Simple Gauge-mediated Models with Local Dynamical Supersymmetry Breaking

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Abstract

We describe a simple class of supersymmetric gauge theories that can act as supersymmetry-breaking sectors for gauge-mediated supersymmetry breaking. The models have a local supersymmetry-breaking minimum along a direction in field space where a singlet gets a large expectation value. The potential along this direction has a runaway behavior stabilized by supersymmetry breaking in the effective low-energy theory. The supersymmetric vacua are at infinite field values, and cosmological bounds on false vacuum decay are easily satisfied. The models have no dimensionful parameters, and all mass scales arise through strong coupling dynamics. Simple variants of the model are compatible with perturbative unification, can naturally have dynamical supersymmetry breaking at a scale as low as 10 TeV, and can solve the $R$ axion problem without appealing to Planck-scale effects.

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Gauge-mediated supersymmetry breaking is an interesting alternative to traditional supergravity-mediated supersymmetry breaking scenarios that has received renewed attention recently [1, 2]. Many explicit models of gauge-mediation have appeared in the literature [2], but most models are very complicated and may best be regarded as existence proofs. (For recent progress in model-building, see Refs. [3, 4, 5].) In this paper, we construct a simple class of gauge-mediated models that naturally generate a local supersymmetry-breaking minimum for a singlet. These models have a classical flat direction along which a singlet $S$ gets a large vacuum expectation value, and many fields get massive. The flat direction is lifted by two competing quantum effects: gaugino condensation in one group factor generates a runaway dynamical superpotential for $S$, while dynamical supersymmetry breaking in another group factor stabilizes the potential. These models have many similarities with the elegant models recently constructed by Dimopoulos, Dvali, Rattazzi, and Giudice [5], which utilize a singlet direction which is flat at tree level and is stabilized perturbatively by the Coleman–Weinberg mechanism.

To illustrate the idea of the present class of models, consider a theory with symmetry group

$$G = SU(N)_P \times SU(5)_B \times [SU(K) \times SU(K')] ,$$

where the group factors on the left are gauge symmetries, and the group in brackets is a global symmetry. (We will eventually gauge part of the global symmetry with the electroweak gauge group.) The matter content is

\begin{align*}
    P & \sim (\Box, 1) \times (\Box, 1), \\
    \bar{P} & \sim (\Box, 1) \times (\Box, 1), \\
    Q & \sim (1, \Box) \times (1, \Box), \\
    \bar{Q} & \sim (1, \Box) \times (1, \Box), \\
    T & \sim (1, \Box) \times (1, 1), \\
    A & \sim (1, \Box) \times (1, 1), \\
    S & \sim (1, 1) \times (1, 1).
\end{align*}

The theory has a superpotential

$$W = \lambda S \text{tr}(P \bar{P}) + \kappa S \text{tr}(Q \bar{Q}),$$

where the trace is over the flavor indices. This is the most general renormalizable superpotential consistent with the symmetries given above together with a $U(1)_R$
symmetry under which $S$ has charge +2, so this theory is strongly natural in the sense of ’t Hooft. As we will see, the role of the $SU(N)_P$ gauge group is to push $S$ away from the origin, and the role of $SU(5)_B$ is to break supersymmetry. The couplings of the two gauge factors may be defined by the scales $\Lambda_P$ and $\Lambda_B$ (analogous to $\Lambda_{QCD}$) where the perturbative gauge couplings become strong. We will assume that $SU(N)_P$ is stronger than $SU(5)_B$, that is, $\Lambda_P \gg \Lambda_B$.

This theory has a classical flat direction where $S$ is nonzero and all other fields vanish. We will show that the theory has a local minimum for large values of $S$ along this direction. For $S \neq 0$, the fields $P, \bar{P}$ and $Q, \bar{Q}$ become massive. We will write a low-energy effective theory for the fields that are massless at the classical level. These include the $SU(N)_P \times SU(5)_B$ gauge multiplets, and the fields $S, T$, and $A$ transforming under $SU(5)_B$. (When we gauge part of the global symmetry with the standard-model gauge group, the standard model fields are of course also light.) It is important that for large $S$, the $SU(5)_B$ $D$ terms lift all flat directions in the fields $A$ and $T$, so we have only a one-parameter flat direction.

