The Heavy Quark Parton Oxymoron
– A mini-review of Heavy Quark Production theory in PQCD

Wu-Ki Tung

Department of Physics and Astronomy, Michigan State University
E. Lansing, Michigan 48824, USA

Abstract. Conventional perturbative QCD calculations on the production of a heavy quark “H” consist of two contrasting approaches: the usual QCD parton formalism uses the zero-mass approximation ($m_H = 0$) once above threshold, and treats $H$ just like the other light partons; on the other hand, most recent “NLO” heavy quark calculations treat $m_H$ as a large parameter and always consider $H$ as a heavy particle, never as a parton, irrespective of the energy scale of the physical process. By their very nature, both these approaches are limited in their regions of applicability. This dichotomy can be resolved in a unified general-mass variable-flavor-number scheme, which retains the $m_H$ dependence at all energies, and which naturally reduces to the two conventional approaches in their respective region of validity. Recent applications to lepto- and hadro-production of heavy quarks are briefly summarized.

1 Introduction

The production of heavy quarks in photo-, lepto-, and hadro-production processes has become an increasingly important subject of study both theoretically and experimentally. For a comprehensive review and references, see Ref. [?]. The theory of heavy quark production in perturbative Quantum Chromodynamics (PQCD) is considerably more subtle than that of light parton (jet) production because of the additional scale introduced by the quark mass. Let us consider the production of a generic heavy quark, denoted by $H$, with non-zero mass $m_H$, in high energy interactions. For definiteness and simplicity, unless otherwise stated, we shall use deep inelastic lepton-hadron scattering as the talking example. A reasonable criterion for a quark to be called “heavy” is $m_H \gg \Lambda_{QCD}$, so that perturbative QCD is applicable at the scale $m_H$. Thus, conventionally, \{c, b, t\} quarks are regarded as heavy.

The relevant energy scales of this problem are: (i) a typical small scale such as $\Lambda_{QCD}$ or masses of light mesons, nucleons, ...; (ii) the highest energy scale $E$ or $\sqrt{s}$; (iii) a typical...
large scale in the physical process, such as $p_t$ of the heavy quark (or the associated heavy flavor hadron), $Q$ of deep inelastic scattering or Drell-Yan processes, or some large mass (such as $m_W, Z, Higgs, SU(2)$) – to be denoted henceforth collectively as $Q$; and (iv) the heavy quark mass $m_H$. By definition, $m_H \gg \Lambda_{QCD}$; and we need $\sqrt{s}$ to be fairly large compared to $m_H$ for the production cross-section to be substantial. Thus the important ratio of scales remaining which determines the physics of the heavy quark production process is that between $m_H$ and $Q$.\footnote{In this talk, we shall not consider “small-$x$” problems associated with logarithms of the large ratio $\sqrt{s}/Q$, cf \cite{7}.} We shall be mainly concerned with $c$ and $b$ quarks for which this ratio can vary over a wide range in practice.

2 Two Contrasting Conventional Approaches

The two conventional approaches to heavy quark production in PQCD can be summarized by the following contrasting master equations used in the calculation\footnote{If a final state particle $C$ is observed, the factorization formula should also contain a fragmentation function $\mathcal{F}(z,\mu)$. We leave out $\mathcal{F}(z,\mu)$ here only for simplicity of discussion. All statements concerning the parton distributions also apply to the fragmentation functions, if present.}

\begin{align}
\text{ZM-VFN: } \sigma_{IA \to CX} &= \sum_{a = \text{all active partons}} f^a_A(x_a, \mu) \otimes \hat{\sigma}_{la \to CX}(\hat{s}, Q, \mu) \bigg|_{m_a \to 0} \\
\text{FFN: } \sigma_{IA \to HX} &= \sum_{a = \text{light partons only}} f^a_A(x_a, \mu) \otimes \hat{\sigma}_{la \to HX}^{FFN}(\hat{s}, Q, m_H, \mu)
\end{align}

