Surface Induced Phase Transition in Quark-Gluon Plasma
Produced in Laboratory

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Abstract

We discuss an outside-inside scenario for a first order quark-hadron transition in the Quark-Gluon Plasma (QGP) expected to be produced in heavy ion collisions, wherein the entire QGP region itself becomes like a subcritical bubble, and starts shrinking. We argue that this shrinking QGP bubble will lead to concentration of baryon number in a narrow beam like region in center, which can be detected by HBT analysis, or as a raised plateau in the rapidity plot of baryon number.

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Possibility of observing signatures of a quark-hadron transition in relativistic heavy-ion collisions has been extensively investigated for sometime now. Lattice calculations allow for the possibility that this transition may be of first order. Earlier studies of such a first order transition have primarily focused on employing homogeneous nucleation of hadronic bubbles in a uniform QGP background. These bubbles expand after nucleation and release latent heat which heats up the plasma, suppressing further nucleation [1].

We discuss an alternative scenario for a first order quark-hadron transition in this paper by focusing on the finite size of the QGP region produced in heavy ion collisions. Outside this region, one either has a hadronic gas (for example, due to energy density gradient in the transverse direction [2]), or simply the QCD vacuum in the confining phase. In either case, there must be a domain wall at the boundary of the QGP region with a non-zero surface tension. For simplicity, we take the region outside this boundary to be (at least a thin layer of) the hadronic phase with roughly same temperature as inside.

We solve the hydrodynamical equations for the longitudinal expansion model of Bjorken [3], and show that transition can proceed entirely due to collapse of this boundary wall. We argue that collapsing boundary wall will lead to concentration of baryons in the central region in a narrow beam like region (of thickness 1-2 fm and length of few tens of fm), which can be detected by the HBT analysis [4] of baryons, or by a raised plateau in the rapidity plot of baryon number.

It is important to appreciate that, in contrast to usual QGP bubbles which nucleate by thermal fluctuations in a superheated hadronic matter, this entire, bubble like, region of QGP does not arise due to any nucleation process. Rather, it forms due to certain initial conditions which create a dense, interacting gas of partons in a collision process and this gas thermalizes later to form a region of QGP. As thermalization proceeds, the order parameter develops a value appropriate for high temperature, leading to the formation of the interface at the boundary.

We start by writing down the hydrodynamical equations governing the evolution of energy density $e$ during the longitudinal expansion [3].

\[
\frac{de}{d\tau} = -\frac{(e + p)}{\tau}.
\]  

Here $\tau$ denotes the proper time and $p$ is the pressure. [We will use natural system of units with $\hbar = c = 1$.] At temperatures well above the critical temperature, $T_c$, QGP will keep expanding in the longitudinal direction without any hindrance. [We neglect any transverse expansion.] As the temperature drops due to the longitudinal expansion via eqn.(1), the outward expansion of the wall (in the local rest frame) will become energetically unfavorable at some temperature greater than $T_c$. Let us first estimate this temperature for a simple case when the QGP region is spherical with radius $R$. The free energy associated with such a bubble is

\[
F = -\frac{4\pi}{3} [p_q(T) - p_h(T)] R^3 + 4\pi R^2 \sigma,
\]

where $p_q$ and $p_h$ are the pressures in the QGP and hadronic phases respectively and $\sigma$ is the surface tension of the wall. Taking the hadronic gas to consist of three massless pions and QGP to consist of gluons and 2 flavors of massless quarks, we have
\[ p_q(T) = \frac{37\pi^2}{90} T^4 - B, \quad p_h(T) = \frac{3\pi^2}{90} T^4, \tag{3} \]

where \( B \) is the bag constant (which is related to \( T_c \) by Gibb’s condition, \( p_q(T_c) = p_h(T_c) \)). For \( T > T_c \), the critical radius of the QGP bubble is,

\[ R_c(T) = \frac{2\sigma}{p_q(T) - p_h(T)}. \tag{4} \]

A QGP bubble of radius larger than \( R_c \) will expand while bubbles of smaller sizes will be subcritical and collapse. By writing \( T = T_c + \Delta T \) and assuming \( \Delta T/T_c << 1 \) (which will be the case of interest to us) we can find the expression for \( \Delta T \) such that for \( T < T_c + \Delta T \) the radius of the QGP region, \( R \), becomes smaller than the critical radius \( R_c \). We find,

\[ \Delta T = \frac{180\sigma}{136\pi^2 RT_c^3} \equiv \Delta T_{sph}. \tag{5} \]

We take \( \sigma = 50 \text{ MeV}/fm^2 \) [5]. With \( T_c = 170 \text{ MeV} \), we find \( \Delta T \simeq 2 \text{ MeV} \), for a spherical QGP region with \( R = 5 \text{ fm} \). So this region starts collapsing when the temperature drops below 172 MeV. For \( T_c = 120 \text{ MeV} \), we get \( \Delta T \simeq 5 \text{ MeV} \). Hereafter, we use \( T_c = 170 \text{ MeV} \).