Since $\Lambda_N \gg \Lambda_5$, the dominant effect that lifts the $S$ flat direction is gaugino condensation in the $SU(N)_P$ gauge theory. This gives rise to a dynamical superpotential for $S$ [6]

$$W_{\text{dyn}} \simeq \frac{1}{16\pi^2} \lambda^3 \left( \frac{\lambda S}{\Lambda} \right)^{K/N} ,$$

where we have omitted an unknown order-1 coefficient. (For an explanation of the factors of $4\pi$ in this formula and throughout this paper, see Refs. [7].) We will assume that $K < N$, so that this superpotential forces $S$ to run away.

The $SU(5)_B$ dynamics becomes strong at a scale that depends on $S$ through the one-loop matching at the scale where $Q, \bar{Q}$ become heavy:

$$\Lambda_{B,\text{eff}}(S) = \Lambda_B^{1-K'/13} (\kappa S)^{K'/13} .$$

The strong $SU(5)_B$ dynamics is believed to break supersymmetry [8]. This theory has no small parameters, and so the supersymmetry breaking is parameterized in the effective lagrangian by the terms

$$\delta L_{\text{eff}} \simeq \frac{1}{16\pi^2} \left\{ |\Lambda_{B,\text{eff}}(S)|^4 + \int d^4\theta |\Lambda_{B,\text{eff}}(S)|^2 \right\} .$$

These terms grow with $S$ and stabilize the runaway potential, and supersymmetry is broken at the (local) minimum.

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1I thank R. Rattazzi for pointing out the existence of the second term in Eq. (6).
To minimize the potential, we use the equation of motion for the $F$ component of $S$:

$$F^\dagger_S \simeq \frac{\partial W_{\text{dyn}}}{\partial S} \left( 1 + \frac{1}{16\pi^2} \left| \frac{\partial \Lambda_{B_{\text{eff}}}(S)}{\partial S} \right|^2 \right)^{-1},$$

(7)

where the last factor comes from the second term in Eq. (6). Note that

$$\frac{1}{16\pi^2} \left| \frac{\partial \Lambda_{B_{\text{eff}}}(S)}{\partial S} \right|^2 = \frac{\kappa_{2K'/13}^2}{16\pi^2} \left( \frac{A_B}{S} \right)^{2(13-K')/13}.$$

(8)

Because $K' < 13$ (the condition that $SU(5)_B$ is asymptotically free), this is negligible as long as $\langle S \rangle \gg \Lambda_B$, which we will find to be the case. We then obtain a local minimum at

$$\langle S \rangle \simeq \Lambda_P \left[ \frac{13K^2(N-K)}{32\pi^2NK'} \frac{\lambda^{2K/N}}{\kappa^{4K'/13}} \left( \frac{A_P}{\Lambda_B} \right)^{4(13-K')/13} \right]^n,$$

(9)

where

$$n = \frac{13N}{2[2K'N + 13(N-K)]}.$$

(10)

$n > 0$ because $K < N$, so $\langle S \rangle \gg \Lambda_P$ as long as $\Lambda_P \gg \Lambda_B$. The $F$ component of $S$ gets a vacuum expectation value

$$\langle F_S \rangle \simeq \frac{\lambda^{K/N} K}{16\pi^2 N} \left[ \frac{32\pi^2N^3K'}{13K^2(N-K)} \frac{\kappa^{4K'/13}}{\lambda^{2K/N}} \left( \frac{A_B}{A_P} \right)^{4(13-K')/13} \right]^{(3N-K)n/N},$$

(11)

We see that $\langle F_S \rangle^2/\langle S \rangle^2$ is proportional to a positive power of $\Lambda_B/A_P$, so $\langle F_S \rangle \ll \langle S \rangle^2$ as long as $\Lambda_B \ll \Lambda_P$. The complicated powers in Eqs. (9) and (11) come from the fact that all mass scales in this model arise from dimensional transmutation.

