HOW THE “H-PARTICLE” UNRAVELS THE QUARK DYNAMICS

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Abstract

It is shown that the short-range part of the Goldstone boson exchange interaction between the constituent quarks which explains baryon spectroscopy and the short-range repulsion in the NN system implies a strong short-range repulsion in the flavour-singlet state of the ΛΛ system with \( J^P = 0^+ \). It then suggests that a deeply bound H-particle should not exist. We also compare our approach with other models employing different hyperfine interactions between quarks in the nonperturbative regime of QCD.

Soon after the suggestion that the hyperfine splitting in hadrons should be due to the colour-magnetic interaction between quarks [1,2] it has been noted by Jaffe [3] that the dibaryon \( uuddss \) with \( J^P = 0^+, I = 0 \), called the H particle, is stable against strong decays. Its mass turned out to be about 80 MeV below the ΛΛ threshold. The reason is that in this case the flavour-singlet state in the 6q system is allowed and the colour-magnetic interaction gives more attraction for the most favourable configuration \([33]_{CS}\) than for two well-separated Λ-hyperons. In Jaffe’s picture the H-particle should be a compact object in contrast to the molecular-type structure of the deuteron.
Since Jaffe’s prediction many calculations in a variety of models have appeared [4]. They give a wide range of predicted masses depending on the model. Realistic calculations usually predict a well-bound H-particle. In particular, the quark-cluster calculations suggest that the implications of the colour-magnetic interaction are radically different in two-nucleon and in coupled YY-YN systems. While in the former case the colour-magnetic interaction between quarks gives rise to a strong short-range repulsion in the NN system, in the latter, there appears a soft attraction or a soft repulsion at short-range [5] when the linear combination of the coupled channels is close to a flavour-singlet state. This soft short-range interaction, reinforced by the medium and long-range attraction coming from the meson exchange between lambdas, provides a bound state with a binding energy of the order 10-20 MeV [6] or even more [7]. However, a simple quark-cluster variational basis is rather poor at short-range. While it is not so important for the baryon-baryon systems with strong repulsion at short-range, this shortcoming becomes crucial for the ΛΛ − ΝΞ − ΣΣ system, with the quantum numbers of H. As soon as a simple quark-cluster variational basis is properly extended, a very deeply bound state is again found [8].

The existence or non-existence of the H-particle has to be settled by experiment. Since approximately 20 years several experiments have been set for “hunting” the H-particle. The very recent high-sensitivity search at Brookhaven [9] gives no evidence for the production of deeply bound H, the production cross section being one order of magnitude below theoretical estimates.

It has been recently suggested that in the low-energy regime, light and strange baryons should be considered as systems of three constituent quarks with QQ interaction (Q is a constituent quark, to be contrasted with a current quark q) that is formed of a central confining part and a chiral interaction that is mediated by Goldstone bosons between constituent quarks [10]. Indeed, at low temperatures and densities, the underlying chiral symmetry of QCD is spontaneously broken by the QCD vacuum. This implies that the valence quarks acquire a constituent (dynamical) mass, which is related to the quark condensates < \bar{q}q > and at the same time the Goldstone bosons π, K, η appear, which couple directly to the con-
stituent quarks [11]. It has been shown that the hyperfine splittings as well as the correct ordering of positive and negative parity states in spectra of baryons with valence u,d,s quarks are produced in fact not by the colour-magnetic part of the one-gluon exchange interaction (OGE) but by the short-range part of the Goldstone boson exchange (GBE) interaction [10,12,13]. This short-range part of the GBE interaction has just opposite sign as compared to the Yukawa potential tail and is much stronger at short interquark separations. There is practically no room for OGE in light baryon spectroscopy and any appreciable amount of colour-magnetic interaction, in addition to GBE, destroys the spectrum [14]. The same short-range part of the GBE interaction, which produces good baryon spectra, also induces a short-range repulsion in the NN system [15]. Thus it is interesting to study the short-range interaction in the ΛΛ system and the stability of the H-particle in the GBE model.