We now estimate the values of \( \Delta T \) suitable for a more realistic cylindrical geometry of the QGP region. The free energy of this QGP region (with \( R_T \) being the transverse radius and \( R_L \) the half length) can be written as

\[ F = -\left[p_q(T) - p_h(T)\right]2\pi R_T^2 R_L + \sigma 4\pi R_T R_L + 2\sigma \pi R_T^2. \tag{6} \]

Here, we have taken the end caps of the cylindrical region to be approximated by flat disks of radius \( R_T \). As the dynamics of wall will be governed by local geometry alone, we only keep relevant terms in eqn.(6), for a given portion under consideration, and discard others. We first determine the critical size for the middle region, near (longitudinal coordinate) \( z = 0 \). For this, we neglect the end portion of the cylinder, i.e. the last term in eqn.(6). The condition for the transverse expansion to be energetically favored is,

\[ \frac{\partial F}{\partial R_T} < 0. \tag{7} \]

This gives a critical value of \( R_T = R_c/2 \) where \( R_c \) is given by eqn.(4). Thus for a given value of \( R_T \), when the temperature drops below a critical value \( T_c + \Delta T_{tr} \) then the transverse dimension becomes subcritical and the wall should start collapsing. Using arguments as for eqn.(5), we find that \( \Delta T_{tr} = \Delta T_{sph}/2 \).

For the longitudinal motion of the wall, we analyze things in the local frame of the plasma. The relevant free energy of a small portion of the cylinder at the end region (with length being \( \Delta z \), say) is given by terms in eqn.(6) with \( R_L \) replaced by \( \Delta z \). The condition for the longitudinal expansion of the wall to be energetically favored is now that the partial derivative of \( F \) with respect to \( \Delta z \) be less than zero (similar to eqn.(7)). This leads to the condition, \( R_T > R_c \). [Here, it is helpful to think of the end caps to have typical curvature determined by \( R_T \), instead of being strictly flat.] Again, for a given value of \( R_T \), when the local temperature drops below a critical value \( T_c + \Delta T_{long} \) then the longitudinal
dimension becomes subcritical and the wall starts shrinking in that direction. Here we find that $\Delta T_{long} = \Delta T_{sph}$.

We now come back to eqn.(1) for the evolution of the energy density. If the total volume of the region (including the QGP region and the hadronic region) is $V_0$ and the volume of the QGP region is $V_q$ then we can write the average energy density as [1],

$$e(\tau) = e_q \frac{V_q}{V_0} + e_h (1 - \frac{V_q}{V_0}), \quad (8)$$

where $e_q$ and $e_h$ are the energy densities in the QGP phase and the hadronic phase respectively,

$$e_q = \frac{37}{30} \pi^2 T^4 + B, \quad e_h = \frac{\pi^2}{10} T^4. \quad (9)$$

By writing a similar expression, as in eqn.(8), for the enthalpy ($e + p$), eqn.(1) leads to the evolution equation for the temperature as a function of proper time. Let us first study the region near $z = 0$. For a boost invariant geometry, the same temperature evolution will be observed at different values of $z$. [Again, for central portion, we neglect any possible effects of the end cap region.] We consider a cylindrical region of a small length $\Delta z$ around $z = 0$. The outer radius of the cylinder is $R_T$ and the radius of the collapsing wall (bounding the QGP region) is denoted by $R_q$. We then have $V_q = \pi R_q^2 \Delta z$ and $V_0 = \pi R_0^2 \Delta z$. As we are neglecting the transverse expansion of the plasma, $R_T$ is time independent while $R_q$ decreases with the velocity of the wall $v(T)$ moving through plasma. That is, $\dot{R}_q = -v(T)$. For $v(T)$ we take the following expression

$$v(T) = a [1 - \frac{T}{T_c}]^b. \quad (10)$$

Here $a$ and $b$ are parameters and $T \leq T_c$. There are many uncertainties in determining the velocity of the wall such as the energy flux through the interface, proper account of surface tension etc. [6]. We will use two sets of values of these parameters. For one set we use $a = 3, b = 3/2$ (valid for $T > 2T_c/3$, see [1,6]) and for the other set we use a sample value, $a = 2$ and $b = 1$ (this should be used for $T > \sim 0.7 T_c$, so that $v(T)$ remains less than the speed of sound). [Velocities for this second set are larger than those used in the literature [1], though linear dependence on $T$ has been discussed in [7]. We use this set as an example, since we expect that the collapse of the wall will be faster with respect to local plasma when one allows for even slightest expansion of the plasma with respect to wall, especially near $T = T_c$.]

