We consider a low-energy supersymmetric scenario in which the effects of heavy supersymmetric interactions are included in a model-independent manner through a series of supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant operators; we explicitly construct all such operators of dimension $\leq 6$. From this set, those operators generated at tree level by the underlying theory are isolated since under weak coupling conditions they are expected to provide the largest effects in low-energy processes. Potential deviations from low energy predictions in processes such as proton decay, neutrinoless double-$\beta$ decay and flavor changing neutral currents are analyzed as illustrations of the method.

I. INTRODUCTION

The Standard Model of particle physics has been successfully tested to great accuracy in the past decade [1], the latest result being the remarkable discovery of the top quark [2] at the Tevatron. Despite the many achievements of the Standard Model at current energies, there are reasons to believe it is but the low-energy manifestation of a more fundamental theory [3]. Supersymmetry provides an appealing extension of the Standard Model, which among other virtues, solves the gauge hierarchy problem naturally [4]. Another argument in favor of supersymmetry is the predicted unification of all gauge coupling constants at approximately $10^{16}$ GeV, which can be taken as a hint of supersymmetric grand unification [5].

In this paper we study a scenario in which low-energy supersymmetry has been found and its details have been determined. Since such low-energy supersymmetric theories often contain indications of some deeper symmetry (such as the unification of the coupling constants mentioned above), we will assume that such a supersymmetric theory represents the low-energy limit of a more fundamental theory (such as theories exhibiting supersymmetric grand unification [6], supersymmetric left-right models [7], etc.). We will then look for signatures of this more fundamental theory using an effective Lagrangian approach [8,9] which will allow us to isolate the observables in the low-energy supersymmetric theory which are most sensitive to the virtual effects of the underlying theory. In our calculations we will not assume a specific high-energy supersymmetric model; we will instead determine the kinds of physics probed by the different observables which we consider and their corresponding scales. We assume that the heavy theory is weakly coupled.

This procedure has been carried out for some particular interactions, such as those relevant to proton decay [10]. In this publication we extend these results by obtaining the complete list (for a given low-energy supersymmetric spectrum) of dimension 5 and 6 operators. We will study the effects of these effective interactions on lepton and baryon number violation and flavor changing processes.

In order to pursue the above program we need to choose a low energy supersymmetric theory. We will use, for simplicity, the minimal extension of the Standard Model (MSSM) [11]. The whole program can be carried out for any other low-energy supersymmetric model (though the presence of new fields considerably lengthens the computations). In the following we will only use the particle content of the MSSM, constraints such as the relationships between the scalar and vector masses [11] will not be imposed.

In the MSSM each Standard Model particle acquires a supersymmetric partner of the same mass. Concerning the scalar sector, (at least) two Higgs doublets with their corresponding supersymmetric partners are required due to the holomorphic character of the superpotential and to achieve anomaly cancelations. In this theory, baryon number is not naturally conserved, which leads to an unacceptably fast rate for proton decay. In order to avoid this problem a new symmetry (R-parity) must be imposed [10,12]. After this is done one finds that the most general renormalizable supersymmetric potential compatible with all the symmetries does not break gauge and super-symmetries. This necessitates the introduction of the so-called “soft breaking terms” [13], i.e., super-renormalizable terms which break supersymmetry and electroweak symmetry. The fact that such terms are needed supports the idea that the MSSM is the low-energy effective model of a more fundamental theory which becomes manifest above a scale $M$ [14].
The low-energy effective Lagrangian of a weakly coupled renormalizable theory can be written as a linear combination of operators in the light fields suppressed by powers of the heavy scale $M$ (as guaranteed by the decoupling theorem [15]). In our case each of these operators is built from the light superfields and respects the low-energy symmetries (i.e. $SU(3)_C \times SU(2)_L \times U(1)_Y$×R-parity). The detailed characteristics of the full theory will be encoded in the set of coefficients multiplying the operators. It is useful to divide the operators into those that can be generated at tree level from vertices of a most general underlying supersymmetric theory. As will be explained in more detail in section IV, the effective supersymmetric Lagrangian has the following structure

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MSSM}} + \frac{1}{M} \left[ \sum_i a_i O_i^{(5)} + \text{h.c.} \right] + \frac{1}{M^2} \left[ \sum_i b_i O_i^{(6)} + \text{h.c.} \right] + \cdots,$$

(1)

where ‘h.c’ denotes complex conjugation, $\{O_i^{(5)}\}$ and $\{O_i^{(6)}\}$ are gauge-invariant operators of dimension 5 and 6 that result after taking F or D components of suitable supersymmetric operators, and $\{a_i\}, \{b_i\}$, etc., are undetermined (and, in general, complex) coefficients; the ellipsis denote higher dimensional contributions.

All the operators are constructed out of the light superfields and must be gauge invariant and respect supersymmetry. To illustrate the procedure by means of which these operators are constructed consider two chiral superfields $\phi_n$ and $\phi_n^*$ transforming respectively as the fundamental representation $\mathbf{n}$ of $SU(N)$ and its complex conjugate $\mathbf{n}^*$, and the corresponding vector superfield $V$. Their explicit transformation laws are

$$\phi_n \to \phi_n' = e^{-iA^a n^a} \phi_n, \quad \phi_n^* \to \phi_n'^* = e^{-iA^a n^a} \phi_n^*, \quad e^V \to e^{V'} = e^{-iA^a n^a} e^V e^{iA^a n^a},$$

(2)

where $t^a$ and $-t^{a*}$ are the (Hermitian) generators for $\mathbf{n}$ and $\mathbf{n}^*$ respectively; it follows that

$$\phi_{n^*}^T \to \phi_{n^*}^{T'} = \phi_{n^*}^T e^{iA^a n^a},$$

(3)

which will be useful below. Note that for the case of local gauge transformations the $\Lambda^a$ must be chiral superfields and therefore are necessarily complex, $\Lambda^a \neq \Lambda^{a*}$.

Using these definitions we can construct two types of invariants,

$$\phi_{n^*}^{T} \phi_n \quad \phi_{n^*}^{T} \phi_n e^V \phi_n.$$  

(4)

Since $\mathbf{n}$ corresponds to the fundamental representation we also have the additional invariant

$$\epsilon_{a_1 \cdots a_N} \phi^{a_1} \cdots \phi^{a_N}.$$  

(5)

1When the (super)fields transform according to other irreducible representations additional invariants can be constructed [17].
All supersymmetric-invariant operators involving these fields are constructed taking products of composites of the form (4) and (5) and extracting their F or D components. In the MSSM both superfields associated with particles and antiparticles are chiral; products of the form $\phi_n^F \phi_n^A$ are then also chiral, and the corresponding F components will appear in the (effective) Lagrangian. The terms of the form $\phi_n^F e^{V \phi_n}$ always involve a chiral-antichiral product, hence its D component would be chosen. A general operator will be a product of these gauge invariant building blocks in which case each operator must be considered separately in order to determine whether its F or D component should be included.

