WSP-IF 97-49

January 1997

Is the standard interpretation of CPT-violation parameter for neutral kaon complex well-founded?

K.Urbanowski

Pedagogical University, Institute of Physics,
Plac Slowianski 6, 65-069 Zielona Gora, Poland.

January 2, 1997

PACS numbers: 03.65.-w., 11.30.Er., 13.20.Eb.

Abstract

Basic assumptions necessary for the proof of the CPT Theorem are considered. It is found that the CPT Theorem is not valid for a physical system with unstable particles decaying exponentially. The Lee, Oehme and Yang (LOY) model of neutral kaons decay is discussed and the conclusion is drawn that CPT-transformation cannot be a symmetry in a system which contains the LOY model as a subsystem, and, thus this model is shown to be incapable of describing possible CPT-violation effects correctly. The approximate formulae for matrix elements of the effective Hamiltonian $H_{||}$ governing the time evolution in neutral kaons subspace (different from those obtained by means Weisskopf-Wigner method and used by LOY) and their implications for an interpretation of the standard CPT-violation parameter $\delta \propto (\langle K_0 | H_{||} | K_0 \rangle - \langle \bar{K}_0 | H_{||} | \bar{K}_0 \rangle)$ are considered. Within this approximation it is shown that $(\langle K_0 | H_{||} | K_0 \rangle - \langle \bar{K}_0 | H_{||} | \bar{K}_0 \rangle) \neq 0$ in the system preserving CPT-symmetry and that $\delta = 0$ is possible only if $(\langle K_0 | H | K_0 \rangle - \langle \bar{K}_0 | H | \bar{K}_0 \rangle) \neq 0$, where H is the total Hamiltonian of the system under consideration, that is, if the system does not preserve CPT-symmetry.

Sw9728

1 Introduction.

The problem of testing CPT-invariance experimentally has attracted the attention of physicists, practically since the discovery of antiparticles. There is a known theorem (called the CPT Theorem) in axiomatic quantum field theory that CPT-invariance must hold. This theorem is based on very general assumptions [1] — [6], which are usually formulated in terms of Wightman functions $W^{(n)}$ [7], [2] — [6]. These will be considered briefly below (details can be found, e.g. in [2] — [4]). If we take, for example, a set of neutral scalar fields $\{\Phi_\Xi(x)\} \ni \phi_A(x)$, $A \in \Xi$, $x \equiv (ct, \vec{r}) \in M_4$ (M_4 is the Minkowski space-time, so $x^2 \equiv c^2t^2 - \vec{r}^2$), — for the sake of simplicity only such fields will be considered here — Wightman functions are vacuum expectation values for products of n field functions

$$W_{ABC\dots}^{(n)}(x_1, x_2, \dots, x_n) \equiv \langle \Omega | \phi_A(x_1) \phi_B(x_2) \phi_C(x_3) \dots | \Omega \rangle, \quad (1)$$

where Ω denotes the vacuum state, which is assumed to be unique up to a constant phase and to be invariant under transformation $U(a, \mathcal{A}) : U(a, \mathcal{A})|\Omega \rangle \equiv |\Omega \rangle$, where $U(a, \mathcal{A})$ is a continuous unitary representation of the inhomogeneous group $SL(2, \mathbb{C}) : \{a, \mathcal{A}\} \rightarrow U(a, \mathcal{A})$, $a \in M_4$, $\mathcal{A} \in SL(2, \mathbb{C})$ associated with the restricted Lorentz group. The basic requirements for quantum fields $\{\Phi_\Xi(x)\}$, which afford possibilities for writing the Wightman functions for fields $\{\Phi_\Xi(x)\}$, besides those of rather strictly mathematical nature (i.e., concerning the state space $\mathcal{H}$ of the system considered, the domain of field operators, its density in $\mathcal{H}$, the hermiticity, the cyclicity of $|\Omega \rangle$, etc. [2] — [6]) are the following:

(A1) Relativistic invariance:

$$U(a, \mathcal{A})\phi_A(x)U(a, \mathcal{A})^{-1} = \phi_A(\mathcal{A}x + a), \quad (2)$$

for any $A \in \Xi$,

(A2) The spectral condition: the eigenvalues of the energy momentum operator $\mathcal{P}_\mu$ lie in or on the plus (forward) light cone V_+ .

The point $p = 0$ is the isolate eigenvalue of $\mathcal{P}$ to the eigenvector $|\Omega \rangle$ — the vacuum. The operator $\mathcal{P}^\mu \mathcal{P}_\mu \equiv m^2$ is interpreted as the square of

the mass (the assumption (A2) is equivalent with requiring all energies to be positive). We have $U(a, 1) \equiv \exp(i\mathcal{P}^\mu a_\mu)$.

(A3) Local commutativity (called also "microscopic causality"): for all pairs $A, B \in \Xi$

$$[\phi_A(x), \phi_B(y)] = 0, \quad (3)$$

if $(x - y)^2 < 0, x, y \in M_4$.

Having a set of fields satisfying the above assumptions and the vacuum state $|\Omega\rangle$, which is cyclic, one has a set of uniquely defined (by the relation (1)) Wightman functions $\{W^{(n)}\}$. Conversely, a knowledge of all Wightman functions $\{W^{(n)}\}$ is sufficient to characterize a quantum field theory completely. Any other quantum field theory with the same values for the $\{W^{(n)}\}$ is equivalent to the field theory from which the original $\{W^{(n)}\}$ were constructed, up to unitary transformation [2] — [7].

CPT-invariance could be shown to hold for any field theory which has properties listed above [2] — [5]. For example, the CPT-symmetry Θ has the property that, e.g., for charged scalar fields φ_A, φ_B

$$\Theta^{-1}\varphi_A(x_1)\varphi_B(x_2)\Theta = \varphi_B(-x_2)^+ \varphi_A(-x_1)^+, \quad (4)$$

(where $\Theta \equiv \mathcal{CPT}$ is antiunitary) and for neutral scalar fields ϕ_A, ϕ_B

$$\Theta^{-1}\phi_A(x_1)\phi_B(x_2)\Theta = \phi_B(-x_2) \phi_A(-x_1), \quad (5)$$

which can be written for vacuum expectation values as follows

$$\langle \Omega | \phi_A(x_1) \phi_B(x_2) | \Omega \rangle = \langle \Omega | \phi_B(-x_2) \phi_A(-x_1) | \Omega \rangle. \quad (6)$$

The requirement of CPT-invariance in terms of Wightman functions is written as follows

$$W_{AB\dots N}^{(n)}(x_1, x_2, \dots, x_n) = W_{N\dots B, A}^{(n)}(-x_n, -x_{n-1}, \dots, -x_1). \quad (7)$$

The essence of the CPT Theorem is contained in the following realization [2] — [5]:

CPT Theorem

If CPT-condition (7) holds for all $x_1, \dots, x_n$ then for every $x_1, \dots, x_n$ such that $x_1 - x_2, \dots, x_{n-1} - x_n$ is a Jost point [2] — [4], the weak local commutativity condition

$$W_{AB\dots N}^{(n)}(x_1, x_2, \dots, x_n) = W_{N\dots B, A}^{(n)}(x_n, x_{n-1}, \dots, x_1) \quad (8)$$

holds. Conversely, if condition (8) holds in a (real) neighborhood of a Jost point, then CPT condition (7) holds everywhere.

Since (A3) implies the condition (8), every field theory of a local hermitean scalar field fulfilling (A1) and (A2) has CPT-symmetry.

Similar theorem is valid for other fields fulfilling the conditions listed above. One should notice that in the course of the proof of CPT Theorem, an essential role is played by the analytical properties of $\{W^{(n)}\}$ generated by (A2) [2] — [6].

