Evaluation of Multiloop Diagrams via Lightcone Integration

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Abstract

When loop momenta of a Feynman or a nonabelian cut diagram are written in lightcone coordinates, integration over their + components can be performed using residue calculus. This yields a number of terms, in which the + components of their internal momenta are evaluated at the poles of appropriate internal lines. The challenge is to find these internal lines for an arbitrarily complicated multiloop diagram. We shall provide two ways for doing that.
I. INTRODUCTION

This work was motivated by the desire to find systematic and efficient ways to evaluate complicated elastic-scattering diagrams at high energies, in the hope of including enough of them to restore unitarity to the BFKL Pomeron \cite{bfkl_pomeron}. A frequent obstacle in this kind of calculations is the tendency for leading-log contributions from individual Feynman diagrams to cancel one another, but this obstacle can be bypassed when Feynman diagrams are replaced by nonabelian cut diagrams, as discussed elsewhere \cite{nonabelian_cut_diagrams}. With either kind of diagrams, the computational challenge remains, though much has been written on it through the years. See, for example, Refs. \cite{feynman_parameters} and \cite{lightcone_coordinates} for discussions and lists of original literature.

It is in applying the method of Cheng and Wu \cite{cheng_wu} that we came upon a modest discovery which we would like to share. The result can be applied to any diagram without high energy approximations, so we shall proceed to discuss it in that context.

There are two popular ways to evaluate loop diagrams. One can introduce Feynman or Schwinger parameters to carry out the momentum integrations. The resulting expression has an electric-circuit interpretation, and allows an analytical expression for the integrand to be written directly from the Feynman diagram \cite{feynman_parameters}. The expression is string-like \cite{string_method} and can be interpreted in a first-quantized formalism \cite{first_quantized_formalism}. The main advantage of this method is its manifest Lorentz covariance, and the main disadvantage is that the integrand no longer factorizes into a product of propagators and vertices.

Alternatively we can simply try to evaluate the integral in its original form. It is often convenient to use lightcone coordinates $q_\pm = q^0 \pm q^3$ for this purpose, because the denominator of a propagator, $q^2 - m^2 + i\epsilon = q_+ q_- - q_\perp^2 - m^2 + i\epsilon$, is linear in $q_+$ and $q_-$ but is quadratic in the usual Cartesian components. Linearity simplifies calculations, allows the $q_+$ (or $q_-$) integrations to be carried out by residue calculus without introducing square roots, thereby completing exactly one quarter of all integrations. The rest of the integrations generally cannot be done analytically except for very simple diagrams, though the $q_-$-integrations giving rise to energy dependence of the diagram can often be carried out in the leading-log
approximation [?].

Our contribution in this paper has to do with these $q_+$ integrations for multiloop diagrams. Integration by residue calculus is straightforward in principle but can be quite complicated in practice when many loops are involved. It is necessary to determine which of the poles are enclosed by the integration contour and evaluated (the ‘contributing poles’), and which are not. An ingenious flow diagram method invented by Cheng and Wu [?] has been used to locate the contributing poles, but in applying this method to a multiloop diagram we encountered complications which we have not seen discussed in the literature: the direction of some flows has to be reversed in subsequent integrations. As a result of this complication, it is generally not possible to identify the relevant contributing poles right away from the flow diagram, though as we shall explain, loop-by-loop computations can be carried out to find them. In the presence of a large number of loops, this computation gets complicated and tedious, but fortunately there is a much simpler ‘path method’ that can be used in those situations. We shall discuss both of them in subsequent sections.

These methods can also be used to evaluate the $q_+$ integrations of nonabelian cut diagrams recently devised [?,?,?].

We shall review flow-diagrams [?] and how they can be used to evaluate $q_+$ integrations of Feynman diagrams in Sec. 2. The problem of flow reversal, the recipe to identify it, and the computation of contributing poles will be discussed in Sec. 3. The path method is discussed in Sec. 4, and the modifications necessary to deal with nonabelian cut diagrams will be discussed in Sec. 5.

II. FLOW DIAGRAM

We review in this section the flow diagram [?], and how it can be used to carry out loop integrations of an $n$-line $\ell$-loop Feynman amplitude

\[
\mathcal{T} = - \left( \frac{i}{2(2\pi)^4} \right)^\ell \int \left( \prod_{a=1}^\ell d^2 k_a \, dk_a_- \, dk_a_+ \right) \frac{N}{\prod_{i=1}^n D_i} \tag{2.1}
\]
with residue calculus. Here $k_a$ are the independent loop momenta, so the internal momenta $q_i$ are linear combinations of $k_a$ and the external momenta $p$’s. In lightcone coordinates $q_\pm = q^0 \pm q^3$, the denominator of a propagator with momentum $q$ is given by $D = q_+ q_- - q_\perp^2 - m^2 + i\epsilon$. Vertex and the other necessary factors are contained in the numerator $N$.