Since both $\phi_n^F \phi_n^A$ and $\phi_n^A \phi_n^F$ are invariant under global gauge transformations (as is (5)), all operators can be obtained by first imposing global gauge invariance and then replacing

$$\phi^a \rightarrow \phi^a e^{V \phi}$$

(6)

for every chiral superfield, where $V\phi = \sum q^{(a)} V^{(a)}$ and the index $a$ runs over the different gauge group factors present. The $q^{(a)}$ are the charges of $\phi$ and $V^{(a)}$ denote the corresponding Lie-algebra valued valued superfields.

It is important to recall at this point that, if an operator $O$ has dimension $d$, then its F-component, $O_F$ has dimension $d + 1$, while its D-component, $O_D$ will have dimension $d + 2$. For example, in order to obtain the contributions to the effective Lagrangian of dimension 5 we need chiral composites of dimension 4 and vector composites of dimension 3. In the next section we construct all dimension 5 and 6 supersymmetric contributions to the effective Lagrangian.

### III. MSSM AND OPERATORS OF DIMENSIONS 5 AND 6

In this section we will construct the operators of dimension $\leq 6$ appearing in the effective Lagrangian. As mentioned in the introduction we assume that the MSSM [11] describes the low energy supersymmetric limit to lowest order in $\mathcal{M}$ (the scale of the underlying theory). This model is invariant under local $SU(3)_C \times SU(2)_L \times U(1)_Y$; the superfields involved are listed in table I, these include two chiral scalar superfields responsible for generating masses for quarks and leptons.

We now construct all operators involving the fields in table I and their covariant derivatives and whose F and D components are of mass dimension 5 and 6. To do this we follow the ideas outlined in the previous section and use the canonical dimensions for superfields and derivatives as follows: [chiral] = 1, [vector] = 0, [gauge field strength] = 3/2 and [covariant derivative] = 1/2 (see, for example, [16]).

Further conservation laws (such as those responsible for the suppression of baryon number violating processes) are related to additional gauge or discrete symmetries [10,12] (eg. R-parity). These additional symmetries will be imposed later. Anticipating these restrictions, however, we will ignore all operators of dimension 4 which violate R-parity (see appendix B) since they imply a short proton lifetime [10,12]. Additional phenomenological constraints will also be considered at the end.

With these preliminaries we can now enumerate all dimension 5 and 6 operators of the MSSM. In constructing this list we have used the equations of motion [18] to eliminate redundant contributions (see appendix A for details). To simplify the expressions it proves convenient to define

$$V_Q = G + W - \frac{1}{6} B$$
$$V_U = -G - \frac{3}{2} B$$
$$V_D = -G + \frac{3}{2} B$$
$$V_L = W + \frac{1}{2} B$$
$$V_E = B$$
$$V_{H_{1,2}} = \pm W + \frac{1}{2} B.$$

(7)
In what follows $O_F$ indicates that the $F$ component of the operator listed is to be included in (1); similarly $O_D$ indicates the $D$ component should appear in (1). For compactness, we will refer to superfields $Q, U^c, D^c, L, E^c$ as “fermions”, $H_1, H_2$ as “scalars” and $G, W, B$ as “vectors”. For some of the operators listed below gauge invariance might allow more than one index contraction; we choose to keep the indices implicit for a simpler presentation. In section V we will calculate the contribution of some of the operators to some physically interesting processes and we will then make explicit the index structure of the relevant operators.

A. Operators of dimension 5

1. Operators involving fermions only

$$O_F^{(1)} = QQU^cD^c$$

$$O_F^{(2)} = QQQL$$

$$O_F^{(3)} = U^cU^cD^cE^c$$

$$O_F^{(4)} = QU^cLE^c$$

2. Operators involving fermions and scalars

$$O_F^{(5)} = QQH_1$$

$$O_F^{(6)} = QU^cE^cH_1$$

$$O_F^{(7)} = LLH_2H_2$$

$$O_F^{(8)} = LH_1H_2H_2$$

3. Operators involving scalars only

$$O_F^{(9)} = H_1H_1H_2H_2$$

4. Operators involving vectors only

These operators must be constructed out of the superfield strengths, which have mass dimension $3/2$ and we denote by $X_\alpha$. Lorentz invariance allows only powers of $X^\alpha X_\alpha$ and its hermitian conjugate. Only $[X^\alpha X_\alpha]_D$ are allowed, but these operators correspond to total divergences.

5. Operators involving fermions and vectors

To obtain this set of operators we follow the procedure outlined in section (II); after the replacement (6) we obtain

$$O_D^{(1)} = QQ(D^c e^V_D)$$

$$O_D^{(2)} = QU^c(L^e e^V_L)$$

$$O_D^{(3)} = U^c(D^c e^V_D)E^c$$

where $V_L$ and $V_D$ are defined in (7).
Again following the procedure described in section II we obtain

\[ O_D^{(4)} = QU^c (H_1^\dagger e^{V_{H_1}}) \] (20)
\[ O_D^{(5)} = QD^c (H_2^\dagger e^{V_{H_2}}) \] (21)
\[ O_D^{(6)} = LE^c (H_2^\dagger e^{V_{H_2}}) \] (22)
\[ O_D^{(7)} = E^c H_1 (H_2^\dagger e^{V_{H_2}}) \] (23)
\[ O_D^{(8)} = (E^{c\dagger} e^{V_E}) H_2 H_2 \] (24)

where \( V_{H_1,2} \) and \( V_E \) are defined in (7).

7. Operators involving scalars and vectors

No operators exist in this category since no gauge invariant expression can be constructed out of three (MSSM) scalar superfields.

B. Operators of dimension 6

1. Operators involving fermions only

\[ O_F^{(10)} = QQQQU^c \] (25)
\[ O_F^{(11)} = QQU^c U^c E^c \] (26)
\[ O_F^{(12)} = U^c U^c U^c E^c E^c \] (27)
\[ O_F^{(13)} = D^c D^c D^c LL \] (28)

2. Operators involving fermions and scalars

\[ O_F^{(14)} = U^c D^c D^c LH_2 \] (29)
\[ O_F^{(15)} = D^c D^c D^c LH_1 \] (30)
\[ O_F^{(16)} = QD^c LH_2 \] (31)
\[ O_F^{(17)} = LLL E^c H_2 \] (32)
\[ O_F^{(18)} = U^c D^c D^c H_1 H_2 \] (33)
\[ O_F^{(19)} = U^c U^c D^c H_2 H_2 \] (34)
\[ O_F^{(20)} = D^c D^c D^c H_1 H_1 \] (35)
\[ O_F^{(21)} = QU^c LH_2 H_2 \] (36)
\[ O_F^{(22)} = QD^c LH_1 H_2 \] (37)
\[ O_F^{(23)} = LLE^c H_1 H_2 \] (38)
\[ O_F^{(24)} = QU^c H_1 H_2 H_2 \] (39)
\[ O_F^{(25)} = QD^c H_1 H_1 H_2 \] (40)
\[ O_F^{(26)} = LE^c H_1 H_1 H_2 \] (41)
\[ O_F^{(27)} = E^c H_1 H_1 H_1 H_2 \] (42)
Chiral operators of this type must contain five chiral scalar superfields; all such composites violate gauge invariance. There are no operators in this category.