2 Exponential decay and CPT Theorem.

Almost all known CPT-invariance tests for unstable particles, e.g., for neutral K mesons, are based on models containing, by assumption, exponentially decaying particles [8] — [14]. The problem of the correctness of such models and their usefulness for designing these tests is explained by the following two theorems. The first one [15] is a simple modification of theorems proved in [16] — [20]:

Theorem 1

If there exists $|\psi\rangle \in \mathcal{H}$, $|\psi\rangle \neq 0$, $\langle \psi | \psi \rangle = 1$, where $\mathcal{H}$ is the total Hilbert state space of the system considered, such that

$$p(t; |\psi\rangle) = |a(t)|^2 \equiv |\langle \psi | \exp(-itH) | \psi \rangle|^2 \leq \exp(-\gamma t), \quad (9)$$

for all $t \geq 0$, where $\gamma > 0$ and H is the total, selfadjoint Hamiltonian of the system under investigation, then the CPT Theorem is not valid for this system.

Using the projection valued measure $E(\lambda)$ for the Hamiltonian H : $H|\psi\rangle = \int \lambda dE(\lambda)|\psi\rangle$, the amplitude $a(t)$ can be expressed as follows

$$a(t) = \int e^{-it\lambda} d\|E(\lambda)|\psi\rangle\|^2. \quad (10)$$

From (9) it follows that

$$\sigma(\lambda) \stackrel{\text{def}}{=} \|E(\lambda)|\psi\rangle\|^2 \quad (11)$$

is analytically in the strip $|\text{Im}\lambda| < \gamma/2$. For real λ , the function $\sigma(\lambda)$ defines the positive Stieltjes measure [21]. So, $a(t)$ is the Fourier transform of this measure $\sigma(\lambda)$. One can show that $\sigma(\lambda)$ is absolutely continuous and has a whole real line $\mathbf{R}^1$ as its support (for details see, e.g., [16]), which means that the spectrum of H is a whole real line [16]. (The proofs that the condition (9) implies unboundedness from below the spectrum of H also can be found in [17, 18, 19]). This is in contradistinction to assumption (A2), which is necessary for the proof of CPT Theorem [2, 3, 4] and concludes the proof of our theorem.

The second Theorem is due to Williams [17]:

Theorem 2

Let $U_{\parallel}(t) = PU(t)P$, where $U(t) = \exp(-itH)$ is a strongly continuous, one parameter, unitary group on a Hilbert space $\mathcal{H} \supset \mathcal{H}_{\parallel} \equiv P\mathcal{H}$ and $P = P^+ = P^2$ is the projection operator, $H = H^+$.

(i) If

$$\|U_{\parallel}(t)|\psi\rangle\| \leq C \exp(-\gamma t) \quad (12)$$

for some $\gamma > 0$ and all $t > 0$, and for some nonvanishing $|\psi\rangle$ in $\mathcal{H}_{\parallel}$, then the spectrum of H is the whole real line.

(ii) The same conclusion holds if we have exponential decrease only in the weak sense that there are two vectors $|\psi_{\alpha}\rangle$ and $|\psi_{\beta}\rangle$ in $\mathcal{H}_{\parallel}$ with $\langle\psi_{\alpha}|\psi_{\beta}\rangle \neq 0$ and

$$|\langle\psi_{\alpha}|U_{\parallel}(t)|\psi_{\beta}\rangle| \leq C \exp(-\gamma t) \quad (13)$$

for some $\gamma > 0$ and all $t > 0$.

For the proof — see [17].

Conclusions following from this theorem are the same as those obtained within Theorem 1: If there exists a multicomponent unstable complex in the system considered, which states are described by multidimensional subspace of unstable states $\mathcal{H}_{\parallel} \subset \mathcal{H}$, and if $|\psi_{\alpha}\rangle, |\psi_{\beta}\rangle \in \mathcal{H}_{\parallel}$ fulfil conditions of the Theorem 2 and the property (13) occurs, then the total Hamiltonian H of the system considered cannot not CPT—invariant.

One should remember that estimations (9), or (12), (13) need not be valid for all $t \geq 0$ in order that conclusions of Theorems 1 and 2 hold: It is sufficient that there exist $T > 0$ such that relations (9) and (12), (13) hold for all $t \geq T$ then also the spectrum of H is the whole real line [16, 20].

Summing up, the CPT Theorem cannot be proved for a system in which H has a spectrum equal to the whole real line, and therefore one finds that CPT—transformation cannot be considered as the symmetry in models containing an unstable state $|\psi\rangle$ (or unstable states $|\psi_{\alpha}\rangle$) with assumed exponential decay laws. Simply, models with exponentially decaying particles can not be CPT—invariant! (This conclusion corresponds also with results of [22]). Therefore such the models cannot be used for studying CPT—invariance properties of physical systems and for designing CPT—invariance tests. For designing such the tests only such models can be used which are CPT—invariant or which are able to be CPT—invariant and CPT—noninvariant depending on parameters of these models. It seems to be logic that models only CPT—noninvariant can not be considered as appropriate for this purpose.

3 Discussion of the LOY model.

CP— and CPT—violation parameters in neutral kaons (or neutral B—mesons, etc.) subsystem can be expressed in terms of matrix elements h_{jk} of the effective Hamiltonian [8] — [14], [23] — [30] (which will be denoted as $H_{\parallel}$) governing the time evolution in subspace $\mathcal{H}_{\parallel} \ni |\psi; t\rangle_{\parallel}$ of state space $\mathcal{H} \ni |\psi; t\rangle$ of the total physical system under consideration, spanned by orthonormal neutral kaon states $|K_0\rangle \stackrel{\text{def}}{=} |1\rangle$, $|\overline{K}_0\rangle \stackrel{\text{def}}{=} |2\rangle$,

$$|\psi; t\rangle_{\parallel} = a_1(t)|1\rangle + a_2(t)|2\rangle, \quad (14)$$

$$|\psi\rangle_{\parallel} \stackrel{\text{def}}{=} |\psi; t=0\rangle_{\parallel} = a_1|1\rangle + a_2|2\rangle. \quad (15)$$

We define $h_{jk} \stackrel{\text{def}}{=} \langle \mathbf{j} | H_{\parallel} | \mathbf{k} \rangle$, $j, k = 1, 2$. The subspace $\mathcal{H} \ominus \mathcal{H}_{\parallel} \stackrel{\text{def}}{=} \mathcal{H}_{\perp} \ni |\psi; t\rangle_{\perp}$ is the subspace of decay products for particles considered: ${}_{\parallel} \langle \psi | \psi; t \rangle_{\perp} \equiv 0$. Time evolution in neutral kaons subspace $\mathcal{H}_{\parallel}$ is described by the Schrödinger-like equation [8] — [14], [23] — [30]:

$$i \frac{\partial}{\partial t} |\psi; t\rangle_{\parallel} = H_{\parallel} |\psi; t\rangle_{\parallel}, \quad (16)$$

for the initial conditions

$$|\psi; t=0\rangle \equiv |\psi\rangle_{\parallel}, \quad |\psi; t=0\rangle_{\perp} \equiv 0. \quad (17)$$

Generally, in the exact theory, the effective Hamiltonian $H_{\parallel}$ is time dependent: $H_{\parallel} \equiv H_{\parallel}(t)$ [31], and then the symbol $H_{\parallel}$ is reserved for $H_{\parallel}(t \rightarrow \infty) \stackrel{\text{def}}{=} H_{\parallel}$, and $h_{jk} \stackrel{\text{def}}{=} h_{jk}(t \rightarrow \infty)$ [32] — [36]. Using solutions of the Eq.(16), changes in time of vectors in subspace $\mathcal{H}_{\parallel}$, can be expressed by means of the evolution operator $U_{\parallel}(t)$ acting in $\mathcal{H}_{\parallel}$:

$$|\psi; t\rangle_{\parallel} = U_{\parallel}(t) |\psi\rangle_{\parallel}, \quad U_{\parallel}(0) = P, \quad (18)$$

where

$$P = |\mathbf{1}\rangle \langle \mathbf{1}| + |\mathbf{2}\rangle \langle \mathbf{2}|, \quad (19)$$

is the projector onto $\mathcal{H}_{\parallel}$.