There is one subtle point here. The velocity as given by eqn.(10) vanishes for $T = T_c$. However, we have argued above that a QGP region of finite size becomes subcritical at a temperature larger than $T_c$. Therefore, in this case, the velocity of the wall (with respect to the local plasma) should be more accurately represented when we replace $T_c$ in eqn.(10) by $T_c + \Delta T$. That is,

$$v(T) = a [1 - \frac{T}{T_c + \Delta T}]^b, \quad (11)$$

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valid for $T \leq T_c + \Delta T$, here, $\Delta T = \Delta T_{tr}$, or $\Delta T_{long}$, depending on whether one is discussing the wall motion in the transverse direction or in the longitudinal direction. [We mention here that the qualitative aspects of our results do not depend on whether one uses eqn.(10) or eqn.(11) for wall velocity.]

Figure 1 shows the evolution of the temperature near $z = 0$ region. Initial conditions, prescribed at (a sample value of) $\tau_0 = 3$ fm, are, $T(\tau_0) = T_c + \Delta T$, and $R_q(\tau_0) = R_T$. The plots are given only up to those specific values of proper time, when the transverse dimension of the QGP region, $R_q$, becomes zero.

For the evolution of temperature near the end regions of the plasma in the longitudinal direction, consider a segment of length $z_0(\tau)$ at the end portion with the transverse dimension of the region being $R_T$. The location of the end portion of the longitudinally moving wall is denoted by $z_q(\tau)$ while the transverse radius of the wall is given by $R_q(\tau)$. The ratio of the volume of the QGP region $V_q$ to the full (local) region $V_0$ is then given by

$$\frac{V_q}{V_0} = \frac{\pi R_q(\tau)^2 z_q(\tau)}{\pi R_T^2 z_0(\tau)}. \quad (12)$$

Here, $z_0(\tau)$ increases due to plasma expansion with the velocity $\dot{z}_0 = z_0/\tau$. Wall velocity in local rest frame of the plasma is again given by eqn.(11), so $\dot{R}_q = -v(T)$ and $\dot{z}_q = z_q/\tau - v(T)$ (as both velocities are small). For simplicity, we use a single value of $\Delta T$ ($= \Delta T_{long}$) for both directions. With the same initial conditions, as in Fig.1, we then determine the evolution of temperature in the end region. We find, that the evolution of temperature is almost similar here as for the $z = 0$ region (Fig.1), apart from minor differences, (thus we do not show these plots). For example, corresponding to the dotted curve in Fig.1, one finds little larger re-heating for the end portion, while for the thick solid curve of Fig.1, one finds smaller reheating. [Note that the temperature obtained from the evolution equation represents an average temperature for the cylindrical segment, and hence, is valid only for time scales larger than the one given by $R_T$.]

FIG. 1. Temperature in the central region as a function of proper time. In both of the figures in this paper, thick solid curve and dashed curve correspond to $R_T = 7$ fm, while thin solid curve and dotted curve correspond to $R_T = 3$ fm. Solid curves correspond to the choice of parameters for the wall velocity (eqn.(11)) as, $a = 3$, $b = 3/2$, while dotted and dashed curves correspond to the parameter set $a = 2$, $b = 1$. 

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We see from Fig.1 that largest reheating temperature is represented by the dotted curve. First, $T$ drops sharply to about $0.9T_c$ due to expansion of the plasma. This value is low enough to cause nucleation of bubbles in the central region, see [1]. Note that the heating, and hence suppression of bubble nucleation, will be somewhat larger near the wall as compared to the center. [We assume that reheating is dominated by the collapsing boundary wall due to large surface area, compared to any nucleated bubbles.] For $\tau >$ about 6 fm, the reheating dominates over cooling due to the expansion, and $T$ starts to rise. It reaches a value of about $0.98T_c$ by $\tau \simeq 16$ fm and $0.99T_c$ by $\tau \simeq 21$ fm. Critical bubble sizes corresponding to these temperatures are, $R_c \simeq 3.4$ fm and 6.7 fm, respectively (eqn.(4)). The entire size of the region is not very large, or, more importantly, the elapsed time is not enough to let any bubbles, nucleated earlier in the interior (when the temperature was lower there), to grow to these sizes. [Note that the wall velocity of the bubbles, eqn.(10), (or eqn.(11) with appropriate $\Delta T$), is very small for temperatures close to $T_c$.] Thus, all these bubbles become subcritical and collapse away. The end result being that the phase transition to the hadronic phase proceeds entirely due to the collapse of the outer wall.

For some other parameters, e.g. those corresponding to the plot given by thick solid curve in Fig.1, the final reheat temperature is not large enough to completely quench nucleation. In this case, the picture of the transition may be somewhat mixed.