4. Operators involving vectors only

To obtain a dimension-6 contribution to the effective Lagrangian we need an \( O_F \) operator of dimension 5, or an \( O_D \) operator of dimension 4. Neither of these can be constructed purely out of powers of the field strength \( X_\alpha \). There are no operators in this category.

5. Operators involving fermions and vectors

\[
\begin{align*}
O_D^{(9)} &= (\bar{Q}^\dagger e^{V_2})(\bar{Q}^\dagger e^{V_2})QQ \\
O_D^{(10)} &= (\bar{Q}^\dagger e^{V_2})Q(U^\dagger e^{V_2})U^c \\
O_D^{(11)} &= (\bar{Q}^\dagger e^{V_2})Q(D^\dagger e^{V_D})D^c \\
O_D^{(12)} &= (U^\dagger e^{V_2})(U^\dagger e^{V_2})U^cU^c \\
O_D^{(13)} &= (U^\dagger e^{V_2})U^c(D^\dagger e^{V_D})D^c \\
O_D^{(14)} &= (D^\dagger e^{V_D})(D^\dagger e^{V_D})D^cD^c \\
O_D^{(15)} &= QQ(U^\dagger e^{V_2})(E^\dagger e^{V_E}) \\
O_D^{(16)} &= Q(U^\dagger e^{V_2})(D^\dagger e^{V_D})L \\
O_D^{(17)} &= (\bar{Q}^\dagger e^{V_2})Q(L^\dagger e^{V_L})L \\
O_D^{(18)} &= (\bar{Q}^\dagger e^{V_2})Q(E^\dagger e^{V_E})E^c \\
O_D^{(19)} &= (U^\dagger e^{V_2})U^c(L^\dagger e^{V_L})L \\
O_D^{(20)} &= (U^\dagger e^{V_2})U^c(E^\dagger e^{V_E})E^c \\
O_D^{(21)} &= (D^\dagger e^{V_D})D^c(L^\dagger e^{V_L})L \\
O_D^{(22)} &= (D^\dagger e^{V_D})D^c(E^\dagger e^{V_E})E^c \\
O_D^{(23)} &= QD^c(L^\dagger e^{V_L})(E^\dagger e^{V_E}) \\
O_D^{(24)} &= (L^\dagger e^{V_L})(L^\dagger e^{V_L})LL \\
O_D^{(25)} &= (L^\dagger e^{V_L})L(E^\dagger e^{V_E})E^c \\
O_D^{(26)} &= (E^\dagger e^{V_E})(E^\dagger e^{V_E})E^cE^c \\
O_D^{(27)} &= (\bar{Q}^\dagger e^{V_2})D^cD^cL \\
O_D^{(28)} &= D^cD^cD^c(E^\dagger e^{V_E}) \\
O_D^{(29)} &= (U^\dagger e^{V_2})D^cLL \\
\end{align*}
\]

where the various \( V_i \) are defined in (7).

6. Operators involving fermions, scalars and vectors

\[
\begin{align*}
O_F^{(28)} &= X_W^2LH_2 \\
O_F^{(29)} &= X_B^2LH_2 \\
O_F^{(30)} &= X_G^2LH_2 \\
O_D^{(30)} &= QQQ(H_2^\dagger e^{VH_2}) \\
\end{align*}
\]
$O^{(31)}_D = (Q^c e_{V\ell}^c) U^c D^c H_2$