The eigenstates of $H_{\parallel}$, $|l\rangle$ and $|s\rangle$, for the eigenvalues μ_l and μ_s respectively [8] — [14], [23] — [30]:

$$\mu_{l(s)} = \frac{1}{2}(h_{11} + h_{22}) - (+)\left[\frac{1}{4}(h_{11} - h_{22})^2 + h_{12}h_{21}\right]^{1/2}, \quad (20)$$

correspond with the long (the vector $|l\rangle$) and short (the vector $|s\rangle$) living superpositions of K_0 and $\overline{K}_0$. Using the eigenvectors $|K_{1(2)}\rangle \stackrel{\text{def}}{=} 2^{-1/2}(|\mathbf{1}\rangle + (-)|\mathbf{2}\rangle)$ of the CP-transformation for the eigenvalues ± 1 , vectors $|l\rangle$ and $|s\rangle$ can be expressed, e.g., as follows [8] — [14], [24] — [30]

$$|l(s)\rangle \equiv (1 + |\varepsilon_{l(s)}|^2)^{-1/2} [|K_{2(1)}\rangle + \varepsilon_{l(s)} |K_{1(2)}\rangle]. \quad (21)$$

This form of $|l\rangle$ and $|s\rangle$ is convenient if one wants to describe possible departures from CP- or CPT-symmetry in the system considered. The following parameters describe the scale of CP- and possible CPT - violation effects [8] — [14], [24] — [30]:

$$\varepsilon \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s + \varepsilon_l), \quad (22)$$

$$\delta \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s - \varepsilon_l), \quad (23)$$

According to the standard meaning, ε describes violations of CP-symmetry and δ is considered as a CPT-violating parameter [8] — [14], [24], [26] — [30]. We have

$$\delta = \frac{h_{11} - h_{22}}{h_{12} + h_{21} + \Delta\mu}, \quad (24)$$

where $\Delta\mu \stackrel{\text{def}}{=} \mu_s - \mu_l$. Such an interpretation of these parameters follows from properties of Lee–Oehme–Yang (LOY) theory of time evolution in the subspace of neutral kaons [23, 25].

The source of The Lee, Oehme and Yang (LOY) model for decay of neutral kaons is the well known Weisskopf–Wigner approach to a description of unstable states. Within this approach, the Hamiltonian H for the problem is divided into two parts $H^{(0)}$ and $H^{(1)}$:

$$H = H^{(0)} + H^{(1)}, \quad (25)$$

such that $|K_0\rangle$ and $|\bar{K}_0\rangle$ are discrete eigenstates of $H^{(0)}$ for 2-fold degenerate eigenvalue m_0 ,

$$H^{(0)}|\mathbf{j}\rangle = m_0|\mathbf{j}\rangle, \quad j = 1, 2; \quad (26)$$

and $H^{(1)}$ induces transitions from these states to other (unbound) eigenstates $|\varepsilon\rangle$ of $H^{(0)}$ and consequently also between $|K_0\rangle$ and $|\bar{K}_0\rangle$.

The condition guaranteeing the occurrence of transitions between subspaces $\mathcal{H}_{\parallel}$ and $\mathcal{H}_{\perp}$, i.e., a decay process of states in $\mathcal{H}_{\parallel}$ can be written as follows

$$[P, H^{(1)}] \neq 0. \quad (27)$$

Usually, in LOY and familiar approaches, it is assumed that

$$[\Theta, H^{(0)}] = 0. \quad (28)$$

It is also obvious that the subspace of neutral kaons $\mathcal{H}_{\parallel}$ should be invariant under Θ :

$$[\Theta, P] = 0. \quad (29)$$

The problem which one usually considers is the time evolution of a state which is prepared initially as a superposition of $|\mathbf{1}\rangle$ and $|\mathbf{2}\rangle$ states.

In the kaon rest-frame, this time evolution is governed by the Schrodinger equation

$$i \frac{\partial}{\partial t} |\Psi; t\rangle = H |\Psi; t\rangle, \quad (30)$$

where

$$|\Psi; t\rangle = a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle + \sum_j F_j(t)|F_j\rangle, \quad (31)$$

$$a_k(t) \equiv \langle \mathbf{k} | \Psi; t \rangle \equiv \langle \mathbf{k} | \exp(-itH) | \Psi; 0 \rangle, \quad (32)$$

$|F_j\rangle \equiv \sum_\varepsilon \langle \varepsilon | F_j \rangle |\varepsilon\rangle = \sum_\varepsilon f_j(\varepsilon) |\varepsilon\rangle$ represents the decay products in the channel j ; $\langle \varepsilon | \mathbf{k} \rangle = 0$, $k = 1, 2$. It is assumed that $F_j(0) = 0$.

Using the interaction representation and rescaling respectively energy ε : defining $\omega = \varepsilon - m_0$ which means that the zero of energy is taken to be rest energy of K , instead of the Schrödinger equation (30) for $|\Psi; t\rangle$, Lee, Oehme and Yang obtained the following equations for amplitudes $a_1(t)$, $a_2(t)$ and $F_j(\omega, t)$ replacing $F_j(t)$ [23, 24]

$$i \frac{\partial}{\partial t} a_k(t) = \sum_{l=1}^2 H_{kl} a_l(t) + \sum_{j,\omega} H_{kj}(\omega) F_j(\omega, t) e^{-i\omega t}, \quad (k=1,2) \quad (33)$$

$$i \frac{\partial}{\partial t} F_j(\omega, t) = e^{i\omega t} [H_{j1}(\omega) a_1(t) + H_{j2}(\omega) a_2(t)], \quad (34)$$

where $H_{kj}(\omega) = H_{jk}(\omega)^*$, ($k = 1, 2$), are the matrix elements responsible for the decay and $H_{kl} = \langle \mathbf{k} | H | \mathbf{l} \rangle$; $k, l = 1, 2$. Equations (33) are exact, equation (34) has been obtained by ignoring the series containing matrix elements of type $\langle \varepsilon | H^{(1)} | \varepsilon' \rangle$. (Eq(34) is exact in the model in which $\langle \varepsilon | H^{(1)} | \varepsilon' \rangle = 0$ — if these matrix elements are very small, then this equation can be treated as a very good approximation for the model considered).

The boundary conditions for Eqs (33), (34) are the following: $a_k(0)$, ($k = 1, 2$), and

$$F_j(\omega, 0) = 0. \quad (35)$$

To solve the system of coupled equations (33), (34), the exponential time dependence for amplitudes $a_k(t)$ has been assumed in [23], i.e., it has been assumed that

$$\frac{a_1(t)}{a_1(0)} = \frac{a_2(t)}{a_2(0)} \equiv e^{-\frac{\Lambda t}{2}}, \quad \text{Re} \Lambda > 0. \quad (36)$$

This crucial assumption is the essence of the approximation which was made in [23] and determines the properties of the so-called LOY model of neutral kaons decay, i.e., the effective Hamiltonian H_{LOY} governing time evolution in neutral kaons subspace, which is a consequence of Eqs (33), (34) and of the requirement (36).

Inserting (36) into Eq(34) and taking into account (35), Eq(34) can easily be solved to obtain

$$\begin{aligned} iF_j(\omega, t) &= [H_{j1}(\omega)a_1(0) + H_{j2}(\omega)a_2(0)] \frac{e^{i\omega t} - \Lambda t/2 - 1}{i\omega - \Lambda/2} \\ &\equiv -i[H_{j1}(\omega)a_1(t) + H_{j2}(\omega)a_2(t)]e^{i\omega t} \frac{1 - e^{\Lambda t/2}e^{-i\omega t}}{\omega + i\frac{\Lambda}{2}}. \end{aligned} \quad (37)$$

Next, one can eliminate the $F_j(\omega, t)$ in (33) by substituting (37) back into (33), and then considering Λ as a very small number: $\Lambda \simeq 0$ one finds the following equation for amplitudes $a_k(t)$ (for details, the reader is referred, e.g., to [24])

$$i\frac{\partial}{\partial t} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = H_{LOY} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}, \quad (38)$$

where (see [23, 24, 25]) $H_{LOY} \equiv M - \frac{i}{2}\Gamma$, and $M = M^+$, $\Gamma = \Gamma^+$ are (2×2) matrices. Standard formulae for matrix elements $h_{kl}^{LOY} \stackrel{\text{def}}{=} \langle \mathbf{k} | H_{LOY} | \mathbf{l} \rangle \equiv M_{kl} - \frac{i}{2}\Gamma_{kl}$ can be found, e.g., in [23, 24, 25]. Those formulae are the frame for almost all calculations of parameters characterizing CP-violation effects in neutral kaons decays, and for searching for possible violations of CPT-symmetry, and for designing CPT-violation tests in such a system [8] — [14].