The Goldstino arises dominantly from the $SU(5)_B$ symmetry breaking sector. The Goldstino decay constant $F$ is therefore given by

$$F \simeq \frac{\Lambda_{B_{\text{eff}}}(\langle S \rangle)}{4\pi} \simeq \left( \frac{13(N-K)}{2NK'} \right)^{1/2} \langle F_S \rangle,$$

(12)

and we see that $F \sim \langle F_S \rangle$.

If we gauge a subgroup of the global symmetry with the standard model gauge group, this model corresponds precisely to a minimal messenger model [2]. The fields $P, \bar{P}$ and/or $Q, \bar{Q}$ carry standard model quantum numbers, and play the role of the
messengers. The soft supersymmetry breaking terms in the observable sector are proportional to the “messenger scale”

\[ M_{\text{mess}} \equiv \frac{\langle F_S \rangle}{\langle S \rangle}. \]  

(13)

Numerically, we require \( M_{\text{mess}} \sim 10 \text{ TeV} \) in order to obtain realistic superpartner masses. It can be checked that the masses of the \( S \) scalars and vectors are also of order the messenger scale in this model. For example, the \( S \) fermion mass is

\[ m_{\psi_S} \simeq \frac{N - K}{N} M_{\text{mess}}. \]  

(14)

These results show that the only important scales in the model are \( \langle S \rangle \) and \( \langle F_S \rangle \). These scales are complicated functions of the parameters of the model (see Eqs. (9) and (11)), but all other mass scales are proportional to simple ratios of these scales.

In order to get some additional intuition about the scales involved, we consider two special cases. The first case is the “minimal scenario” where

\[ N = 2, \quad K = 1, \quad K' = 5. \]  

(15)

This is the smallest model that has an \( SU(5) \) global symmetry that can be gauged with the electroweak gauge group. (The electroweak gauge group \( SU(3)_C \times SU(2)_W \times U(1)_Y \) is imbedded into \( SU(5) \) in the standard way, and we refer to this imbedding as \( SU(5)_{\text{EW}} \) for brevity.) The model contains \( 5 \times (\Box \oplus \bar{\Box}) \) under \( SU(5)_{\text{EW}} \), so this model is marginally compatible with perturbative unification for low values of \( \langle S \rangle \). In this case,

\[ \frac{\langle S \rangle}{\Lambda_P} \simeq 0.07 \frac{\lambda^{0.39}}{\kappa^{0.61}} \left( \frac{\Lambda_P}{\Lambda_B} \right)^{0.97}, \quad \frac{\langle F_S \rangle}{\langle S \rangle^2} \simeq 3 \frac{\kappa^{1.5}}{\lambda^{0.48}} \left( \frac{\Lambda_B}{\Lambda_P} \right)^{2.4}. \]  

(16)

The second case is the “maximal scenario”, where the \( SU(5)_B \) gauge factor has so many matter fields that it is barely asymptotically free:

\[ N = 2, \quad K = 1, \quad K' = 12. \]  

(17)

In this model, the \( SU(5)_B \) gauge coupling runs very slowly until the scale \( \kappa_S \), where many \( SU(5)_B \) charged fields become massive; below this scale, the effective \( SU(5)_B \) coupling becomes strong quickly. This mechanism is interesting because the effective \( SU(5)_B \) dynamical scale is tied closely to \( \Lambda_P \), and (to a good approximation) the model has only one scale. If we gauge an \( SU(5)_{\text{EW}} \) subgroup of the global \( SU(12) \) symmetry, then this model also contains \( 5 \times (\Box \oplus \bar{\Box}) \) under \( SU(5)_{\text{EW}} \). In this case,

\[ \frac{\langle S \rangle}{\Lambda_P} \simeq 0.2 \frac{\lambda^{0.21}}{\kappa^{0.79}} \left( \frac{\Lambda_P}{\Lambda_B} \right)^{0.066}, \quad \frac{\langle F_S \rangle}{\langle S \rangle^2} \simeq 0.2 \frac{\kappa^{2.0}}{\lambda^{0.033}} \left( \frac{\Lambda_B}{\Lambda_P} \right)^{0.16}. \]  