$O^{(32)}_D = (Q^c e_{V\ell}^c) D^c D^c H_1$

$O^{(33)}_D = (Q^c e_{V\ell}^c) Q L H_2$

$O^{(34)}_D = (U^c e_{V\ell}) U^c L H_1$

$O^{(35)}_D = (D^c e_{V\ell}^c) D^c L H_2$

$O^{(36)}_D = Q U^c E^c (H_2^c e_{V\ell}^c)$

$O^{(37)}_D = Q D^c (E^c e_{V\ell}^c) H_2$

$O^{(38)}_D = (U^c e_{V\ell}) D^c L H_1$

$O^{(39)}_D = (L^c e_{V\ell}^c) L L H_2$

$O^{(40)}_D = Q (U^c e_{V\ell}) (D^c e_{V\ell}) H_1$

$O^{(41)}_D = Q (D^c e_{V\ell}^c) (D^c e_{V\ell}) H_2$

$O^{(42)}_D = (Q^c e_{V\ell}) Q L (H_1^c e_{V\ell}^c)$

$O^{(43)}_D = (U^c e_{V\ell}) U^c L (H_1^c e_{V\ell}^c)$

$O^{(44)}_D = (D^c e_{V\ell}) D^c L (H_1^c e_{V\ell}^c)$

$O^{(45)}_D = Q D^c (E^c e_{V\ell}^c) (H_1^c e_{V\ell}^c)$

$O^{(46)}_D = U^c (D^c e_{V\ell}^c) (L^c e_{V\ell}^c) H_2$

$O^{(47)}_D = (L^c e_{V\ell}^c) L L (H_1^c e_{V\ell}^c)$

$O^{(48)}_D = L (E^c e_{V\ell}^c) E^c (H_1^c e_{V\ell}^c)$

$O^{(49)}_D = L (E^c e_{V\ell}^c) E^c H_2$

$O^{(50)}_D = (Q^c e_{V\ell}^c) Q H_1 H_2$

$O^{(51)}_D = (U^c e_{V\ell}) U^c H_1 H_2$

$O^{(52)}_D = (D^c e_{V\ell}^c) D^c H_1 H_2$

$O^{(53)}_D = (U^c e_{V\ell}) D^c H_1 H_1$

$O^{(54)}_D = U^c (D^c e_{V\ell}^c) H_2 H_2$

$O^{(55)}_D = L L (H_1^c e_{V\ell}^c) H_2$

$O^{(56)}_D = (L^c e_{V\ell}^c) L H_1 H_2$

$O^{(57)}_D = (E^c e_{V\ell}^c) E^c (H_1^c e_{V\ell}^c) H_1$

$O^{(58)}_D = (Q^c e_{V\ell}^c) Q (H_1^c e_{V\ell}^c) H_1$

$O^{(59)}_D = (Q^c e_{V\ell}^c) Q (H_2^c e_{V\ell}^c) H_2$

$O^{(60)}_D = (U^c e_{V\ell}) U^c (H_2^c e_{V\ell}^c) H_2$

$O^{(61)}_D = (U^c e_{V\ell}) U^c (H_1^c e_{V\ell}^c) H_1$

$O^{(62)}_D = (D^c e_{V\ell}^c) D^c (H_2^c e_{V\ell}^c) H_2$

$O^{(63)}_D = (D^c e_{V\ell}^c) D^c (H_1^c e_{V\ell}^c) H_1$

$O^{(64)}_D = (U^c e_{V\ell}) D^c H_1 (H_2^c e_{V\ell}^c) H_2$

$O^{(65)}_D = L L (H_1^c e_{V\ell}^c) (H_1^c e_{V\ell}^c)$

$O^{(66)}_D = (L^c e_{V\ell}^c) L (H_1^c e_{V\ell}^c) H_1$

$O^{(67)}_D = (L^c e_{V\ell}^c) L (H_2^c e_{V\ell}^c) H_2$

$O^{(68)}_D = (E^c e_{V\ell}^c) E^c (H_2^c e_{V\ell}^c) H_2$
where $X_B$, $X_W$ and $X_G$ denote the field strengths associated with the U(1), SU(2) and SU(3) gauge fields.

7. Operators involving scalars and vectors

\begin{align}
O_F^{(31)} &= X_W^2 H_1 H_2 \\
O_F^{(32)} &= X_B^2 H_1 H_2 \\
O_F^{(33)} &= X_G^2 H_1 H_2 \\
O_D^{(70)} &= (H_1^1 e^{V_{h_1}}) H_1 H_2 \\
O_D^{(71)} &= (H_2^1 e^{V_{h_1}}) H_1 H_2 \\
O_D^{(72)} &= (L^1 e^{V_{c}}) H_1 H_2 \\
O_D^{(73)} &= (H_1^1 e^{V_{h_1}}) (H_2^1 e^{V_{h_2}}) H_2 \\
O_D^{(74)} &= (H_2^1 e^{V_{h_2}}) (H_2^1 e^{V_{h_2}}) H_2 \\
\end{align}

where $X_B$, $X_W$ and $X_G$ denote the field strengths associated with the U(1), SU(2) and SU(3) gauge fields.

IV. TREE-LEVEL EFFECTIVE OPERATORS

When considering the phenomenological effects of the above operators it must be emphasized that the contributions of an operator will be suppressed whenever it is generated by loops in the underlying theory. Within our assumption of a weakly coupled underlying theory the coefficient of any loop-generated operator will be suppressed by a factor $1/(16\pi^2)$ together with a product of coupling constants. Operators generated at tree level will only be suppressed by a product of couplings. Because of this it becomes important to determine which of the operators of the previous section are generated at tree level.

Following [19] we first enumerate all possible vertices present in a general renormalizable supersymmetric gauge theory. Using these vertices we construct all possible graphs corresponding to effective contributions of dimensions 5 and 6. Since we are interested in the effective theory at low energies we consider only graphs with light external and heavy internal lines.

A general renormalizable superpotential contains terms of the form $\phi^2$ and $\phi^3$ where $\phi$ denotes a fermion or scalar chiral superfield. Fermion-vector and scalar-vector interactions are generated by $\phi^1 e^V \phi$, where $V$ denotes a vector superfield. The expansion of the exponential will bring interactions of the form $\phi^1 V^n \phi$ with $n$ an integer. Finally, there are vector self-interactions coming from the kinetic terms; the corresponding vertices contain three or more vector lines.

Given this set of vertices we now discriminate between heavy and light fields. We denote by $\psi_i, s_i, V_i$ with $i = l, h$ light (or heavy) fermion, scalar and vector superfields respectively.

A first restriction concerns effective operators involving vector superfields only; they are generated by the kinetic terms and take the general form (see for example [16]),

\begin{equation}
\begin{aligned}
f_{abc}(\bar{D}^2 D^a V_a)(\bar{D}^2 V_b) (D_\alpha V_c), & \quad f_{abc} f_{cde}(\bar{D}^2 V_a)(D^a V_b) \bar{D}^2 V_c (D_\alpha V_d)
\end{aligned}
\end{equation}

Since the light generators form a subalgebra any structure constant of the form $f_{lhh}$ must vanish. Therefore, as in the non-supersymmetric case [19], the allowed vertices are of the form

\begin{equation}
\begin{aligned}
V_l V_l V_l, & \quad V_l V_h V_h, \quad V_h V_h V_h.
\end{aligned}
\end{equation}
For four-vector interactions only the following combinations of structure constants are non-vanishing \[ f_{iui}, f_{hhi}, f_{ihh}, f_{hh}h, f_{hhi}, f_{hhi}h, f_{hhi}h, f_{hh}f_{hh}, f_{hh}f_{hh} \] leading to
\[ V_i V_i V_i V_i, V_i V_i V_i V_A, V_i V_i V_i V_h, V_i V_h V_h V_h. \]

A second restriction follows from the fact that the generators of the light algebra cannot mix light with heavy chiral superfields. This disallows vertices of the form \( \psi_1 V_i^n \psi_h \) and \( s_i V_i^n s_h \) (n integer); similarly no vertex of the form \( \psi_1 V_i^n s \) is present in the theory, which follows from the initial diagonalization of the kinetic terms.

The remaining restrictions apply to vertices containing a single scalar superfield, which are generated by the corresponding kinetic term when a heavy scalar gets a vacuum expectation value \( v_h = \langle 0 | s_h | 0 \rangle \). Being interested in operators of dimension \( \leq 6 \) we need only consider diagrams with one heavy internal vector line. This means that we can replace
\[ e^{V_i v_h} \rightarrow \int_0^1 ds \ e^{s V_i v_h} e^{(1-s) V_i} \]
(123)
Using the fact that the light generators annihilate \( v_h \) we have \( V_i v_h = 0 \). We also note that given an broken generator \( T_{\text{broken}} \) the vector \( T_{\text{broken}} v_h \) is the direction of a would-be Goldstone boson; since these excitations transform among themselves under the unbroken group we have \( s_i T_{\text{broken}} v_h = 0 \) and \( s_i V_i T_{\text{broken}} v_h = 0 \) for all physical scalar superfields \( s \). Collecting these results we get
\[ s_i e^{V_i v_h} = s_i \int_0^1 ds \ e^{s V_i v_h} e^{(1-s) V_i} v_h + O(V_h^2) \]
\[ = O(V_h^2) \]
(124)
so that there are no vertices of the form \( s_i V_i^n s_h \). There are no other general constraints to be derived from gauge invariance.