To make it easier to describe we shall always choose to close the integration contours in the lower-half plane, thus taking the contributing poles from the lower half planes. Since $k_{a+}$ appears in each denominator $D$ multiplied by a $q_-$, whether the pole of this denominator in the $k_{a+}$ variable is in the upper or the lower half plane depends on the sign of $q_-$. Which is which does not matter since the two half planes can be interchanged by changing the integration variable from $k_{a+}$ to $-k_{a+}$, if necessary.

The flow diagram [?] is a graphical way to keep track of that sign. More specifically, it is a Feynman diagram in which the direction of the ‘−’ momentum flow is indicated by an arrow. Momentum conservation requires each vertex to have some incoming arrows and some outgoing arrows. For reasons to be explained later, flow diagrams with arrows all in the same direction (all clockwise or all anticlockwise) around a closed loop can also be ignored.

A single Feynman diagram generally possesses many flow diagrams, each representing a region of the ‘−’ momenta. It is easy to see that those with flow arrows pointing in one direction around a loop give rise to poles in one half plane, and those with flow arrows pointing in the other direction give rise to poles in the opposite half plane. Again it does not matter which is which.

Now we can see why a flow diagram with all arrows running in the same directions around a closed loop can be ignored. In that case all the poles are in the same half plane, so closing the contour in the other half plane makes it zero so it can be neglected.

This then is a brief summary of the flow-diagram technique of Cheng and Wu [?]. When this is carried out in a multiloop diagram, a number of complications emerges. It is the purpose of this paper to point out these complications and discuss how they can be resolved, first in a straightforward but relatively complicated way, and then with a much simpler ‘path
III. FLOW REVERSAL

The location of the poles are governed by the flow diagram as explained above only for real contours. However, after the integration over loop-\(a\) is performed, resulting in a pole at line \(j\) being encircled by a complex contour and evaluated at \(q_{j+} = (q_{j\perp}^2 + m^2 - i\epsilon)/q_{j-}\), the ‘+’ component of every momentum in the same loop will acquire a small imaginary part via its dependence on \(k_{a+}\). The remaining denominators \(D_i = q_{i+}q_{i-} - a_i + i\epsilon (i \neq j)\) of the loop now have two sources of \(i\epsilon\): the original one and the one inherited from the \(a\)-integration. If the combined sign of \(i\epsilon\) remains positive, then the original flow diagram can be used in subsequent integrations to determine the pole locations, as originally intended. If however the combined sign is negative, then the pole of \(D_i\) would instead belong to the other half plane, a consequence that can be simulated by reversing the arrow direction of the \(i\)th line. This is flow reversal.

When does flow reversal take place? To be definite we shall assume all \(\epsilon\) to be finite and identical in every loop just before its integration. The combined coefficient of \(i\epsilon\) in \(D_i\) is then \(\mp q_{i-}/q_{j-} + 1\), for line \(i\) running in the same/opposite direction as line \(j\). The flow of the \(i\)th line should therefore be reversed iff this coefficient is negative, which happens when line \(i\) belongs to the same loop, runs in the same direction as line \(j\), and carries a larger ‘−’ momentum than line \(j\). This then is the recipe to determine whether a flow should be reversed or not.

Let us illustrate this recipe and the subsequent calculations to obtain the contributing poles with two explicit examples: a two-loop diagram, and a four-loop diagram. In the process we will see how important it is to take flow reversals into account.
A. A two-loop example

Fig. 1 is one of two possible flow diagrams for a two-loop Feynman diagram; the other has line 6 reversed.

Let $a$ denote the loop with lines (1536) and $b$ the loop with lines (2647). The big loop with lines (153472) is the union of these two loops and will be denoted by $a.b$. Only two of the three loop-momenta are independent.

Suppose we choose the loop momentum for $a$ to be one of the two independent momenta, and decide to carry out its (+ component) integration first. In this loop the flow along line 1 and the flow along lines 5,3,6 are opposite, so their respective poles lie in opposite half planes of $k_{a^+}$. We choose (the pole on line) 1 to be in the lower half plane so 5,3,6 appear in the upper half plane. Integration picks up the pole at 1; but we also have to worry whether this integration reverses any flow for other lines in the same loop. According to the recipe discussed above, lines 5,3,6 will not suffer a flow reversal because their arrows all point in the opposite direction to 1.

To simplify later descriptions we shall abbreviate this whole process by the notation $a(1)$, which means that a loop integration on loop $a$ has been carried out and the pole of line 1 is
singled out to be evaluated. This notation also implies no flow reversal for other lines in the loop. If lines \( p, q, \cdots \) did get reversed, then we would have used the notation \( a(1|p,q,\cdots) \) instead.