(18)
As expected, the physical results are very insensitive to $\Lambda_B$. However, our approximations are only valid if $\lambda(S) \gg \Lambda_P$ and $\langle F_S \rangle \ll \langle S \rangle^2$. Both of these conditions are satisfied for $\lambda \sim 1, \kappa \ll 1$. We can also consider the regime where $\lambda \sim \kappa \sim 1$, assuming that the local minimum persists despite the fact that our approximations are no longer under control. This is interesting because in this regime the model effectively has a single scale and no small parameters, and the supersymmetry-breaking dynamics takes place at energies as low as $\Lambda_P \sim 10$ TeV. This holds out the hope that dynamical supersymmetry breaking may be accessible to direct experimental study.

We now consider the global minimum in this model. The classical space of vacua has two branches. For $S \neq 0$, we have the one-parameter space of classical flat directions analyzed above; this branch has no supersymmetric minimum when quantum effects are included. We now consider the $S = 0$ branch, taking $K = 1$ for simplicity. Along this branch, the $SU(N)_P$ gauge dynamics gives rise to a dynamical superpotential in the direction parameterized by the gauge-invariant $X \equiv P \bar{P}$. The effective superpotential below the scale $X$ is then

$$W_{\text{eff}} \simeq \lambda S X + \kappa S \text{tr}(Q \bar{Q}) + \frac{\Lambda^3_P}{16\pi^2} \left( \frac{\Lambda^2_P}{X} \right)^{1/(N-1)} .$$

The description in terms of $X$ is smooth because we are considering vacua with $X \neq 0$. We can therefore integrate out $S$ and $X$ to obtain an effective $SU(5)_B$ gauge theory with superpotential

$$W_{\text{eff}} \simeq \frac{\Lambda^3_P}{16\pi^2} \left( \frac{\lambda \Lambda^2_P}{\kappa \text{tr}(Q \bar{Q})} \right)^{1/(N-1)} .$$

One can check that all supersymmetric vacua in this model have at least one runaway direction. This is already an interesting result, since it shows that the local supersymmetry-breaking minimum is closer to the origin of field space than the supersymmetric vacua, making it more likely that the universe ends up in the false vacuum if it cools from temperatures larger than $\Lambda_P$. (In an inflationary universe, these considerations apply only if the reheat temperature is larger than $\Lambda_P$.)

We would like to know the lifetime of the false vacuum. The potential in this model is far too complicated to do an honest calculation, so we will limit ourselves

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2 For the dynamics of the theory with $K' \leq 4$, see Refs. [9, 10]. The dynamics of this theory for $K' \geq 5$ is presently not understood. (One can write a “dual” for this theory using deconfinement [10], but it is not weakly coupled.) However, symmetry and holomorphy do not allow the moduli space to be modified at the quantum level for $K' \geq 5$, so it is clear that the theory has a moduli space of supersymmetric vacuum states with $\text{tr}(Q \bar{Q}) \to \infty$, even if we do not understand the physics of many of these vacua.
to simple estimates. Because the energy difference between the false vacuum and the true vacuum is small compared to the distance in field space to the classical escape point, we can bound the tunnelling rate by approximating the potential as completely flat. In that case, the Euclidean tunnelling action is \[ I_E \simeq 2\pi^2 \frac{(\Delta \phi)^4}{V}, \] (21)
where \( \Delta \phi \) is the distance in field space to the classical escape point, and \( V \) is the value of the potential in the false vacuum. The value of \( Q \) in the \( S = 0 \) branch where the energy density is equal to that of the false vacuum is
\[ Q \sim \langle S \rangle \left( \frac{\Lambda_P}{\langle S \rangle} \right)^{[2N+K(N-1)]/[N(N+1)]} \ll \langle S \rangle. \] (22)