Using the allowed vertices we now construct all possible diagrams corresponding to contributions to the effective Lagrangian of dimension 5 and 6 which are generated at tree level; the procedure is straightforward, although lengthy. The diagrams are depicted in figures 1 and 2, the corresponding operators are presented in table II.

<table>
<thead>
<tr>
<th>operator</th>
<th>diagrams</th>
<th>equation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d = 5 ) F-type</td>
<td></td>
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</tr>
<tr>
<td>( \psi^4 )</td>
<td>(1.a)</td>
<td>(8) – (11)</td>
</tr>
<tr>
<td>( \psi^3 s )</td>
<td>(1.b)</td>
<td>(12) – (13)</td>
</tr>
<tr>
<td>( \psi^2 s^2 )</td>
<td>(1.c)</td>
<td>(14)</td>
</tr>
<tr>
<td>( \psi s^3 )</td>
<td>(1.d)</td>
<td>(15)</td>
</tr>
<tr>
<td>( s^4 )</td>
<td>(1.e)</td>
<td>(16)</td>
</tr>
<tr>
<td>( d = 6 ) F-type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \psi^3 )</td>
<td>(1.f)</td>
<td>(25) – (28)</td>
</tr>
<tr>
<td>( \psi^2 s )</td>
<td>(1.g)</td>
<td>(29) – (32)</td>
</tr>
<tr>
<td>( \psi^3 s^2 )</td>
<td>(1.h)</td>
<td>(33) – (38)</td>
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<tr>
<td>( \psi s^3 )</td>
<td>(1.i)</td>
<td>(39) – (41)</td>
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<tr>
<td>( s^4 )</td>
<td>(1.j)</td>
<td>(42)</td>
</tr>
<tr>
<td>( d = 6 ) D-type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \psi^4 e^V )</td>
<td>(1.l)</td>
<td>(43) – (60)</td>
</tr>
<tr>
<td>( \psi^3 s e^V )</td>
<td>(1.m)</td>
<td>(77) – (85)</td>
</tr>
<tr>
<td>( \psi^3 s^2 e^V )</td>
<td>(1.n)</td>
<td>(95) – (105)</td>
</tr>
<tr>
<td>( \psi s^3 e^V )</td>
<td>(1.o)</td>
<td>(107), (110)</td>
</tr>
<tr>
<td>( s^4 e^V )</td>
<td>(1.p)</td>
<td>(117) – (119)</td>
</tr>
</tbody>
</table>

**TABLE II.** Tree-level generated operators; \( \psi, s, V \) denote, respectively, fermion, scalar and vector superfields.

\( O_D \) operators of dimension 5 are not generated at tree-level since they involve superfields with opposite chiralities, a type of vertex that we have seen is not present in a supersymmetric potential.

In terms of components, operators (8-16) contribute to the effective Lagrangian (1) terms with 2 scalars and 2 fermions and involve either sfermions or higgsinos. All these operators violate chiral symmetry and, assuming
naturality [20], are suppressed by powers of the fermion masses. Similarly (25-42) generate terms with three scalars and two fermions violating chiral symmetry and all involve sfermions or higgsinos.

V. DEVIATIONS FROM THE MSSM

We have argued above that observables modified by tree-level-generated operators are most sensitive to the heavy physics. We have also argued that additional symmetries are required in order to prevent fast proton decay. In this section we impose these further symmetries and determine the observability of physics at the scale $M$ through the effects of operators that are both R invariant and tree-level-generated. The list of these operators is given in table III.

<table>
<thead>
<tr>
<th>operator</th>
<th>diagrams</th>
<th>equation(s)</th>
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<tbody>
<tr>
<td>$d = 5$ F-type</td>
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<td></td>
</tr>
<tr>
<td>$\psi^4$</td>
<td>(1.a)</td>
<td>(8) - (11)</td>
</tr>
<tr>
<td>$\psi^2 s^2$</td>
<td>(1.c)</td>
<td>(14)</td>
</tr>
<tr>
<td>$s^4$</td>
<td>(1.e)</td>
<td>(16)</td>
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<tr>
<td>$d = 6$ F-type</td>
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<td></td>
</tr>
<tr>
<td>$\psi^4 s$</td>
<td>(1.g)</td>
<td>(29) - (32)</td>
</tr>
<tr>
<td>$\psi^2 s^3$</td>
<td>(1.i)</td>
<td>(39) - (41)</td>
</tr>
<tr>
<td>$d = 6$ D-type</td>
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<tr>
<td>$\psi^4 e^V$</td>
<td>(1.l)</td>
<td>(43) - (60)</td>
</tr>
<tr>
<td>$\psi^2 s^2 e^V$</td>
<td>(1.n)</td>
<td>(95) - (105)</td>
</tr>
<tr>
<td>$s^4 e^V$</td>
<td>(1.p)</td>
<td>(117) - (119)</td>
</tr>
</tbody>
</table>

TABLE III. R-invariant tree-level generated operators; $\psi$, $s$, $V$ denote, respectively, fermion, scalar and vector superfields.
FIG. 2. Diagrams corresponding to tree-level generated operators of type D (all of dimension 6). Same conventions for fermions and scalars as in Fig.1. Wavy lines with indices $i_l, l = 1, 2$ on these diagrams represent $i_l$ supervector lines converging to the same vertex. Heavy chiral field propagators are denoted by heavy solid lines, heavy vector propagators by heavy wavy lines.

The operators of type D in table III contribute three types of interactions (in terms of the component fields): 4-fermion interactions, terms with 2 fermions 2 scalars and one covariant derivative, and terms with 4 scalars and two covariant derivatives; none violate chiral symmetry. The operators (95-101) and (103-105) modify the couplings of the vector-boson to the quarks and leptons and are correspondingly constrained by existing data which implies $M > 2$ TeV (taking the corresponding $b_i \sim 1$ in (1) and assuming no cancelations) [21]. Four-fermion interactions provide similar bounds [1,22]. Finally (117)-(119) contribute two the $\rho$ parameter which also provides a bound of the same order [1]. It is interesting to note that supersymmetry and gauge invariance allows us to bound operators containing only sparticles through current data.

We note that all these bounds merely state that $M$ is greater than (roughly) the scale at which low-energy supersymmetry is expected to become manifest. Higher sensitivity can be achieved by considering processes which are strictly forbidden with in the MSSM, but which can be generated by the heavy dynamics. This possibility will be considered in the next section.

A. Rare processes

The operators in table III which respect the global symmetries of the MSSM are subdominant compared to similar vertices induced by the MSSM interactions themselves (this is the basic reason for above mild bounds on $M$). There are a few operators, however, which violate some of the global symmetries of the MSSM and the corresponding observables are much more sensitive to $M$.

We will study three important examples: operators contributing to proton decay, to flavor changing neutral currents (FCNC) and to processes with lepton number violation.