Defining projector

$$Q = I - P \equiv \sum_{\epsilon} |\epsilon\rangle\langle\epsilon|, \quad (39)$$

(where I is the unit operator in $\mathcal{H}$), the matrix elements h_{jk}^{LOY} of H_{LOY} can be written in more convenient form for examining their CPT-transformation properties than the standard one:

$$h_{jk}^{LOY} = H_{jk} - \Sigma_{jk}(m_0), \quad (j, k=1, 2), \quad (40)$$

where

$$H_{jk} \equiv m_0 \delta_{jk} + \langle \mathbf{j} | H^{(1)} | \mathbf{k} \rangle, \quad (41)$$

and $\Sigma_{jk}(\varepsilon) = \langle \mathbf{j} | \Sigma(\varepsilon) | \mathbf{k} \rangle$, and

$$\begin{aligned} \Sigma(\varepsilon) &= PHQ \frac{1}{QH Q - \varepsilon - i0} QHP \\ &\stackrel{\text{def}}{=} \Sigma^R(\varepsilon) + i\Sigma^I(\varepsilon), \end{aligned} \quad (42)$$

where for ε real, $\Sigma^R(\varepsilon) = \Sigma^R(\varepsilon)^+$ and $\Sigma^I(\varepsilon) = \Sigma^I(\varepsilon)^+ \equiv \frac{1}{2}\Gamma(\varepsilon)$. This means that simply

$$H_{LOY} = m_0 P + PH^{(1)}P - \Sigma(m_0) \equiv PHP - \Sigma(m_0). \quad (43)$$

It should be noticed that the assumption (36) does not only change integration properties of Eq(34) essentially, making the integration of this equation easier, but also it essentially changes the analytical properties of the amplitude $F_j(\omega, t)$ and thus amplitudes $a_k(t)$ leading to definite properties of matrix elements of H_{LOY} . According to the Theorems 1 and especially 2 of Sec.2 (the amplitudes $a_k(t)$ (32) are strictly connected by requirement (36) with the expression (13) in Theorem 2), these changes of analytical properties of amplitudes considered cause the model obtained to behave like a system with the Hamiltonian unbounded from below. Therefore CPT transformation cannot be the symmetry for such an obtained model, even if it was the symmetry for the initial system. This means that solutions of Eq(30), (or Eqs (33), (34)), and solutions of Eq(38) should have different CPT-transformation properties. (This reservation need not take place in the case of the CP- transformation).

Assuming that CPT is the symmetry for the system under investigation:

$$[\Theta, H] = 0, \quad (44)$$

i.e., that $[\Theta, H^{(1)}] = 0$ additionally, one easily finds that

$$H_{11} \equiv H_{22}, \quad (45)$$

and

$$\Sigma_{11}^R(m_0) \equiv \Sigma_{22}^R(m_0), \quad \Sigma_{11}^I(m_0) \equiv \Sigma_{22}^I(m_0). \quad (46)$$

Therefore the obvious conclusion, exploited widely in the literature, is that

$$\Sigma_{11}(m_0) \equiv \Sigma_{22}(m_0), \quad (47)$$

which mean that

$$h_{11}^{LOY} \equiv h_{22}^{LOY}. \quad (48)$$

in CPT-invariant system (44), (29). This is the standard result of the LOY approach and this is the picture which one meets in the literature [8] — [14], [23] — [30] and which one obtains searching only for properties of matrix elements of above obtained H_{LOY} . (In the light of the conclusions following from above Theorems 1 and 2 this relation need not be true for real systems). Property (48) leads to the conclusion that

$$\delta \simeq \delta^{LOY} \equiv 0, \quad (49)$$

Now, let us imagine the following situation: an experimenter is equipped only with apparatus measuring δ -parameter exclusively, so he is unable to measure other parameters characterizing properties of the neutral kaon complex. Next, let us assume that he knows only the LOY theory and that as a result of his experiment the value $\delta = 0$ is obtained. Then the problem of how to interpret this result arises. Within the LOY approximation the following interpretations are admissible: Firstly that the system under investigation is CPT-invariant, i.e., that the relation (44) holds. Secondly: the system is CP-invariant but it is not CPT-invariant. (It is easy to verify that if $[C\mathcal{P}, H] = 0$ then also $h_{11}^{LOY} \equiv h_{22}^{LOY}$, i.e., $\delta^{LOY} = 0$). Thirdly: the system is CP- and CPT-invariant (or, equivalently, CP- and T-invariant). Without other experiments (i.e., without measuring other characteristic parameters, e.g., ε , for neutral K mesons) it is impossible to decide which interpretation is correct.

The standard interpretation (founded on the experimental result that $[C\mathcal{P}, H] \neq 0$ for neutral kaon decays [37] and on the properties of the LOY model) of the parameter δ is as follows: the experimentally obtained value $\delta = 0$ is equivalent to the property: $[\Theta, H] = 0$ [8] — [14], [23] — [30]. Taking into account conclusions of Sec. 2) and results obtained in [27], [32] — [36] one finds that relation (48) and this interpretation may not be true for real systems. Only more accurate approximation can guarantee proper interpretation of the relations (48), (49). An example of such the

approximation has been given in [27]. Another example has been considered in [33] — [35]. Assuming (44) $h_{11} \neq h_{22}$ was found in [27] and in [33] — [35] within those approximations.

4 Beyond the LOY approximation.

The aim of this Section is describe briefly, for readers convenience, the approximate formulae for the effective Hamiltonians $H_{\parallel}(t)$ and $H_{\parallel}$ found in [33] — [35], and to show that these formulae enable us to find some estimation for $(h_{11} - h_{22})$ and lead to a new conclusion that the value zero can not be allowable for the parameter δ if the total Hamiltonian is CPT-invariant, contrary to the conclusions of the LOY theory [8] — [14], [23] — [30]. The explanation of different predictions of the LOY theory and of the approach exploited in [33] — [35] is as follows: the approximation used in these papers is more exact than the Weisskopf-Wigner approximation leading to the LOY model. The approximate formulae for $H_{\parallel}(t)$ has been derived in [34, 35] using the Krolkowski-Rzewuski equation for the projection of a state vector [38, 39] and [32] — [35], [40] — [42], which results from the Schrödinger equation for the total system under consideration [38, 39], and, in the case of initial conditions (17), takes the following form

$$\begin{aligned} (i\frac{\partial}{\partial t} - PHP)U_{\parallel}(t) &= -i\int_0^{\infty} K(t-\tau)U_{\parallel}(\tau)d\tau, \\ U_{\parallel}(0) &= P, \end{aligned} \quad (50)$$

where:

$$\begin{aligned} K(t) &= \Theta(t)PHQ \exp(-itQHQ)QHP, \\ \Theta(t) &= \{1 \text{ for } t \geq 0, \ 0 \text{ for } t < 0\}. \end{aligned} \quad (51)$$

Putting

$$H_{\parallel}(t) \equiv PHP + V_{\parallel}(t). \quad (52)$$

one finds from (16), (18) and (50)

$$V_{\parallel}(t)U_{\parallel}(t) = -i\int_0^{\infty} K(t-\tau)U_{\parallel}(\tau)d\tau \stackrel{\text{def}}{=} -iK * U_{\parallel}(t). \quad (53)$$