The Zero-mass Variable-flavor-number (ZM-VFN) scheme formula, Eq.\footnote{In this talk, we shall not consider “small-$x$” problems associated with logarithms of the large ratio $\sqrt{s}/Q$, cf \cite{7}.}, is used routinely in most high energy calculations: in global analyses of parton distributions, from EHLQ \cite{5} to MRS \cite{5} and CTEQ \cite{5}, as well as in all analytic or Monte Carlo programs for generating SM and new physics cross-sections. In this equation, the parton label $a$ is summed over all possible active parton species; $\mu$ is the factorization and renormalization scale; and $\hat{\sigma}_{la \to CX}$ is the perturbatively calculable hard cross section involving partons only. “Active” partons include all quanta which can participate effectively in the dynamics at the relevant energy scale $\mu (\sim Q)$ \cite{5,5,5}, including charm and bottom quarks at current collider energies. Thus, the active flavor number $n_f$ depends on the energy scale ("resolving power") of the problem; it is not fixed at any particular value. The hard cross-section $\hat{\sigma}_{la \to CX}(\hat{s}, Q, \mu)$ is calculated in the limit of zero mass for all the partons, and it is made infra-red safe by dimensional regularization in the $\overline{\text{MS}}$ scheme – hence the name Zero-mass Variable-flavor-number (ZM-VFN) scheme.

The advantage of the ZM-VFN scheme is that it is quite simple to implement. For the light partons $a = \{g, u, d, s\}$, $m_a \to 0$ is a valid approximation for all hard scale $Q$ (since, by definition, $Q \gg m_a$). But for a heavy quark $H$, it is a reasonable approximation only in the high energy regime $\mu (\sim Q) \gg m_H$; and it clearly becomes unreliable in the intermediate region $Q \sim \mathcal{O}(m_H)$.\footnote{In this talk, we shall not consider “small-$x$” problems associated with logarithms of the large ratio $\sqrt{s}/Q$, cf \cite{7}.}
In contrast to the above, the fixed-flavor-number (FFN) scheme, Eq. ?? has been used in most recent fixed-order perturbative calculations of heavy quark production [?,?,?]. In this scheme, by definition, only light partons (e.g., u, d, s and g for charm production) are included in the initial state: the number of parton flavors \( n_f \) is kept at a fixed value regardless of the energy scales involved \( (n_f = 3, 4 \text{ for } c, b \text{ production respectively}). \) The main feature here is: \( H \) is pictured as a heavy particle - much in the same way as \( W, Z, \) and other new heavy particles, and very different from the zero-mass light partons; hence the mass \( m_H \) is kept exactly in the hard cross-section \( \sigma_{ab \rightarrow HX}^{FFN}(\hat{s}, Q, m_H, \mu). \) Typically, the perturbative \( \sigma_{ab \rightarrow HX}^{FFN}(\hat{s}, Q, m_H, \mu) \) will contain logarithm factors of the form \( a_s^2(\mu) \ln^{n-k-m}(Q/m_H) \ln^m(\hat{s}/Q^2). \) If \( Q \sim m_H \) (and \( x \sim Q^2/s \) is not too small), these factors are under control; and we have effectively a one large scale hard process. Hence, the FFN scheme is the natural scheme to use in the energy region \( Q \sim m_H \) - this is precisely where the ZM VFN scheme is expected to be inappropriate.

From the heavy particle perspective, this approach also has the advantage of being conceptually simple, even if the NLO calculation requires considerable amount of work. However, the sharp distinction drawn between the \( H \) quark and the other light quarks, say between \( c \) and \( s \), in this formalism appears quite unnatural as the hadron system is probed at the scale \( \mu \sim Q \) available in current high energy processes. And it has been known since the next-to-leading order (NLO) calculations in the FFN scheme were completed [?,?] that, for both charm and bottom production, there are two disconcerting features about the results: (i) the NLO corrections turn out to be of the same numerical magnitude as (in fact, generally larger than) the leading order (LO) result; and (ii) the uncertainty of the theoretical calculation, as measured by the dependence of the calculated cross section on the unphysical scale parameter \( \mu \), is as large in NLO as in LO - contrary to what is expected from a good perturbation expansion [?]. These features mean that the truncated perturbative series in this scheme has left out important physics effects. Experimentally, it is also known that the measured charm and bottom production cross sections do not agree with the NLO theoretical predictions, at least in the overall normalization, even when the scale \( \mu \) is allowed to vary within a reasonable range [?].