1. Proton decay

Baryon and lepton number conservation are automatic in the Standard Model after gauge invariance and renormalizability. However, this is not so in supersymmetric models, where we need to invoke additional symmetries [10,12] in order to forbid fast baryon decay through dimension-four operators. Discrete, global and gauge symmetries have been considered in the literature, though only gauge symmetries are respected by gravitational interactions [23]. Therefore only the low-energy remnants of such extra gauge interactions appear to be feasible candidates. Here we use R-parity
to forbid these dangerous operators [10,12]. After this is done, there are still operators of dimensions 5 and 6 which violate baryon and lepton number conservation (but conserve B-L), namely (9), (10), (49) and (50); which involve only quark and lepton superfields

\[
d = 5: \quad [QQQL]_F, \quad [U^cU^cD^cE^c]_F
\]

\[
d = 6: \quad [QQ(U^c\bar{e}V^c)(\bar{E}^c\bar{e}V^c)]_D, \quad [Q(U^c\bar{e}V^c)(D^c\bar{e}V^c)]_D.
\]

which are precisely the ones described in Refs. [10]. The \(d = 5\) operator \([QQQL]_F\) violates baryon number but is not \(R\)-parity invariant and is therefore not included in the above list. There are various other operators, such as (17-19), which also violate baryon number; these terms, however, are loop generated and will give subdominant contributions.

The \(d = 5\) operators are generated through the exchange of a heavy chiral superfield, whose mass we denote by \(M_S\), (diagrams (1.a)). The coefficient of these operators will then be suppressed by the corresponding Yukawa couplings \([10]\), denoted by \(y\); these operators have a prefactor \(\sim y^2/M_S\). A specific example is furnished by the \(SU(5)\) GUT [24] where the heavy chiral superfield corresponds to two scalar superfields transforming according to the \(5\) and \(\bar{5}\) irreducible representations of \(SU(5)\).

The \(d = 6\) operators can be generated through the vector superfield exchanges of diagrams (1.i) \(^2\); their coefficient have the form \(\sim g^2/M_V^2\), where \(M_V\) denotes the mass of the heavy vectors and \(g\) the corresponding gauge coupling constant. Within the \(SU(5)\) GUT [24] such operators are generated by the exchange of heavy vector superfields.

The baryon-violating vertices contained in the \(d=5\) effective operators involve two sparticle external lines and therefore induce proton decay only at loop-level (see for example [11,25,10] and references therein). For example, for the decay \(p \rightarrow K^+\bar{\nu}\) the diagram consists of one loop with two squarks and one gaugino propagator, and one effective vertex of order \(M^{-1}\). The corresponding amplitude is

\[
I_{(5)} \sim y^2 g^2 f/(16\pi^2 M_S), \quad \text{where } g \text{ denotes a gauge coupling constant and } f \text{ a factor depending on the light particles in the loop, explicitly [26]} \quad f \sim m_\tilde{q}/m_\tilde{q}^2 \text{ where } m_\tilde{q} \text{ and } m_\tilde{q} \text{ are the gluino and squark masses respectively. For a rough estimate we will take } I_{(5)} \sim y^2 g^2/(16\pi^2 M_S m_{\text{susy}}) \text{ where } m_{\text{susy}} \text{ denotes a typical scale of the light supersymmetry.}
\]

Regarding \(d=6\) operators, only \([Q(U^c\bar{e}V^c)(D^c\bar{e}V^c)]_D\) contributes to the decay \(p \rightarrow K^+\bar{\nu}\). Expanding this operator in terms of its components we obtain a term

\[
[QU^cD^cL]_D = \epsilon_{abc}(u_a c - d_a \nu)\bar{w}_b \bar{s}_d
\]

where the Latin indices correspond to \(SU(3)\) and \(s\) denotes the \(s\)-quark field. The order of magnitude for the amplitude generated by this vertex is \(I_{(6)} \sim g^2/M_V^2\) where \(M_V\) denotes the vector mass and \(g\) the corresponding gauge coupling constant (we will assume that all gauge coupling constants are of the same order of magnitude).

The proton width is \(\sim |I_{(6)} + I_{(5)}|^2 N_c^2/8\pi\), where \(N_c\) denotes the number of quark flavors. The current lifetime limit of \(10^{32}\text{yr}\) then implies \(M_V \sim 2 \times 10^{15}\text{ GeV}\) and \(M_S \sim 2 \times 10^{17}\text{ GeV}\), having taken \(m_{\text{susy}} = 1\text{ TeV}\), and using \(g = 0.656\), \(y = 10^{-4}\) [10].

The contribution from the \(d=5\) operators dominate whenever \(M_V^2/M_S > 2 \times 10^{13}\text{ GeV}\). A specific realization of this scenario occurs in \(SU(5)\) GUT models [24] where \(M_S < M_V\) is assumed [10] (the estimates presented here may be modified in specific models due to mixing-angle suppression or cancelations). There also are scenarios in which the contributions corresponding to \(d = 5\) operators are suppressed such as in some \(SO(10)\) grand unified theories [27]. In such cases the dominant effects is generated by \(d = 6\) which occurs when \(M_V^2/M_S < 2 \times 10^{13}\text{ GeV}\); taking, for example, \(M_S \sim 10^{18}\text{ GeV}\) [27] implies \(M_V < 5 \times 10^{15}\text{ GeV}\).

\section*{2. Flavor changing neutral currents}

Flavor changing neutral currents are another important means to obtain information regarding the physics underlying the MSSM [28]. As it is well known, FCNC are suppressed in the Standard Model through the GIM mechanism [29]. In the MSSM, detailed analysis of the radiatively induced FCNC shows that these are very small also (see [28] and references therein). In supersymmetric theories embedded in an underlying GUT, large FCNC may be induced by off-generational Yukawa couplings [30].

\(^2\)The chiral exchanges in fig. (1.i) contribute only to operators containing one or more vectors.
In what follows we consider the MSSM dominant contribution to $K^0 - \bar{K}^0$ mixing at the one-loop level to compare with the tree-level contribution coming from effective vertices. We compare these effects with the ones induced by the effective operators (43), (45) and (48)

\[ (Q^c e^{V_Q}) (Q^c e^{V_Q}) Q Q, \quad (Q^c e^{V_Q}) Q (D^{c\dagger} e^{V_D}) D^c, \quad (D^{c\dagger} e^{V_D}) (D^{c\dagger} e^{V_D}) D^c \]

(128)

which contribute through a 4-fermion contact interaction; the operators (95–101) contribute through a modification of the couplings of the vector-bosons to the quarks (including a possible flavor-changing $Z$ coupling). No dimension 5 or 6 F-type operator can contribute since these provide 2 or 3 sparticles, while we need 4 external fermion lines for this process.