(Here the star $*$ denotes the convolution: $f * g(t) = \int_0^\infty f(t - \tau)g(\tau) d\tau$). Next, using this relation and a retarded Green's operator $G(t)$ for the equation (50)

$$G(t) = -i\Theta(t) \exp(-itPHP)P, \quad (54)$$

one obtains [34, 35, 42]

$$V_{\parallel}(t) U_{\parallel}(t) = -iK * \left[1 + \sum_{n=1}^{\infty} (-i)^n L * \dots * L \right] * U_{\parallel}^{(0)}(t), \quad (55)$$

where L is convoluted n times, $1 \equiv 1(t) \equiv \delta(t)$,

$$L(t) = G * K(t), \quad (56)$$

and

$$U_{\parallel}^{(0)} = \exp(-itPHP) P \quad (57)$$

is a "free" solution of Eq.(50). Of course, the series (55) is convergent if $\|L(t)\| < 1$. If for every $t \geq 0$

$$\|L(t)\| \ll 1, \quad (58)$$

then, to the lowest order of $L(t)$, one finds from (55) [34, 35, 42]

$$V_{\parallel}(t) \cong V_{\parallel}^{(1)}(t) \stackrel{\text{def}}{=} -i \int_0^\infty K(t - \tau) \exp[i(t - \tau)PHP] d\tau. \quad (59)$$

In case (19) of the projector P , this approximate formula for $V_{\parallel}(t)$ leads to the following expressions for the matrix elements $v_{jk}(t \rightarrow \infty) \stackrel{\text{def}}{=} v_{jk}$ of $V_{\parallel}(t \rightarrow \infty) \stackrel{\text{def}}{=} V_{\parallel} \cong V_{\parallel}^{(1)}(\infty)$ [34, 35],

$$\begin{aligned} v_{j1} = & - \frac{1}{2} \left(1 + \frac{H_z}{\kappa} \right) \Sigma_{j1}(H_0 + \kappa) - \frac{1}{2} \left(1 - \frac{H_z}{\kappa} \right) \Sigma_{j1}(H_0 - \kappa) \\ & - \frac{H_{21}}{2\kappa} \Sigma_{j2}(H_0 + \kappa) + \frac{H_{21}}{2\kappa} \Sigma_{j2}(H_0 - \kappa), \end{aligned} \quad (60)$$

$$\begin{aligned} v_{j2} = & - \frac{1}{2} \left(1 - \frac{H_z}{\kappa} \right) \Sigma_{j2}(H_0 + \kappa) - \frac{1}{2} \left(1 + \frac{H_z}{\kappa} \right) \Sigma_{j2}(H_0 - \kappa) \\ & - \frac{H_{12}}{2\kappa} \Sigma_{j1}(H_0 + \kappa) + \frac{H_{12}}{2\kappa} \Sigma_{j1}(H_0 - \kappa), \end{aligned} \quad (61)$$

where $j, k = 1, 2$,

$$H_0 = \frac{1}{2}(H_{11} + H_{22}), \quad H_z = \frac{1}{2}(H_{11} - H_{22}), \quad (62)$$

$$\kappa = (|H_{12}|^2 + H_z^2)^{1/2}. \quad (63)$$

Hence, by (52)

$$h_{jk} = H_{jk} + v_{jk}. \quad (64)$$

These formulae for v_{jk} and thus for h_{jk} have been derived without assuming any symmetries of the type CP-, T-, or CPT-symmetry for the total Hamiltonian H of the system considered. It should also be emphasized that all components of the expressions (60), (61) have the same order with respect to $\Sigma(\varepsilon)$.

In the case of preserved CPT-symmetry (44), property (45) and the other similar one imply that $\kappa \equiv |H_{12}|$, $H_z \equiv 0$ and $H_0 \equiv H_{11} \equiv H_{22}$, and [34, 35]

$$\Sigma_{11}(\varepsilon = \varepsilon^*) \equiv \Sigma_{22}(\varepsilon = \varepsilon^*) \stackrel{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*). \quad (65)$$

Therefore matrix elements v_{jk}^Θ of operator $V_\parallel^\Theta$ ($V_\parallel^\Theta$ denotes $V_\parallel$ when (44) occurs) take the following form

$$\begin{aligned} v_{j1}^\Theta = & - \frac{1}{2} \left\{ \Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) \right. \\ & \left. + \frac{H_{21}}{|H_{12}|} \Sigma_{j2}(H_0 + |H_{12}|) - \frac{H_{21}}{|H_{12}|} \Sigma_{j2}(H_0 - |H_{12}|) \right\}, \end{aligned} \quad (66)$$

$$\begin{aligned} v_{j2}^\Theta = & - \frac{1}{2} \left\{ \Sigma_{j2}(H_0 + |H_{12}|) + \Sigma_{j2}(H_0 - |H_{12}|) \right. \\ & \left. + \frac{H_{12}}{|H_{12}|} \Sigma_{j1}(H_0 + |H_{12}|) - \frac{H_{12}}{|H_{12}|} \Sigma_{j1}(H_0 - |H_{12}|) \right\}, \end{aligned}$$

Assuming

$$|H_{12}| \ll |H_0|, \quad (67)$$

we find

$$v_{j1}^\Theta \simeq -\Sigma_{j1}(H_0) - H_{21} \frac{\partial \Sigma_{j2}(x)}{\partial x} \Big|_{x=H_0}, \quad (68)$$

$$v_{j2}^\Theta \simeq -\Sigma_{j2}(H_0) - H_{12} \frac{\partial \Sigma_{j1}(x)}{\partial x} \Big|_{x=H_0}, \quad (69)$$

where $j = 1, 2$. One should stress that due to a presence of resonance terms, derivatives $\frac{\partial}{\partial x} \Sigma_{jk}(x)$ need not be small, and so the products $H_{jk} \frac{\partial}{\partial x} \Sigma_{jk}(x)$ in (68), (69).

Finally, assuming that (67)) holds and using relations (68),(69),(64) and the expression (40), we obtain for the CPT-invariant system

$$h_{j1}^\Theta \simeq h_{j1}^{\text{LOY}} - H_{21} \frac{\partial \Sigma_{j2}(x)}{\partial x} \Big|_{x=H_0}, \quad (70)$$

$$h_{j2}^\Theta \simeq h_{j2}^{\text{LOY}} - H_{12} \frac{\partial \Sigma_{j1}(x)}{\partial x} \Big|_{x=H_0}, \quad (71)$$

where $j = 1, 2$ and h_{jk}^Θ denotes matrix elements of $H_\parallel^\Theta$ ($H_\parallel^\Theta$ is the operator $H_\parallel$ when the property (44) occurs). From this formulae we conclude that, e.g., the difference between diagonal matrix elements of $H_\parallel^\Theta$, which plays an important role in designing CPT-invariance tests for the neutral kaons system, equals

$$2h_z^\Theta \equiv h_{11}^\Theta - h_{22}^\Theta \simeq 2H_{12} \frac{\partial \Sigma_{21}(x)}{\partial x} \Big|_{x=H_0} - H_{21} \frac{\partial \Sigma_{12}(x)}{\partial x} \Big|_{x=H_0} \neq 0. \quad (72)$$

Analogously, to the lowest order of $|H_{12}|$, for eigenvalues μ_l, μ_s (20) of $H_\parallel$, we obtain [35]

$$\begin{aligned} \mu_s^\Theta &\simeq \mu_s^{\text{LOY}} - \frac{1}{2} \left[H_{12} \left(\frac{\partial \Sigma_{21}(x)}{\partial x} \Big|_{x=H_0} + a \frac{\partial \Sigma_0(x)}{\partial x} \Big|_{x=H_0} \right) \right. \\ &\quad \left. + H_{21} \left(\frac{\partial \Sigma_{12}(x)}{\partial x} \Big|_{x=H_0} + a^{-1} \frac{\partial \Sigma_0(x)}{\partial x} \Big|_{x=H_0} \right) \right], \\ \mu_l^\Theta &\simeq \mu_l^{\text{LOY}} - \frac{1}{2} \left[H_{12} \left(\frac{\partial \Sigma_{21}(x)}{\partial x} \Big|_{x=H_0} - a \frac{\partial \Sigma_0(x)}{\partial x} \Big|_{x=H_0} \right) \right. \\ &\quad \left. + H_{21} \left(\frac{\partial \Sigma_{12}(x)}{\partial x} \Big|_{x=H_0} - a^{-1} \frac{\partial \Sigma_0(x)}{\partial x} \Big|_{x=H_0} \right) \right], \end{aligned} \quad (73)$$

where,

$$a \stackrel{\text{def}}{=} \left(\frac{h_{21}^{LOY}}{h_{12}^{LOY}} \right)^{1/2}, \quad (74)$$

and so on.