Let us now discuss about some possible observable consequences of this type of scenario of phase transition. Here we apply ideas used in the early Universe in the context of baryon number inhomogeneity generation at the end of quark-hadron transition [8]. Due to large masses of baryons in the hadronic phase and much smaller masses of quarks in the QGP phase, the baryon number density in the QGP phase is larger than in the hadronic phase. The ratio $R = n_{Bq}^B/n_{Bh}^B$ of the baryon number densities in the two phases has been estimated by many people. A rough estimate may give $R \simeq 10$. Evolution of the value of $R$ in a collapsing QGP droplet has also been extensively discussed in literature in the context of the early Universe [8]. It seems reasonable that a good fraction of the net baryon number inside the droplet may get trapped inside. We will simply assume this to be the case, at this preliminary stage of exploration, instead of getting into details of the mechanism of trapping.

In our model of the phase transition in a heavy ion collision, the shrinking of boundary wall leads to a picture which is identical to that of a shrinking QGP droplet in the early Universe. Immediate consequence is that the net baryon number of the region (which will be expected to be non-zero, though small, even for RHIC and LHC energies) will become concentrated near the central region of the plasma due to the collapse of the boundary wall. [We mention here that people have discussed formation of strangelets due to strangeness enhancement of the QGP droplet in heavy ion collisions by evaporating hadrons [9]. However, as we show below, the longitudinal collapse of the wall is insignificant by the time transition ends, i.e. when transverse collapse of wall is completed. Thus, the picture of any, strangeness enriched, exotic lump of matter forming at the center of the collision, or in different portions due to local fluctuations, needs to be modified.]

We calculate the longitudinal collapse of the wall, in the center of mass frame, by taking its velocity with respect to local plasma to be given by eqn.(11). In the longitudinal expansion model, the plasma velocity at longitudinal distance $z$ is given by $z/t$ [3]. Taking the end portion of the wall to have initial rapidity equal to some maximum rapidity, the
The evolution of frame rapidity at wall end is shown in Fig. 2, until the time when transverse collapse is completed. The final value of this frame rapidity gives the rapidity interval inside which most of the net baryon number may be expected to be concentrated. We see that this interval can shrink by up to one unit by the time the transition is completed, i.e. when \( R_q \) becomes zero.

![Rapidity vs proper time](image)

**FIG. 2.** Evolution of frame rapidity at the end portion of the wall due to its longitudinal collapse in the center of mass frame.

Large values of the final rapidity of the wall imply that the longitudinal extent of the wall is large when the transverse collapse is completed. This concentrates all (or most) of the baryon number in a narrow beam like structure at the center of the cylindrical region. Transverse dimension of this region should be of the order of wall thickness, i.e. 1-2 fm, while its length can be very large, of the order of \( \tau \) when \( R_q \) becomes zero in Figs. 1 and 2. Such a structure for baryon source can be easily detected by the HBT analysis.

Further, the reduction in the frame rapidity at the end portion of wall implies that in the rapidity plot of net baryon number, one will expect a raised plateau near zero rapidity with width determined by the final rapidity in Fig. 2. Equivalently, one may expect a depression in the rapidity plot of antibaryons due to annihilations with increased number of baryons. Uniformity of this plateau will depend on whether some (or all) of the fragmentation region lies inside the boundary wall initially.

We conclude by summarizing our main results. We have presented a novel scenario of first order quark-hadron phase transition relevant for the situation in heavy ion collisions. Here, the phase transition proceeds by the shrinking of the interface surrounding the entire QGP region, starting at a temperature slightly above \( T_c \) itself (due to the QGP region becoming subcritical). Collapsing boundary wall heats up the entire region due to the release of latent heat which inhibits bubble nucleation in the enclosed region. The reheat temperature can be close enough to \( T_c \) so that if any bubbles were nucleated in the central region, they become subcritical and shrink away. Thus, finally, the entire phase transition proceeds due to collapse of the boundary wall only. We argue that collapsing boundary wall can concentrate the net baryon number of the QGP region in center in a long beam like region of very small thickness (of order 1-2 fm), which can be detected by HBT analysis. Also, as baryons get concentrated in a smaller rapidity interval, it should be observable as a raised plateau in the rapidity plot of baryon number (or a depressed plateau in the rapidity...
plot of antibaryons). We emphasize that a signature of a pencil shaped source of baryon number will not only demonstrate the existence of QGP state, it will also show the transition to be of first order, and will provide experimental evidence for the baryon concentration by shrinking QGP bubbles as has been proposed for the early Universe.

Other possible signatures of this picture of phase transition can arise in rapidity plots of strange particles, in transverse momentum distributions of particles (e.g. baryons) reflecting concentration in the transverse direction, etc. It will be interesting to see if the collapsing boundary wall can leave some imprints also on the spectrum of photons and dileptons.

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