The pure supersymmetric contribution generated by the MSSM corresponds to the gluino-box diagram which has an amplitude of order $I_{MSSM} \sim \xi^2/(4\pi M_\chi)^2$. The factors $(4\pi)^{-2} \sim 1/160$ and $M_\gamma^{-2}$ come from the loop integration and $\xi$ is an additional suppression factor due to super-GIM mechanism cancelations [28] (which is also dependent on the squark mass difference). According to experiments, this amplitude is expected to be $I_{MSSM} \leq 10^{-15}$ GeV$^{-2}$ (see for example [28, 31]), while the amplitude generated by the dimension 6 operators is of order $I_{(6)} \sim M^{-2}$. If we assume that there are no cancelations among contributions from the three operators above, we obtain $M \geq 10^4$ TeV.

If flavor-changing heavy physics enters below this threshold there must be GIM-like cancelations in the coefficients of the dimension 6 operators contributing to FCNC. This added uncertainty is always present when we consider operators which mix generations.

We note that all the four-Fermi effective operators in table which give rise to FCNC are generated by the exchange of chiral superfields. The flavor changing gauge vertices are produced either by chiral or vector superfields.

We briefly mention that the same approach can be used in lepton-flavor violation. The relevant operators are (103-105) generated by heavy chiral or vector superfield exchanges. These operators can induce $\mu \to e\gamma$ with an amplitude $\sim e v^2/M^2$ ($e$ denotes the electron charge), the corresponding width is of order $\sim \alpha m_u (v/M)^4$. The experimental limits on the branching ratio are [1] $B(\mu \to e\gamma) \leq 4.9 \times 10^{-11}$, and correspond to $M > 5.6 \times 10^8$ GeV. For a heavy mass $\sim 10^{16}$ GeV the estimated branching ratio lies 29 orders of magnitude below the experimental limit. Note that these considerations do not apply to flavor violating mass terms which can give measurable effects [32].

3. Lepton number violation

Of the entries in table III the operators (14), (31), (32) and (102) violate lepton number by two units but conserve baryon number. Of these

\[ [LLH_2 H_2]_F \]

(129)

contains terms involving only known particles and generates a Majorana mass for the neutrinos. This operator is generated through the diagrams (1.c) corresponding to the exchange of a heavy scalar or fermion superfield, the coefficient for this operator is then proportional to a Yukawa coupling. The Majorana mass will then be of order $y v^2/M$, where $v$ denotes the vacuum expectation value of the $H_2$ and $y$ a Yukawa coupling; $M$ corresponds to the mass of the heavy superfield $^3$. There are several other operators which violate lepton number such as (64-66), these terms are loop generated (and the corresponding contributions to the effective Lagrangian necessarily involve sparticles) and will give small contributions, assuming all lepton number violations have the same characteristic scale.

Assuming no significant cancelations among the various contributions to neutrinoless double-$\beta$ decay current data implies [1] $y v^2/M \la 1$ eV which corresponds to $M/y \ga 6 \times 10^{13}$ GeV. As a concrete example take the underlying theory to be an $SO(10)$ GUT [33] with the heavy chiral field identified as the one containing the right-handed neutrino $\nu_R$. In this theory the coupling $y$ is related to the up-quark Yukawa coupling so that $y \sim 1, 0.006, 2 \times 10^{-5}$ for the third, second and first generation respectively; correspondingly we have $M \ga 6 \times 10^{13}, 4 \times 10^{11}, 10^9$ GeV.

VI. CONCLUSIONS

In this paper an effective Lagrangian approach is presented which allows the calculation of the contributions to supersymmetric low-energy process due to a hypothesized underlying supersymmetric heavy theory. This is a phenomenological approach in which a minimal amount of assumptions regarding the heavy physics are made.

3 The operator (129) is R-parity invariant but violates other discrete symmetries [12] which can be used to eliminate it.
We have assumed that a low-energy theory such as the minimal supersymmetric extension of the Standard Model (MSSM) is valid. Corrections to this low-energy limit are introduced by non-renormalizable operators constructed from MSSM superfields. These operators must have the symmetries of the low-energy theory, in this case $SU(3) \times SU(2) \times U(1)$ and R-parity, the latter which must be included to avoid fast nucleon decay.

For phenomenological reasons we divided the operators into those that can be generated at tree level by the underlying physics and those which must be generated via loops. Under weak coupling conditions of the underlying theory, tree-level operators are expected to be dominant (for a given dimension) compared to loop-generated ones. Having done this we can estimate the relevant contributions for a process, or compare the coefficients of the operators and determine their relative (estimated) strengths.

Existing data puts, for the most part, no stringent bounds on the scale of heavy supersymmetry. We studied three exceptions: proton decay, FCNC and lepton number violation. For the case of baryon number violation the corresponding contributions have been extensively studied in the literature \cite{11,25,28,31}; the above estimates (validated by these detailed calculations) $M_S \sim 10^{17}$ GeV and $M_V \sim 10^{15}$ GeV for the masses of the heavy scalars and vectors respectively. The constraints on FCNC imply $M_S \sim 10^{13}$ GeV. Finally neutrinoless double-$\beta$ decay data requires $M_S > 6 \times 10^{13}$ GeV. In obtaining these bounds we assumed that there are no significant cancelations among the various contributions, when this is relaxed the bounds are weakened. Should the MSSM (or an alternative model) be discovered, the elucidation of the various couplings and masses should considerably improve this picture allowing for a variety of bounds obtained by considering processes involving sfermions and other superparticles. The study of rare processes, however, will provide the best bounds unless low-lying mass thresholds exist.

The bounds derived constrain different types of heavy fields depending on the diagrams responsible for the relevant operators. It is important to note that although the bounds obtained for $M$ using L violating operators constrains the physics at which lepton number is violated, this does not necessarily imply that all physics beyond low-energy supersymmetry is similarly bounded. Thus, while the bounds obtained using rare processes are quite severe, they do not necessarily extend to the whole set of operators obtained.

ACKNOWLEDGMENTS

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APPENDIX A: EQUATIONS OF MOTION

When writing all possible effective operators from a given set of light fields and covariant derivatives there may be redundant information; operators which differ by terms vanishing when the equations of motion are used are equivalent at the level of the S matrix (see for example \cite{18} and references therein). One can then use the classical equations of motion to reduce the actual set of operators to a minimal set of inequivalent operators. In particular, for operators that involve covariant derivatives, the equations of motion can be used to reduce them to operators without covariant derivatives. In what follows we sketch the way the substitution proceeds, without trying to be exhaustive. We denote the supersymmetry-gauge covariant derivative by $\nabla^\alpha = e^{-V} D^\alpha e^V$ and $D^\alpha$ for the covariant spinorial derivative.