So, in a general case, in contradistinction to the property (48) obtained within the LOY theory, one finds for diagonal matrix elements of $H_{\parallel}$ calculated within the above described approximation that in CPT-invariant system (see (66) — (71))

$$h_{11}^{\Theta} \neq h_{22}^{\Theta}, \quad (75)$$

(The other approximation improving WW formulae for h_{jk} and used in [27] lead to the same result).

Assuming condition (67) one finds that ($2h_z^{\Theta} = 0$ only if $[\mathcal{CP}, H] = 0$). This means that if CPT-symmetry is conserved in the system considered and (67) holds then the parameter δ (24) takes the following form

$$\delta^{\Theta} \equiv \frac{2h_z^{\Theta}}{D^{\Theta}}, \neq 0, \quad (76)$$

where

$$D^{\Theta} \simeq D^{LOY} - (H_{12} + H_{21}a^{-1})(1+a)\frac{\partial \Sigma_0(x)}{\partial x} \Big|_{x=H_0},$$

and $D^{LOY} = h_{12}^{LOY} + h_{21}^{LOY} + \Delta\mu^{LOY}$, $\Delta\mu^{LOY} \stackrel{\text{def}}{=} \mu_s^{LOY} - \mu_l^{LOY}$, μ_s^{LOY} and μ_l^{LOY} are eigenvalues of H_{LOY} for eigenstates $|K_s\rangle$ and $|K_l\rangle$ respectively.

Using the above, briefly described formalism, one can find $(h_{11} - h_{22})$ for generalized Fridrichs-Lee model [43]. Assuming CPT-invariance (i.e., (44), it has been found in [34] (see formulae (152), (144) and (145) in [34]) that

$$\begin{aligned} 2h_z &\equiv h_{11} - h_{22} = \lim_{t \rightarrow \infty} (h_{11}(t) - h_{22}(t)) \\ &\equiv \frac{m_{21}\Gamma_{12} - m_{12}\Gamma_{21}}{|4m_{12}|} (F_0(m_0 - \mu - |m_{12}|) - (F_0(m_0 - \mu + |m_{12}|))), \end{aligned} \quad (77)$$

where, Γ_{12}, Γ_{21} can be identified with those apperaing in the LOY theory, $m_0 \equiv H_{11} = H_{22}$ can be considered as kaon mass [43], $m_{jk} \equiv H_{jk}$ ($j, k = 1, 2$), μ can be traeted as the mass of the decay products of the neutral kaon [43], and

$$\begin{aligned} F_0(m) &= im^{-1/2}a_1(0), \\ a_1(0) &\equiv (m_0 - \mu)^{1/2}, \end{aligned}$$

which, in the case of conserved CPT-symmetry, lead to the following estimation for $|m_{12}| \ll (m_0 - \mu)$ (which corresponds to (67)):

$$\begin{aligned} h_{11} - h_{22} &\simeq i \frac{m_{21}\Gamma_{12} - m_{12}\Gamma_{21}}{4(m_0 - \mu)} \\ &\equiv \frac{\text{Im.}(m_{12}\Gamma_{21})}{2(m_0 - \mu)} \stackrel{\text{df}}{=} 2h_z^{FL}. \end{aligned} \quad (78)$$

For neutral K-system, to evaluate $2h_z^{FL}$ one can follow, e.g., [43] and one can take $\frac{1}{2}\Gamma_{21} = \frac{1}{2}\Gamma_{12}^* \sim \frac{1}{2}\Gamma_s \sim 5 \times 10^{10}\text{sec}^{-1}$ and $(m_0 - \mu) = m_K - 2m_\pi \sim 200 \text{ MeV} \sim 3 \times 10^{23}\text{sec}^{-1}$ [43]. Thus

$$2|h_z^{FL}| \sim 1,7 \times 10^{-13} |m_{12}| \equiv 1,7 \times 10^{-13} |H_{12}|. \quad (79)$$

Relation (78) is the numerator in the formula (24) for the parameter δ . To evaluate the denominator of this expression one has to know estimations of the sum $(h_{12} + h_{21})$ and $\Delta\mu$. For generalized Fridrichs–Lee model one finds in the case of preserved CPT-symmetry (see [34], formula (146))

$$\begin{aligned} h_{12} + h_{21} &= m_{12} + m_{21} \\ &- \frac{1}{4}[(\Gamma_{12} + \frac{m_{12}}{|m_{12}|}\Gamma_{11}) \\ &+ (\Gamma_{21} + \frac{m_{21}}{|m_{12}|}\Gamma_{22})]F_0(m_0 - \mu + |m_{12}|) \\ &- \frac{1}{4}[(\Gamma_{12} - \frac{m_{12}}{|m_{12}|}\Gamma_{11}) \\ &+ (\Gamma_{21} - \frac{m_{21}}{|m_{12}|}\Gamma_{22})]F_0(m_0 - \mu - |m_{12}|). \end{aligned} \quad (80)$$

(where $\Gamma_{11} = \Gamma_{22} \stackrel{\text{def}}{=} \Gamma_0$ if (44) holds). Within assumptions (44), and, analogously to the estimation (78), $|m_{12}| \ll (m_0 - \mu)$ relation (79) can be replaced by the following one:

$$\begin{aligned} h_{12} + h_{21} &\simeq m_{12} - \frac{i}{2}\Gamma_{12} + m_{21} - \frac{i}{2}\Gamma_{21} \\ &+ \frac{i}{4} \frac{m_{12} + m_{21}}{m_0 - \mu} \Gamma_0 \stackrel{\text{def}}{=} h_{12}^{FL} + h_{21}^{FL}. \end{aligned} \quad (81)$$

Using these same assumptions the following estimation for $\Delta\mu$ can be found

$$\begin{aligned}\Delta\mu &\simeq 2[(m_{12} - \frac{i}{2}\Gamma_{12})(m_{21} - \frac{i}{2}\Gamma_{21})]^{1/2} \\ &+ \frac{1}{8} \frac{m_{12}\Gamma_{21} + m_{21}\Gamma_{12} + i4|m_{12}|^2}{[(m_{12} - \frac{i}{2}\Gamma_{12})(m_{21} - \frac{i}{2}\Gamma_{21})]^{1/2}} \frac{\Gamma_0}{m_0 - \mu} \\ &\stackrel{\text{def}}{=} \Delta\mu^{FL}.\end{aligned}\tag{82}$$

In the literature, the assumption $h_{12} + h_{21} \simeq \Delta\mu$ is considered to be an obvious condition in the course of derivation of the formula for the parameter δ (24) [8] — [14]. Therefore in the denominator of the expression (24) the term $(h_{12} + h_{21})$ is usually replaced by $\Delta\mu$, which yields [8] — [14]:

$$\delta \simeq \frac{h_{11} - h_{22}}{2\Delta\mu} \stackrel{\text{def}}{=} \delta^{st}.\tag{83}$$

This is the standard formula for the CPT-violation parameter δ exploited in the large literature. The expression (83) for δ^{st} is the basic formula for a discussion and interpretations of many experiments with neutral kaons.

