CP–VIOLENTING PARAMETERS FOR NEUTRAL B–MESONS
AND THEIR COMPLETE MEASUREMENT

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Phenomenological CP-violating parameters in decays of neutral B-mesons are
discussed with special attention to the degree of their measurability. Important role
of the sign of \( \Delta m_B \) is emphasized. We briefly describe how it could be determined
experimentally.

1 Introduction

Twenty years ago the statement

- The origin of CP-violation will not be established till its manifestations
  are known only for neutral kaons

could be considered as somewhat heretical. Now it and one more statement:

- The most promising testing ground for detailed studies of CP-violation
  is provided by decays of neutral B-mesons,

became generally accepted (see, e.g., reviews 2,3,4).

As a result, many papers have been devoted to discussion of B-meson decay
modes favorable for CP-violation searches and to experimental manifestations
of possible sources of the violation (see, e.g., references in reviews 2,3,4). A more
straightforward problem, degree of measurability of phenomenological parameters
describing CP-violation in B-meson decays, has not been considered. One possible reason
could be a close similarity of neutral B-mesons to neutral kaons. However, heavier masses of the third quark generation produce various
differences, sometimes rather essential, in the meson decay properties. Here
we discuss basic CP-violating parameters for neutral B-mesons with special
attention to the question how one could achieve their complete measurement.
The presentation is essentially based on the papers 5,6.
2 Standard $CP$–violating parameters

Time evolution of neutral $B$-mesons is known to be determined by the two eigenstates

\[ B_\pm = \frac{1}{\sqrt{2(1 + |\varepsilon_B|^2)}} \left[ (1 + \varepsilon_B)B^0 \pm (1 - \varepsilon_B)\overline{B^0} \right]. \]  

(1)

If we apply the phase convention $\overline{B^0} = (CP)B^0$, exact $CP$-conservation would imply $\varepsilon_B = 0$ and the states $B_\pm$ having the definite $CP$-parities equal $\pm 1$. Generally, they are eigenstates of an effective (non-Hermitian) Hamiltonian. Since $\varepsilon_B$ changes under rephasing $B^0$ and $\overline{B^0}$ (without influencing $B_\pm$), it cannot be measurable itself. Only $\frac{1 + \varepsilon_B}{1 - \varepsilon_B}$ is rephasing-invariant and admits measurement in experiment. The value

\[ \delta_B = \frac{|1 + \varepsilon_B|^2 - |1 - \varepsilon_B|^2}{|1 + \varepsilon_B|^2 + |1 - \varepsilon_B|^2} = \frac{2\text{Re}\varepsilon_B}{1 + |\varepsilon_B|^2}, \]  

(2)

directly similar to the quantity $\delta_K$ for neutral kaons, may be considered as the measure of $CP$-violation in $B^0\overline{B^0}$ mixing.

The Standard Model leads to an extremely small, really unmeasurable, value of $\delta_B$ (see, e.g., discussion in Ref. 7), much smaller than $\delta_K$. More promising are studies of decays

\[ B^0(\overline{B^0}) \rightarrow f \]  

(3)

with final states $f$ of definite $CP$-parities 7,8. To measure $CP$-violation in a particular decay mode one can use deviation of the parameter

\[ \lambda_{B}^{(f)} = \frac{1 - \varepsilon_B}{1 + \varepsilon_B} \cdot \frac{\langle f | \overline{B^0} \rangle}{\langle f | B^0 \rangle}, \]  

(4)

from the $CP$-parity value of the state $f$. Any $\lambda_{B}^{(f)}$ is rephasing-invariant and, hence, its complete measurement (i.e., of both the absolute value and phase) should be possible.

Parameters $\lambda_B$ are similar to analogous parameters $\lambda_K$ in nonleptonic kaon decays which are equivalent to more familiar parameters $\eta$. Both sets satisfy many relations if $CPT$ is conserved (see discussion in Ref. 6). Unitarity gives one more relation for each set 9. Those relations, together with the fact that a single partial width of neutral kaons (for the mode $K^0 \rightarrow (2\pi)_{I=0}$) is more than 3 orders above any other, provide the known structure of kaon decays
with only one parameter of $CP$-violation being independent and large enough for measurement. An essential preference of $B$-physics is that having many decays with more comparable probabilities it can reveal many independent and measurable $CP$-violating parameters.

Standard calculations for the decay (3) with the initially pure $B^0$-meson lead to the time distribution

$$W^{(f)}_B(t) \sim \left| \frac{1 + \lambda^{(f)}_B}{2} \right|^2 \exp(-\Gamma_+ t) + \left| \frac{1 - \lambda^{(f)}_B}{2} \right|^2 \exp(-\Gamma_- t)$$

$$+ \exp\left( -\frac{\Gamma_+ + \Gamma_-}{2} t \right) \left( 1 - \left| \frac{\lambda^{(f)}_B}{2} \right|^2 \cos \Delta m_B t - \text{Im} \lambda^{(f)}_B \sin \Delta m_B t \right)$$

(5)

which exhibits degree of measurability of parameters $\lambda_B$. Here we denote $\Delta m_B = m_+ - m_-$; $m_+ , \Gamma_+$ and $m_- , \Gamma_-$ are the mass and width of the corresponding state $B_{\pm}$. Eq.(5) has the same structure as, e.g., decay yield of $K^0(t) \to \pi\pi$. The first two terms are contributions of the eigenstates $B_{\pm}$, the last two terms describe their interference.

