellent; at least as good as that achieved in the recent global analyses. When we repeat the analysis with the kinematic constraint omitted we see that the description (given by the dashed curves in Fig. 2) is not so good and, moreover, the extrapolation of the gluon to $x \approx 0.4$ no longer describes the WA70 prompt photon data. How important are the $\ln(1/x)$ effects? First we replace the BFKL kernel in (3) by the standard DGLAP splitting function, $P_{gg}$. We find that the gluon is not changed by much in the HERA domain $x \gtrsim 10^{-4}$, compare the dashed and dotted curves in Fig. 3. This effect is well known. In the power series expansion of the gluon anomalous dimension in $\alpha_s/\omega$ the coefficients of the 2nd, 3rd and 5th terms are zero, where $\omega$ is the moment variable. On the other hand, when we use pure DGLAP evolution for the quark singlet, as well as the gluon, the difference is pronounced; compare the dashed and dot-dashed
curves in Fig. 3.

We would like to conclude that we have a theoretically well-grounded and consistent formalism which, with the minimum of non-perturbative input, is able to give a good perturbative description of the observed structure of $F_2$. Moreover the BFKL/DGLAP components of $F_2$ are decided by dynamics. In this way we have made a determination of the universal gluon distribution $f(x, k_T^2)$ which can be used, via the $k_T$ factorization theorem, to predict the behaviour of other observables at small $x$. The predictions for $F_2$ (charm) and $F_L$ can be found in [2].

REFERENCES