For a qualitative insight it is convenient first to consider a schematic quark-quark interaction which neglects the radial dependence of the GBE interaction. In this model, the short-range part of the GBE interaction between the constituent quarks is approximated by:

\[ V_\chi = -C_\chi \sum_{i<j} \lambda_i^F \cdot \lambda_j^F \bar{\sigma}_i \cdot \bar{\sigma}_j, \]  

(1)

where \( \lambda_i^F \) with an implied summation over \( F (F=1,2,...,8) \) are the quark flavour Gell-Mann matrices and \( \bar{\sigma} \) the spin matrices. The minus sign of the interaction (1) is related to the sign of the short-range part of the GBE interaction (which is opposite to that of the Yukawa potential tail), crucial for the hyperfine splittings in baryon spectroscopy.

In a harmonic oscillator basis, \( \hbar \omega \) and the constant \( C_\chi \) implied by the schematic model (1), can be determined from \( \Delta N \) and \( N(1440) - N \) splittings to be \( C_\chi = 29.3 \text{ MeV} \), \( \hbar \omega \sim 250 \text{ MeV} \) [10].

The colour- and flavour-singlet \( uuudds \) states are described by \([222]_C\) and \([222]_F\) Young diagrams respectively. For the S-wave relative motion of two \( s^3 \) clusters, the spatial symmetries of the 6Q system are \([6]_O, [42]_O\) and for the spin \( S=0 \) the corresponding spin symmetry is \([33]_S\). The antisymmetry condition requires \([f]_{FS} = [\bar{f}]_{OC} \), where \([\bar{f}]\) is the conjugate of \([f]\). Thus, among the states given by the inner products:
\[ [33]_S \times [222]_F = [33]_{FS} + [411]_{FS} + [2211]_{FS} + [1^6]_{FS}, \]  
(\text{2})

\[ [6]_O \times [222]_C = [222]_{OC}, \]  
(\text{3})

\[ [42]_O \times [222]_C = [42]_{OC} + [321]_{OC} + [222]_{OC} + [3111]_{OC} + [21111]_{OC}, \]  
(\text{4})

only the four states are allowed:

\[ |1> = |[6]_O[33]_{FS}[222]_{OC}> \]
\[ |2> = |[42]_O[33]_{FS}[222]_{OC}> \]
\[ |3> = |[42]_O[411]_{FS}[3111]_{OC}> \]
\[ |4> = |[42]_O[2211]_{FS}[42]_{OC}> \]  
(\text{5})

For these states the expectation values of the interaction (1) can be easily calculated in terms of the Casimir operators eigenvalues for the groups \( SU(6)_{FS}, SU(3)_F \) and \( SU(2)_S \) using the formula given in Appendix A of Ref. [15]. The corresponding matrix elements are given in Table 1. Thus the interaction (1) is attractive for the states \(|1> - |3>\) and repulsive for \(|4>\). This suggests that it is a good approximation to restrict the basis to \(|1>, |2>\) and \(|3>\) for the diagonalization of a more realistic Hamiltonian. Keeping in mind that the spatial symmetry \([6]_O\) is compatible with the lowest non-excited \(s^6\) configuration, one can roughly evaluate the energy of the lowest state relative to the \(2\Lambda\) threshold as:

\[ <s^6[6]_O[33]_{FS}|H_0 + V_\chi |s^6[6]_O[33]_{FS}> - 2 <s^3[3]_O[3]_{FS}|H_0 + V_\chi |s^3[3]_O[3]_{FS}> = 4C_\chi + 3/4\hbar\omega = 305 \text{ MeV}, \]  
(\text{6})

where \(H_0\) is the kinetic energy in the 6Q system. While here and below we use notations of the shell model, it is always assumed that the center of mass motion is removed. In the kinetic energy, \(3/4\hbar\omega\), we have neglected the mass difference between \(u,d\) and \(s\) constituent quarks. The pair-wise colour electric confinement contribution is exactly the same for \(s^6\) configuration and for two well separated \(s^3\) clusters, so it cancels out.