Distribution (5) contains contributions of $|\lambda^{(f)}_B|^2$, $\text{Re} \lambda^{(f)}_B$ and $\text{Im} \lambda^{(f)}_B$ multiplied by different functions of time. So, at first sight, all the three quantities can be easily extracted if the distribution is found experimentally with necessary accuracy. It is just the case in two-pion decays of neutral kaons where parameters $\lambda^{(\pi\pi)}_K$ have been completely measured indeed.

But $\text{Re} \lambda^{(f)}_B$ would not appear at all in distribution (5) if $\Gamma_+$ and $\Gamma_-$ coincided. So, a very small expected difference of $\Gamma_+$ and $\Gamma_-$, contrary to neutral kaons, may prevent direct measurement of $\text{Re} \lambda^{(f)}_B$. Indirect measurement is still possible, of course, since $|\text{Re} \lambda^{(f)}_B|$ can be calculated from $|\lambda^{(f)}_B|$ and $|\text{Im} \lambda^{(f)}_B|$, and the sign of $\text{Re} \lambda^{(f)}_B$ may be fixed by choosing approximate $CP$-parities of the neutral $B$ eigenstates (detailed discussion of the procedure and its relation to experiments see in Refs. 5,6).

The situation for $\text{Im} \lambda^{(f)}_B$ is less simple. Eq.(5) contains it multiplied by $\sin \Delta m_B t$, which sign is still unknown since only $|\Delta m_B|$ has been measured. For kaons, special experiments on $K_S$ regeneration in several plates have allowed to measure the sign of $\Delta m_K = m_L - m_S$ in respect to the known sign of the regeneration phase (collection of results see in Ref. 10). Similar experiments for $B$-mesons are impossible because of too small lifetime. However, without knowing the sign of $\Delta m_B$ one cannot achieve the complete measurement of $CP$-violating parameters in neutral $B$-meson decays. Below we discuss how to find the sign on the base of the method suggested in Ref. 5.
3 Unusual properties of heavy meson decays

Specific feature of neutral $B$-mesons, having no analogues for neutral kaons, is the existence of decays

$$B^0 (\bar{B}^0) \to f K^0 (\bar{K}^0) ,$$

with $f$, again, being definite $CP$-parity states. They are mainly induced by the quark decay $b \to c \bar{c} s$. The most popular final state of such a kind is $J/\psi K^0 (\bar{K}^0)$. Decays (6) generate a new set of $CP$-violating parameters

$$\chi_{BK}^{(f)} = \frac{1 - \varepsilon_B}{1 + \varepsilon_B} \cdot \frac{1 + \varepsilon_K}{1 - \varepsilon_K} \cdot \frac{\langle f \bar{K}^0 | B^0 \rangle}{\langle f K^0 | B^0 \rangle} ,$$

similar to (4). They are invariant under rephasing of both $B$ and $K$ mesons and should also be completely measurable.

However, the most interesting and unique property of decays (6) is the coherence of neutral $B$ and neutral $K$ evolutions [5]. It leads to double flavor oscillations which, as was recently emphasized [11], are similar to the long-known EPR effect [12].

Let us consider, as an example, the cascades

$$B^0 (\bar{B}^0) \to J/\psi K^0 (\bar{K}^0) , \quad K^0 (\bar{K}^0) \to \pi \pi (\pi \mp l^\pm \nu (\bar{\nu})) .$$

Coherence arises here since only transitions $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$ are possible. As a result, kaon evolution is an immediate continuation of $B$-evolution (though they do not coincide, of course). This produces unusual properties of such cascades [5]. E.g., their double-time distributions over $B$ and $K$ decay times $t_B$ and $t_K$ are non-factorisable. What is most essential for our present purposes is their sensitivity to the sign of $\Delta m_B$ in respect to the known sign of $\Delta m_K$. Corresponding terms in time distributions are generated by interference at both stages of the evolution (i.e., we need interference between both $B_+, B_-$ and $K_S, K_L$).

Manifestations of the sign of $\Delta m_B$ have been considered in more detail for the B-factory [13] and LHC [6] environments. Necessary experiments require very high statistics. Indeed, from available data [14] we find that any of cascades (8), appended by decays $J/\psi \to l^+ l^-$, has small $(Br)_{eff} \approx 5 \cdot 10^{-5}$. Due to necessity of $K_S, K_L$ interference the sign effect always contains an additional small factor of order $10^{-3}$ ($CP$-violation in two-pion kaon decays, or small semileptonic branching ratios of $K_S$). Required statistics seems to be unreachable at the projected B-factories. LHC may be promising, but statistics of LHC-B at moderate luminosity [15] also looks insufficient [6]. Effect of the sign of $\Delta m_B$
could be searched for either by other detectors at LHC or by LHC-B working at full luminosity. Accurate consideration shows as well that semileptonic kaon decays in cascades (8) might be more favorable than two-pion ones, but detailed studies of experimental efficiencies are still necessary.

In summary, we have demonstrated that $CP$-violating parameters in neutral $B$-meson decays can be completely measured only if the sign of $\Delta m_B$ is known. This sign might be determined in special experiments, e.g., at LHC.

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References