Therefore, our estimate is dominated by the tunnelling from the false vacuum to \( S \sim \Lambda_P \). In this way we obtain the bound
\[ I_E \gtrsim 2\pi^2 \frac{\langle S \rangle^4}{\langle F_S \rangle^2}, \] (23)
which is always large. Since this is an extremely conservative estimate, we believe that decay into the supersymmetric vacuum is not a problem in these models.

It should be clear that the ideas involved in the model described above are very robust, and it is easy to construct variants. Many different gauge theories could act as the “push” and supersymmetry-breaking sectors of a model of this type. We close by mentioning some generalizations that have additional phenomenologically attractive features. First, if we want to preserve perturbative one-step unification, we can introduce a single \( \oplus \oplus \) of \( SU(5)_{EW} \) coupled to \( S \) instead of gauging a subgroup of the global \( SU(F_1) \times SU(F_2) \) symmetry. These messengers get mass for \( S \neq 0 \), and the analysis of the local minimum proceeds exactly as above.

Another variant of the above model solves the \( R \)-axion problem without appealing to Planck-scale effects [12]. The idea is to introduce several “push” group factors with different numbers of colors and flavors to break the \( U(1)_R \) symmetry explicitly. For example, a model with “push” group \( SU(N_1) \times SU(N_2) \) with \( K_1 \) and \( K_2 \) flavors, respectively, has no anomaly-free \( U(1)_R \) symmetry as long as \( K_1/N_1 \neq K_2/N_2 \). If \( K_1 < N_1 \) and \( K_2 < N_2 \), the dynamical superpotential along the \( S \) classical flat direction is given by gaugino condensation in the groups \( SU(N_1) \) and \( SU(N_2) \):
\[ W_{\text{eff}} \simeq \frac{1}{16\pi^2} \left[ \Lambda^3_{P1} \left( \frac{\lambda_1 S}{\Lambda_{P1}} \right)^{K_1/N_1} + \Lambda^3_{P2} \left( \frac{\lambda_2 S}{\Lambda_{P2}} \right)^{K_2/N_2} \right]. \] (24)
This clearly has no $U(1)_R$ symmetry. In general, there will be a hierarchy between the scales $\Lambda_{P1}$ and $\Lambda_{P2}$, so there will be an approximate $U(1)_R$ symmetry and the $R$ axion will be a pseudo Nambu–Goldstone mode. In this model, if we assume $\Lambda_{P2} \ll \Lambda_{P1}$, the $R$ axion mass is

$$m_R \sim M_{\text{mess}} \frac{\lambda_2^{K_2/(2N_1)}}{\lambda_1^{K_1/(2N_2)}} \Lambda_{P2}^{(3N_2-K_2)/N_2} \left( \frac{\langle S \rangle}{\Lambda_{P1}} \right)^{K_2/N_2-K_1/N_1}. \quad (25)$$

This can be phenomenologically acceptable for a wide range of parameters. Ref. [12] showed that fine-tuning the cosmological constant in supergravity breaks $U(1)_R$ explicitly, leading to an $R$ axion mass. In the present model, this mass is

$$m_{R,\text{grav}} \sim \left( \frac{\langle F_S \rangle^2}{\langle S \rangle M_{\text{Planck}}} \right)^{1/2} \sim M_{\text{mess}} \left( \frac{\langle S \rangle}{M_{\text{Planck}}} \right)^{1/2}, \quad (26)$$

which is always larger than 100 MeV even if $\langle S \rangle \sim \langle F_S \rangle^{1/2} \sim 10$ TeV. This is safe from supernova constraints, so it is not clear that the $U(1)_R$ breaking mechanism discussed above is required in these models.

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References