This simple estimate shows that the lowest “compact” flavour-singlet 6Q state with quantum numbers \(J^P = 0^+, I = 0, S = -2\) lies a few hundreds MeV above the \(\Lambda\Lambda\) threshold.

In a more quantitative calculation we use the Hamiltonian [10,12]:

\[ \ldots \]
\[ H = \sum_{i=1}^{6} m_i + \sum_i \frac{\vec{p}_i^2}{2m} - \frac{(\sum_i \vec{p}_i)^2}{12m} + \sum_{i<j} V_{\text{conf}}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij}) \] (7)

where the confining interaction is:

\[ V_{\text{conf}}(r_{ij}) = -\frac{3}{8} \lambda_i^c \cdot \lambda_j^c C r_{ij} \] (8)

and the spin-spin component of the GBE interaction between the constituent quarks i and j reads:

\[ V_\chi(\vec{r}_{ij}) = \left\{ \sum_{F=1}^{3} V_\pi(\vec{r}_{ij}) \lambda_i^F \lambda_j^F \right. \\
\left. + \sum_{F=4}^{7} V_K(\vec{r}_{ij}) \lambda_i^F \lambda_j^F + V_\eta(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j \] (9)

where \( \lambda^0 = \sqrt{2/3} \mathbf{1} \) (\( \mathbf{1} \) is the 3 \( \times \) 3 unit matrix). The interaction (9) includes \( \pi, K, \eta \) and \( \eta' \) exchanges. In the large-\( N_c \) limit, when axial anomaly vanishes [16], the spontaneous breaking of the chiral symmetry \( U(3)_L \times U(3)_R \rightarrow U(3)_V \) implies a ninth Goldstone boson [17], which corresponds to the flavour singlet \( \eta' \). Under real conditions, for \( N_c = 3 \), a certain contribution from the flavour singlet remains and the \( \eta' \) must thus be included in the GBE interaction.

The parameters of the Hamiltonian (7)-(9) can be found in Ref. [18]. The Hamiltonian (7)-(9) with constituent masses \( m_u,d = 340 \text{ MeV}, m_s = 440 \text{ MeV} \) provides a very satisfactory description of the low-lying N and \( \Delta \) spectra in a fully dynamical nonrelativistic 3-body calculation [18] as well as of the strange baryon spectra [19]. However, this parametrization should be considered as an effective one only. Within the semi-relativistic parametrization [13] the parameters of the potential, as well as the form of the short-range part of \( V_\chi \) is very different. Since in this letter we study only qualitative effects, related to the spin-flavour structure and sign of the short-range part of the GBE interaction, we consider the present nonrelativistic parametrization as a reasonable framework. The nonrelativistic parametrization [18] is especially convenient for our purpose since it allows to use the \( s^3 \) harmonic oscillator function as a variational ansatz for all the ground state octet and decuplet baryons. In particular, \( \langle \Lambda|H|\Lambda > \) takes its minimal values of 1165.4 MeV at a harmonic
oscillator parameter value of $\beta = 0.449$ fm, i.e. only about 40 MeV above the actual value, obtained in the dynamical 3-body calculations [19].

At zero separation between two $s^3$ clusters in S-wave relative motion, in an adiabatic cluster approximation, only $s^6$ and $s^4p^2$ configurations survive [20]. The former one has $[6]_O$ spatial symmetry while the latter $[42]_O$. We diagonalize the Hamiltonian (7)-(9) in the basis of the first three states defined by (5) and find a very strong repulsion since the lowest eigenvalue of the flavour-singlet state $J^P = 0^+$ is 847 MeV above the $\Lambda\Lambda$ threshold. It then definitely suggests that within the physical picture under discussion the compact (well bound) H-particle should not exist.