Operators that involve

$$\nabla^2 \phi = e^{-V} D^2 e^V \phi$$

(A.1)
can be simplified by using the equations of motion. The classical equation for $\phi$ has the form

$$e^V \nabla^2 \phi = D^2 e^V \phi = (\frac{\partial W}{\partial \phi})^\dagger$$

(A.2)

where $W$ is the superpotential and involves operators up to dimension three.

As an example we can consider operator (A.6) below. From the MSSM superpotential (see appendix B), we obtain

$$\frac{\partial W}{\partial Q} = aU^c H_2 + bD^c H_1$$

(A.3)

where $a, b$ are constants, and therefore

$$(Q^\dagger e^V \nabla^2 Q)^\dagger = aQU^c H_2 + bQ D^c H_1.$$
The operators on the right are exactly (B.1) and (B.2). We also note that, in this particular case, the operators are a total divergence after taking the D-component on both sides, since the right-hand side is a chiral operator.

Operators that involve the combination
\[ V_\alpha \nabla^\alpha = \bar{D}^2 \left[ e^{-V} (D_\alpha e^V) \right] \nabla^\alpha \] (A.5)
can be similarly simplified by using the equations of motion.

In what follows we enumerate all the operators involving covariant derivatives and superfield strengths. We use the notation \( V_{\alpha I} \) to represent the superfield strength corresponding to superfields \( V_I \) defined in (7).

1. \( d = 5 \) operators

a. Operators involving fermions, vectors and covariant derivatives

\[
\begin{align*}
O_D^{(80)} &= (Q^1 e^{VQ}) \nabla^2 Q \\
O_D^{(81)} &= (U^{c1} e^{VU}) \nabla^2 U^c \\
O_D^{(82)} &= (D^{c1} e^{VD}) \nabla^2 D^c \\
O_D^{(83)} &= (L^c e^{VL}) \nabla^2 L \\
O_D^{(84)} &= (E^c e^{VE}) \nabla^2 E^c
\end{align*}
\]
where the expression for the covariant derivative involves those vector superfields that correspond to each case.

b. Operators involving scalars, vectors and covariant derivatives

\[
\begin{align*}
O_F^{(34)} &= H_1 V_\alpha \nabla \nabla H_2 \\
O_D^{(85)} &= (H^{c1} e^{VH}) \nabla^2 H_1 \\
O_D^{(86)} &= (H^2 e^{VH}) \nabla^2 H_2
\end{align*}
\]

C. Operators involving fermions, scalars, vectors and covariant derivatives

\[
\begin{align*}
O_F^{(35)} &= L V_\alpha \nabla \nabla H_2 \\
O_D^{(87)} &= (L^c e^{VL}) \nabla^2 H_1
\end{align*}
\]
Similar comments as above for \( V_{\alpha I} \).

2. \( d = 6 \) operators

a. Operators involving fermions, vectors and covariant derivatives

\[
\begin{align*}
O_F^{(36)} &= Q \nabla^2 U^c Q \nabla \nabla D^c \\
O_F^{(37)} &= Q \nabla^2 U^c L \nabla \nabla E^c \\
O_D^{(88)} &= Q^1 \nabla^2 e^{VQ} \nabla^2 Q \\
O_D^{(89)} &= U^{c1} \nabla^2 e^{VU} \nabla^2 U^c \\
O_D^{(90)} &= D^{c1} \nabla^2 e^{VD} \nabla^2 D^c \\
O_D^{(91)} &= L^c \nabla^2 e^{VL} \nabla^2 L \\
O_D^{(92)} &= E^c \nabla^2 e^{VE} \nabla^2 E^c
\end{align*}
\]
\[ O_D^{(93)} = (Q^c e^V Q) \nabla_\alpha Q \]  
\[ O_D^{(94)} = (U^c e^V U) \nabla_\alpha U^c \]  
\[ O_D^{(95)} = (D^c e^V D) \nabla_\alpha D^c \]  
\[ O_D^{(96)} = (L^c e^V L) \nabla_\alpha L \]  
\[ O_D^{(97)} = (E^c e^V E) \nabla_\alpha E^c \]  

\[ O_D^{(98)} = (Q \nabla_\alpha U^c) \]  
\[ O_D^{(99)} = (U^c \nabla_\alpha Q) \]  
\[ O_D^{(100)} = (D^c \nabla_\alpha D) \]  
\[ O_D^{(101)} = (L^c \nabla_\alpha L) \]  
\[ O_D^{(102)} = (E^c \nabla_\alpha E) \]

b. Operators involving scalars, vectors and covariant derivatives

\[ O_D^{(38)} = H_1 \nabla_\alpha H_1 H_2 \nabla_\alpha H_2 \]
\[ O_D^{(98)} = H_1 \nabla_\alpha U^c \nabla_\alpha H_1 \]
\[ O_D^{(99)} = H_2 \nabla_\alpha U^c \nabla_\alpha H_2 \]
\[ O_D^{(100)} = (H_1^c e^V H_1) \nabla_\alpha H_1 \]
\[ O_D^{(101)} = (H_2^c e^V H_2) \nabla_\alpha H_2 \]

\[ O_D^{(39)} = Q H_1 \nabla_\alpha U^c \nabla_\alpha H_1 \]
\[ O_D^{(40)} = L H_2 \nabla_\alpha U^c \nabla_\alpha H_2 \]
\[ O_D^{(41)} = L H_1 \nabla_\alpha H_2 \nabla_\alpha H_2 \]
\[ O_D^{(102)} = L \nabla_\alpha U^c \nabla_\alpha H_1 \]
\[ O_D^{(103)} = L \nabla_\alpha U^c \nabla_\alpha H_2 \]

\[ O_F^{(39)} = Q \nabla_\alpha U^c E \nabla_\alpha H_1 \]
\[ O_F^{(40)} = L \nabla_\alpha L H_2 \nabla_\alpha H_2 \]
\[ O_F^{(41)} = L \nabla_\alpha H_1 H_2 \nabla_\alpha H_2 \]
\[ O_F^{(102)} = L \nabla_\alpha V \nabla_\alpha H_1 \]
\[ O_F^{(103)} = L \nabla_\alpha V \nabla_\alpha H_2 \]

APPENDIX B: DIMENSION-FOUR OPERATORS

We need dimension-four operators of the F-type to construct the superpotential. The list is the following:

\[ O_F^{(42)} = Q U^c H_2 \]
\[ O_F^{(43)} = Q D^c H_1 \]
\[ O_F^{(44)} = L E^c H_1 \]
\[ O_F^{(45)} = Q D^c L \]
\[ O_F^{(46)} = U^c D^c D^c \]
\[ O_F^{(47)} = L L E^c \]
\[ O_F^{(48)} = H_1 H_1 E^c \]

The first three operators conserve baryon and lepton number and enter the MSSM Lagrangian. Operators (B.4)-(B.7) violate these symmetries and induce fast proton decay. They are excluded by imposing and additional symmetry such as R parity (see section IV)