Now we are ready to tackle the second integration. Since the ‘+’ momentum on line 1 is already determined by the \( a \)-integration, \textit{no subsequent loop is allowed to contain it}. For that reason the second loop must be \( b \) and not \( a.b \). Referring to Fig. 1, the result of the \( a \) and the \( b \) integrations may be taken to be \( a(1)b(2) \) and \( a(1)b(6|2) \). This is indicated in Fig. 2, where a cross (\( \times \)) is the pole taken at the first integration, and a heavy dot (\( \bullet \)) is the pole taken at the second integration. Figs. 2(a) and 2(b) give respectively these two terms.

![Diagram](image)

\textbf{FIG. 2.} Contributing poles of Fig. 1 obtained by first carrying out the \( a \)-loop integration. Pole of the first integration, for this and for subsequent diagrams, is indicated by a cross (\( \times \)), and the pole of the second integration is indicated by a heavy dot (\( \bullet \)). Figs. 2(a) and 2(b) represent respectively the terms \( a(1)b(2) \) and \( a(1)b(6|2) \).

We conclude that the contributing poles for this diagram are lines (1,2) and lines (1,6). Since no flow reversal occurred in this situation until all the integrations are done, we could have read off these contributing poles directly from the flow diagram in Fig. 1. This corresponds to situations under which flow diagrams are normally used.

\textit{The contributing poles must be independent of the order of integrations and the choice of the independent loops.} We shall illustrate this point explicitly, and in the process demonstrate
how important flow reversals can be.

So let us try next to integrate $b$ first to get $b(2)$ and $b(6|2)$. No flow reversal takes place on $b(2)$ because lines 4 and 7 are in the opposite direction, and line 6, though in the same direction, has $q_6 = q_2 - q_1 < q_2 - q_1$. For the other term, the same inequality shows that a line reversal occurs on line 2. Hence $b(6|2)$.

For the term $b(2)$, the second integration must not contain line 2, so it has to be over loop $a$. The result is $b(2)a(1)$, giving rise to $(2,1)$ as the contributing pole for this term. This is drawn in Fig. 3(a). Again the cross denotes the pole resulting from the first integration, and the heavy dot the pole resulting from the second integration.

For the term $b(6|2)$, the remaining integration must be over loop $a.b$ in order not to contain line 6. With the reversal of line 2, all the lines in $a.b$ except 1 run in the same direction, so we may again just pick off the pole at 1, yielding $b(6|2)a.b(1)$, with the contributing pole at $(6,1)$. This is shown in Fig. 3(b). Notice that compared to Fig. 3(a) the direction of line 2 has been reversed.

We therefore obtain the same contributing poles as before. Note however that if line 2 was not reversed after $b(6)$, then the second integration over $a.b$ would have picked up an
erroneous term with relevant poles at 6 and 2.

FIG. 4. Contributing poles of Fig. 1 obtained by first carrying out the $a.b$-loop integration.

Finally suppose we carry out $a.b$ first, getting $a.b(1|2)b(6)$ and $a.b(2)a(1)$. In the first case line 2 is reversed, so the subsequent integration gives $b(6)$, making the total result $a.b(1|2)b(6)$, with the contributing pole (1,6). This is shown in Fig. 4(a). For the term $a.b(2)$, it requires no flow reversal so the second integration gives $a(1)$, making it $a.b(2)a(1)$, with contributing pole (1,2). This is shown in Fig. 4(b). The result is once again the same with the other two calculations. If flow reversals were not taken into account, the result would have been different and wrong.

The main lesson learned from this very simple example is that generally detailed loop-by-loop calculation must be performed, with proper flow reversals taken into account, in order to obtain the correct locations of the contributing poles.

B. A four-loop example

The task of obtaining the contributing poles becomes more arduous for diagrams with a larger number of loops. The calculation must be carried out loop by loop, with more and more terms and flow reversals to keep track of. Besides, with multiloops there is a
huge number of ways in choosing the independent loops and their order of integrations, each giving very different intermediate results though at the end they must all yield the same contributing poles. It is not known a priori how to make the best choice to maximally simplify the intermediate calculations.

To illustrate these points we shall work out in this subsection a four-loop example and obtain its contributing poles in two different ways.

Consider Fig. 5, with the following choice of independent loops: \(a = (4, 8, 12, 13, 7), b = (5, 9, 3, 11, 12, 8), c = (13, 12, 10, 1, 6),\) and \(d = (10, 12, 11, 2).\) We shall carry out the integrations in the order \(a, b, c, d\) as much as possible.