This situation may not be all that surprising: for \( c \) and \( b \) quarks, the condition \( Q \sim m_H \) is not well satisfied in most practical cases. In fact, current experimental ranges for lepto-and hadro-production of these heavy flavors mostly lie in a region between those appropriate for the ZM VFN \( (Q \gg m_H) \) and FFN \( (Q \sim m_H) \) schemes. We need a well-defined theory which applies over the full \( Q \) range! Other possible sources for these problems are: (i) large corrections due to large logarithms of \( (s/Q^2) \)—the small-\( x \) problem [?]; (ii) inadequate understanding of the hadronization of heavy quarks in comparing PQCD calculations with experiment; and (iii) existence of non-perturbative components of \( H \) inside the nucleon which are, by definition, excluded by the FFN scheme. In this talk, we shall concentrate on physics issues pertaining to the changing role of the heavy quark \( H \) over the full \( Q \) range. It is particularly interesting because the interplay between the two independent scales \( m_H \) and \( Q \) embodies much interesting QCD physics which is amenable to precise treatment.
When the energy scale becomes large, $Q/m_H \gg 1$, the FFN scheme becomes suspect because large logarithm factors of $\ln(Q/m_H)$ in the hard cross section $\sigma_{ab \rightarrow HX}^{FFN}$ becomes increasingly singular, and higher-order terms containing higher powers of the same can no longer be omitted. In other words, the truncated perturbation series in this scheme can become rather unreliable as $Q$ becomes large. The clue for addressing this problem is already contained in Eq.\ref{master}:

$$\ln^{-k}(Q/m_H)$$ in the hard cross section $\sigma_{ab \rightarrow HX}^{FFN}$ can be resummed to all orders in $\alpha_s$ to become the parton distribution $f_A^H(x, \mu)$ (evolved to $(k+1)$-loops). The $H$ parton should be included in the sum over parton flavors; it participates in the hard scattering on the same footing as the other partons. After removing these potentially dangerous logarithm terms, the remaining hard cross section $\sigma_{ab \rightarrow HX}$ becomes infra-red safe as $Q/m_H \rightarrow \infty$. It is important to note, however, the resummation of large $\ln(Q/m_H)$ logarithms does not require taking the $m_H \rightarrow 0$ limit for the remaining ("mass-subtracted") hard cross-section as is done in the conventional ZM VFN formulation, Eq.\ref{master}. In fact, by retaining the $m_H$ dependence in the mass-subtracted (hence infra-red safe) $\sigma_{ab \rightarrow HX}(s, Q, m_H, \mu)$, one arrives at a consistent theory for heavy quark production which is valid over the entire energy range from $Q \lesssim m_H$ to $Q \gg m_H$:\footnote{Factorization of any applicable fragmentation functions is implicitly assumed.}

$$\sigma_{1A \rightarrow CX}(s, Q, m_H) = \sum_{a = \text{all active partons}} f_A^a(x_a, \mu) \otimes \sigma_{la \rightarrow CX}(s, Q, m_H, \mu)$$

A program to systematically implement this intuitive physical picture has been developed in a series of papers in [\ref{1}, \ref{2}, \ref{3}, \ref{4}]. The resulting formalism constitutes a natural generalization of the conventional zero-mass QCD parton formalism to correctly include general quark mass effects, hence will be called the general-mass variable-flavor-number (GM-VFN) scheme. (In some recent literature it has also been called the ACOT scheme, Ref.\cite{5}.)

More precisely, this formalism is based on a well defined renormalization scheme \cite{5} which provides a natural transition from the threshold region $Q \sim \mathcal{O}(m_H)$ to the high energy region $Q \gg m_H$; and the validity of the generalized factorization theorem can be established order-by-order in perturbation theory \cite{5}. The key points are:

- the renormalization scheme is a composite of two simple schemes, natural for $Q \lesssim m_H$ and $Q \geq m_H$ respectively, with matching conditions that make the schemes equivalent in the domain of overlap $Q \sim m_H$ where they are equally valid for practical low order calculations \cite{5};
- one scheme utilizes a subtraction procedure (BPHZ) which leads to manifest decoupling of the heavy particle in the region $Q \ll m_H$, thereby gives precise meaning to the FFN scheme (with no heavy quark partons);
- the other scheme is ordinary $\overline{MS}$ as regards the definition of the coupling $\alpha_s(\mu)$ and the parton densities $f_A^a(x, \mu)$, hence retains the normal ($m_H = 0$) evolution equations