From (81) and (82) it only follows that the order of expressions $(h_{12}^{FL} + h_{21}^{FL})$ and $\Delta\mu^{FL}$ is the same. This means that quantity $(h_{12} + h_{21})$ should not be replaced by $\Delta\mu$ when calculating the parameter δ (24) more accurate than it is possible within the LOY approach. Therefore one can also suppose that the use of δ^{st} (83) instead of δ (24) can lead to wrong conclusions.

Searching for another properties of the Friedrichs–Lee model it has been also found in [34] that $h_{jk}(t) \simeq h_{jk}$ practically for $t \geq T_{as} \simeq \frac{10^2}{\pi(m_0 - \mu - |m_{12}|)}$ (see [34], formula (153)). Therefore, within assumptions used above we obtain $T_{as} \sim 10^{-22}$ sec.

5 Final remarks.

The problem, which seems to be worth investigating, is what consequences result for the total Hamiltonian H from the fact that the parameter δ (24) is equal to zero. It is obvious that this problem can be reduced to the search for the value of the numerator of the expression (24): $\delta = 0$ only and only if $(h_{11} - h_{22}) = 0$. Therefore an examination of conditions under which

$(h_{11} - h_{22}) = 0$ is crucial for this question. Approximate formulae (64) and (60), (61) for h_{jk} seem to be too complicated for the simple realization of this purpose. It is much easier to come to some conclusions about the implications for H of the zero value of the difference of diagonal matrix elements of $H_{||}$, i.e. of $(h_{11} - h_{22}) = 0$, following the method used in [35] and described in Sec. 4, i.e., assuming (analogously to (67)) that κ (63) is relatively small. So, let

$$\kappa < |H_0|, \quad (84)$$

then expanding Σ_{jk} in (60), (61) into Taylor series, one obtains to the lowest, nontrivial order of κ :

$$h_{11} - h_{22} = \alpha + \beta + \zeta + \mathcal{O}(\kappa^2), \quad (85)$$

where

$$\alpha = (H_{11} - H_{22}) \left\{ 1 - \frac{1}{2} \frac{\partial}{\partial x} [\Sigma_{11}(x) + \Sigma_{22}(x)] \right|_{x=H_0} \right\}, \quad (86)$$

$$\beta = \Sigma_{22}(H_0) - \Sigma_{11}(H_0), \quad (87)$$

$$\zeta = H_{12} \frac{\partial \Sigma_{21}(x)}{\partial x} \Big|_{x=H_0} - H_{21} \frac{\partial \Sigma_{12}(x)}{\partial x} \Big|_{x=H_0}. \quad (88)$$

From the properties of $\mathcal{CP}$, $\mathcal{T}$ and Θ transformations (see, eg., [1] — [6], [44] — [48]), one can easily infer that if the system considered is CPT-invariant but CP-noninvariant, i.e., if the property (44) holds but $[\mathcal{CP}, H] \neq 0$ then $\alpha = \beta \equiv 0$ but $\zeta \neq 0$ (see (65), and (72), (78)). So, in this case it is necessary that $(h_{11} - h_{22}) \neq 0$ and $\delta \neq 0$.

Taking into account the interpretation problems of the experimenter equipped with an apparatus measuring only δ parameter described in Sec. 3, one concludes from (85) that, as in the case of the LOY model discussed above, the case $\alpha = \beta = \zeta = 0$, i.e., $(h_{11} - h_{22}) = 0$, can occur if:

i) $[\mathcal{CP}, H] = [\mathcal{T}, H] = 0$ (but, as has already been mentioned, the experimental result is $[\mathcal{CP}, H] \neq 0$ for neutral kaons decays [37], [8] — [14], [24] — [30]),

or

ii) $[\Theta, H] \neq 0$.

This last conclusion results from the fact that if $[\mathcal{CP}, H] \neq 0$ then for $[P, \Theta] = 0$ in any case, independently of whether $\Theta H \Theta^{-1} = H$ or $\Theta H \Theta^{-1} \neq H$, there are $\zeta \neq 0$, and $\zeta \neq \pm\beta \neq \pm\alpha$, $\zeta \neq \pm\alpha$. So, if $\alpha + \beta + \zeta = 0$ then $\alpha \neq 0$, which is possible only if $H_{11} \neq H_{22}$. If $[\mathcal{CP}, H] \neq 0$ then the case $(h_{11} - h_{22}) = 0$ is possible only if such a nontrivial theory (i.e., Hamiltonian H) exists in which the condition $\Theta H \Theta^{-1} = H$ would imply that $(H_{11} - H_{22}) = 0$, $\beta = 0$ simultaneously with $\zeta = 0$. Conversely, if $[\mathcal{CP}, H] \neq 0$ but $[\Theta, H] = 0$, which implies that $H_{11} = H_{22}$, then $(h_{11} - h_{22}) \neq 0$, that is $\delta \neq 0$.

Usually, as was mentioned, the form (25) of a total Hamiltonian H is considered, and properties (28), (27) are assumed. (There are no assumptions about $[\Theta, H^{(1)}]$). In such a case it is possible that it may be $(H_{11} - H_{22}) = 0$ and thus the first term (i.e., α) of the sum (85) can be equal to zero: $\alpha = 0$. (In this case $h_{jk} = h_{jk}^{\text{LOY}}$ if, additionally, $H_{12} = H_{21} = 0$). As was mentioned above, $[\mathcal{CP}, H] = [\mathcal{CP}, H^{(1)}] \neq 0$ in neutral kaons system, which means that $\zeta \neq 0$ (only the nontrivial case $H_{12} = H_{21}^* \neq 0$ is considered).

Taking these conclusions and (85) — (88) into account, and accepting the point of view of the above mentioned experimenter, one finds that $(h_{11} - h_{22}) = 0$ can occur only if $\beta \neq 0$ in (85) but this is possible only if $[\Theta, H^{(1)}] \neq 0$, i.e., $[\Theta, H] \neq 0$. (Similar conclusions can be drawn using the method based on some extension of the Weisskopf-Wigner approximation discussed in [27] and different from the approximation described in Sec. 4).

In the light of the above discussion, only the experimental result $(h_{11} - h_{22}) \simeq 2h_z^{FL}$, or $(h_{11} - h_{22}) \simeq 2h_z^\Theta$, means that CPT-symmetry is preserved in the system under consideration. On the other hand, results $|h_{11} - h_{22}| \ll 2|h_z^{FL}|$, (or $|h_{11} - h_{22}| \ll 2|h_z^\Theta|$) and $|h_{11} - h_{22}| \gg 2|h_z^{FL}|$, (or $|h_{11} - h_{22}| \gg 2|h_z^\Theta|$) signal that CPT-symmetry can be violated in the system considered, i.e., that $[\Theta, H] = 0$.

Analogous consequences will follow from the following results of experiments: Results $|\delta| \ll |\delta^\Theta|$ or $|\delta| \gg |\delta^\Theta|$ will prove that CPT-symmetry is violated by interactions causing decays of neutral K-mesons or similar systems. Only the result $\delta \simeq \delta^\Theta \neq 0$ can be considered as the confirmation of CPT-invariance of the tested system.

The LOY theory is unable to give a correct interpretation of such hypothetical, experimental results. Conclusions obtained in Sec. 2 explain of such the property of LOY approximation. Similar reservations concern models of neutral kaons decay in which Bell-Steinberger unitarity relation is assumed

to be valid: the exponential decay laws for states $|K_0\rangle$ and $|\bar{K}_0\rangle$ was also assumed in the course of the derivation of this relation [26, 49, 22].

Summing up, the general conclusion is the following: formulae for the effective Hamiltonian governing the time evolution in neutral kaons subspace obtained within the LOY model should not be used for designing CPT-invariance tests. Also, an interpretation of such tests within the LOY model is under question. (This conclusion agrees with the results of [50]).