The value of the repulsion given above depends on the way the kinetic energy was calculated. For simplicity, in the 6Q system, we considered in the kinetic energy term only, that $u,d$ and $s$ quarks have the same mass $\bar{m} = (4m_u + 2m_s)/6$. We have also carried calculations in the extreme limits $\bar{m} = m_u$ and $\bar{m} = m_s$ and obtained 1050 MeV and 531 MeV respectively, which is the energy above the $\Lambda\Lambda$ threshold. This extreme values just prove that the strong repulsion persists in any case.

We consider this result as an additional evidence in favour of the GBE model to be the dominant hyperfine interaction between the constituent quarks. Indeed, a deeply bound H-particle is definitely excluded by experiment [9] and the colour-magnetic interaction, at variance with GBE, implies such a deeply bound state.

There are suggestions that the instanton-induced ('t Hooft) interaction could be important for the hyperfine splittings in baryons [21]. Assuming that the instanton-induced interaction is responsible for the essential part of $\Delta - N$ hyperfine splitting, the deeply bound H-particle should also disappear [22]. This interaction is very strong and attractive in colour-singlet $q\bar{q}$ pseudoscalar channel and could be indeed responsible for the chiral symmetry spontaneous breaking in QCD vacuum [23] and be the most important interaction in mesons; however, it is much weaker in $qq$ pairs. In addition, this interaction is related to the existence of zero modes of light current quarks in the instanton field. Thus there is no justification to use this interaction between the constituent quarks which are rather
heavy and are in fact quasiparticles, but not localized particles. There are also indications from lattice QCD that the instanton-induced interaction cannot be responsible for $\Delta - N$ splitting. For example, the $\Delta - N$ splitting disappears after cooling [24] (only instantons survive the cooling procedure), while it is appreciable before cooling. There is also direct evidence from lattice QCD that the hyperfine splittings are related mostly to $q\bar{q}$ excitations in baryons, but not to forces mediated by gluonic fields in $qq$ pairs [25]. There are also simple symmetry arguments showing that t’Hooft interaction cannot provide correct ordering of lowest positive and negative parity states in light and strange baryon spectra [10] (for baryon spectra obtained in such a model in a nonperturbative calculation see second paper in Ref. [21]. From the results obtained in this paper, one can see, indeed, that the lowest positive parity excitations in all parts of the spectrum -N(1440),$\Delta$(1600)$,\Lambda$(1600),$\Sigma$(1660),...- lie much above the negative parity excitations).

One should also mention the QCD sum rule estimate for the H-particle [26]. It is shown there that there is no qualitative difference between the two-nucleon system and the two-lambda one (including the flavor singlet channel), which strongly supports our point of view.

Here we have considered the 6Q system in a flavour singlet state only. The same analysis can be extended to the $\Lambda\Lambda$ system in the $^1S_0$ partial wave by adding to the flavour singlet other allowed flavour states. Then, similarly to the NN system [15] there will appear a strong short-range repulsion coming from the same short-range of the GBE interaction. There is however an attraction in the $\Lambda\Lambda$ system at medium- and long-range, coming from the Yukawa potential tail of the GBE interaction as well as from correlated two-pseudoscalar-meson exchange. At the moment, one cannot exclude that this interaction could weakly bind $\Lambda\Lambda$ in a molecule-like system. However, this attraction should be similar in its nature to the attraction in the $^1S_0$ partial wave of the NN system, which is too weak to bind the system. A firm prediction about the existence or non-existence of a weakly bound $\Lambda\Lambda$ system of nuclear nature can only be made in a fully dynamical calculation.
REFERENCES


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TABLE I. Expectation values of the operator (1) in $C_\chi$ units corresponding to the states (5).

| $[f|O|f]_{FS}[f|O]_{OC}$ | $<V_\chi>/C_\chi$ |
|--------------------------|-------------------|
| $[6]|O|33]_{FS}[222]_{OC}$ | -24               |
| $[42]|O|411]_{FS}[3111]_{OC}$ | -24               |
| $[42]|O|33]_{FS}[222]_{OC}$ | -24               |
| $[42]|O|2211]_{FS}[42]_{OC}$ | 8                 |