The magnitude of $(h_{11} - h_{22})$ depends on the model of interactions considered. So, the formalism described in Sec. 4 gives a new possibility of the verification of models of interactions leading to decay processes in neutral kaon complex.

Above remarks and results of Sec. 4 lead also to the conclusion that one should be very careful when interpreting results of experiments with neutral kaons, in which violations of quantum mechanics are tested [51, 52]. (The interpretation of such experiments, which has been generally accepted till now, is founded on the standard properties of the LOY approximation).

References

- [1] W. Pauli, in: "Niels Bohr and the Development of Physics", ed. W. Pauli (Pergamon Press, London 1955) pp. 30 -51. G. Luders, K. Dan. Vidensk. Selsk. Mat. Fys. Medd., **28**, No5, p.1(1954); Ann. Phys. (NY) **2**, (1957) 1.
- [2] R. F. Streater and A. S. Wightman, CPT, Spin, Statistics and All That (Benjamin, New York, 1964).
- [3] R. Jost, Helv. Phys. Acta **30**, (1957) 409 ; The General Theory of Quantized Fields, (American Mathematical Society, Providence - Rhode Islands 1965).
- [4] N. N. Bogolubov, A. A. Logunov and I. T. Todorov, Introduction to Axiomatic Field Theory (Benjamin Inc., New York, 1975).

- [5] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Petersson and Co.; Evanston - Ill., Elmsford - N. Y. , 1961).
- [6] R. H. Dalitz, *Nucl. Phys. B (Proc. Suppl.)*, **24A**, (1991) 3.
- [7] A. S. Wightman, *Phys. Rev.* **101**, (1956) 860.
- [8] J.W.Cronin, *Rev. Mod. Phys.* **53**, 373(1981).
- [9] J.W.Cronin, *Acta Phys. Polon.*, **B15**, 419(1984).
- [10] V.V.Barmin, *et al.*, *Nucl. Phys.* **B247**, 293(1984).
- [11] L.Lavoura, *Ann. Phys. (NY)*, **207**, 428(1991).
- [12] C.Buchanan, *et al.*, *Phys. Rev.* **D45**, 4088(1992).
- [13] C.O.Dib and R.D.Peccei, *Phys. Rev.*, **D46**, 2265(1992).
- [14] R.D.Peccei, *CP and CPT Violation: Status and Prospects*; preprint UCLA/93/TEP/19, University of California, June 1993.
- [15] K. Urbanowski, "Why the LOY model cannot be used for designing CPT-invariance tests in the neutral kaons system? On CPT-noninvariance of systems with exponentially decaying particles.", Preprint of the Pedagogical University No WSP-IF 94-38, Zielona Gora, May 1994.
- [16] K. Sinha, *Helv. Phys. Acta* **45**, (1972) 619.

- [17] D. N. Williams, Commun. math. Phys., **21**, (1971) 314.
- [18] L. P. Horwitz, J. A. La Vita and J. P. Marchand, Journ. Math. Phys., **12**, (1971) 2537.
- [19] E. B. Davies, Quantum Theory of Open Systems, (Academic Press, London, 1976); Chap. 7.
- [20] P. Exner, Open Quantum Systems and Feynman Integrals, (Reidel, Dordrecht, 1985).
- [21] S. Bochner, Lectures on Fourier Integrals, (Princeton University Press, 1959).
- [22] P. K. Kabir and A. Pilaftsis, Phys. Rev. **A 53**, (1996) 66.
- [23] T. D. Lee, R. Oehme and C. N. Yang, Phys. Rev., **106**, (1957) 340.
- [24] Ed.: M. K. Gaillard and M. Nikolic, Weak Interactions, (INPN et de Physique des Particules, Paris, 1977); Chapt. 5, Appendix A.
- [25] T. D. Lee and C. S. Wu, Annual Review of Nuclear Science, **16**, (1966) 471.
- [26] S.M. Bilenkij, Particles and nucleus, vol. 1. No 1 (Dubna 1970), p. 227 (in Russian). P. K. Kabir, The CP-puzzle, Academic Press, New York 1968.
- [27] L.A.Khalfin, The theory of $K_0, \bar{K}_0$ ($D_0, \bar{D}_0$ and $T_0, \bar{T}_0$ mesons beyond the Weisskopf-Wigner approximation and the CP-problem, preprint LOMI

P-4-80, Leningrad, February 1980 [in Russian].

- [28] P.K.Kabir, *The CP-puzzle* (Academic Press, New York, 1968).
- [29] E.D. Comins and P.H.Bucksbaum, *Weak Interactions of Leptons and Quarks* (Cambridge University Press, 1983).
- [30] T.P.Cheng and L.F.Li, *Gauge Theory of Elementary particle Physics* (Clarendon, Oxford, 1984).
- [31] L.P. Horwitz and J.P Marchand, *Helv. Phys. Acta.* **42** (1966) 801.
- [32] K. Urbanowski, *Phys. Lett.* **A171**, (1992) 151.
- [33] K.Urbanowski, *Int. J. Mod. Phys.* **A7**, 6299 (1992).
- [34] K.Urbanowski, *Int. J. Mod. Phys.* **A8**, 3721 (1993).
- [35] K.Urbanowski, *Int. J. Mod. Phys.* **A 10**, 1155 (1995).
- [36] K.Urbanowski, *Phys. Lett.* **B 313**, 374 (1993).
- [37] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, *Phys. Rev. Lett.* **13**, 138 (1964).
- [38] W.Krolikowski and J.Rzewuski, *Bull. Acad. Polon. Sci.*, **4**, 19 (1956).

- [39] W.Krolikowski and J.Rzewuski, *Nuovo Cimento* **B25**, 739 (1975) and references one can find therein.
- [40] K.Urbanowski, *Acta Phys. Polon.* **B14**, 485 (1983).
- [41] K.Urbanowski, *Int. J. Mod. Phys.* **A6**, 1051 (1991).
- [42] K.Urbanowski, *Phys. Rev.* **A 50** 2847 (1994).
- [43] C.B. Chiu and E.C.G. Sudarshan, *Phys. Rev.* **D42**, (1990) 3712, E. C. G. Sudarshan, C. B. Chiu and G. Bhamathi, Unstable Systems in Generalized Quantum Theory, Preprint DOE-40757-023 and CPP-93-23, University of Texas, October 1993.
- [44] A.Messiah, *Quantum Mechanics*, vol.2, (Wiley, New York, 1966).
- [45] A.Bohm, *Quantum Mechanics: Foundations and Applications*, 2nd ed. (Springer, New York, 1986).
- [46] E.P.Wigner, in: *Group Theoretical Concepts and Methods in Elementary Particle Physics*, Ed. F.Gürsey, (New York 1964).
- [47] Yu.V.Novozhilov, *Introduction to the Theory of Elementary Particles* (Nauka, Moscow, 1972), [in Russian].
- [48] W.M.Gibson and B.R.Pollard, *Symmetry Principles in Elementary Particle Physics* (Cambridge University Press, 1976).
- [49] J.S. Bell and J. Steinberger, in: Oxford Int. Conf. on Elementary Particles 19/25 September 1965: Proceedings, Eds. T.R. Walsh, A.E.Taylor,

R.G.Moorhouse and B.Southworth, (Rutheford High Energy Lab., Chilton, Didcot 1966), pp. 195 — 222.

- [50] K. Urbanowski, Is the new interpretation of some standard CPT-violation parameters necessary?, Preprint of the Pedagogical University, No WSP-IF 94-39, Zielona Gora, May 1994.

- [51] J. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Nucl. Phys. **B241**, (1984) 381. J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. **B 293** (1992) 142. J. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos, Precision Tests of CPT Symmetry and Quantum Mechanics in the Neutral Kaon System, Preprint: CERN-TH.95-99, May 1995.

- [52] P. Huet and M. E. Peskin, Nucl. Phys. **B 434**(1995) 3. P. Huet, Testing Violation of CPT and Quantum Mechanics in the $K_0 - \bar{K}_0$ system, Preprint: SLAC-Pub-6491, May 1994.