![FIG. 5. A four-loop (Feynman) flow diagram.](image)

The first integration over loop \(a\) yields \(a(4)\) and \(a(7|4)\). Since \(q_{7-} = q_{4-}\), whether 7 reverses the flow of 4, or 4 reverses 7, is quite arbitrary. We shall imagine a small ‘−’ momentum to be fed in at \(p_2\) to make \(q_{4-} > q_{7-}\), whence it is 7 that reverses 4.

After the \(b\)-integration we get four terms, which for brevity shall be written together as additions: \([a(4) + a(7|4)][b(3) + b(11|3)]\). After the \(c\)-integration there are now eight terms equal to \([a(4) + a(7|4)][b(3) + b(11|3)][c(1) + c(10|1)]\). The final \(d\)-integration is a bit complicated because loop \(d\) contains some of these poles from previously integrations so we are sometimes forced to take the loop \(d,c\) or the loop \(d,b\) instead of \(d\) itself. The final result contains 10 terms:
\[ [a(4) + a(7|4)]b(3)c(1)[d(10|11x) + d(11|10x)] + \\
[a(4) + a(7|4)]b(3)c(10|1)d.c(2) + \\
[a(4) + a(7|4)]b(11|3)[c(1)d.b(2) + c(10|1)d.c(2)]. \tag{3.1} \]

Momenta 10 and 11 are in the same direction in loop \(d\), but they are not strictly ordered. Either of these two ‘\(-\)’ momenta can be the smaller one, and therefore has the potential of reversing the flow of the other. This ‘uncertainty’ is indicated in the formula by attaching the symbol ‘\(x\)’ behind the potentially reversed lines. Fortunately such complicated reversals occurs at the very last integration so there is no need to worry about its details. On the other hand, if the \(d\)-integration was taken sooner, the computation would have been considerably more complicated because these flow reversals and their effect on subsequent integrations must be computed in detail. This illustrates the statement made earlier that there are many independent loops and many possible orders to carry out the integrations, and that some would lead to simpler computations than the other.

To summarize, we have obtained ten contributing poles: \((7,3,1,10), (7,3,1,11), (7,3,10,2), (7,11,2,1), (7,11,2,10)\), as well as another five with line 7 replaced by line 4.

Let us now illustrate another way to get the same result, by choosing this time the four independent loops to be \(a = (4, 8, 12, 13, 7), b = (5, 9, 3, 11, 12, 8), e = c.d = (1, 6, 13, 11, 2)\), and \(d = (10, 12, 11, 2)\), and try to carry out the integration in the order \(a, b, c.d, d\) as much as possible.

These loops are what we shall call later on the natural loops for the contributing pole \((7,3,1,10)\). They are obtained first by removing the lines 7,3,1,10 from the original diagram, and then inserting one of them back to get the loops.

The first two integrations are identical to those before, so we get \([a(4) + a(7|4)]b(3) + b(11|3)\). Now \(e = c.d\) contains the line 11 but not 3, so the next integration involving \(b(3)\) gives \(e(1) + e(2|1)\) but the next integration involving \(b(11|3)\) gives \(c(1) + c(10|1)\), as \(c = d.(c.d)\). The last loop \(d\) contains lines 2 and 11, so for some terms the integration over \(d\) has to be changed into integration over \(d.b\) or \(d.e = c\). The final answer is
\[ a(4) + a(7|4)]b(3)e(1)[d(10|11x) + d(11|10x)] + \\
b(3)e(2|1)c(10) + b(11|3)c(1)d.b(2) + \\
b(11|3)c(10|1)d.b(2). \quad (3.2) \]

This results in the same ten contributing poles as before, as it should.

**IV. PATH METHOD FOR FINDING CONTRIBUTING POLES**

In this section we propose a simple method to obtain the contributing poles. With this method there is no need to declare the independent loops and their order of integrations, so there is no need to keep track of the complicated flow reversals either. This makes the method most useful in the presence of a large number of loops.

We begin by choosing a path \( P \) in the flow diagram. By a path we mean a continuous line (no branches, no loops) running from beginning to end, with all the arrows on it pointing in the same direction. The solid lines in Figs. 6 and 7 are examples of such paths. By adding branches to the path we can construct trees. A class of these trees, \( T[P] \), turns out to be in one-one correspondence with the contributing poles. The path method of finding contributing poles is actually a method to construct the trees in \( T[P] \).

From an \( \ell \)-loop diagram one can obtain trees by removing \( \ell \) lines. We shall refer to these removed lines as the *missing lines* for the tree. The set of all trees so obtained with path \( P \) as their common backbone will be denoted by \( S[P] \). From \( S[P] \) we select a subset \( T[P] \) satisfying the following *directional rule*: when any one of the \( \ell \) missing lines is inserted into the tree, a loop is formed. If the direction of any of the inserted line around this loop is the same as the direction of the lines along path \( P \), this tree is rejected. If it is opposite, then this tree is retained to be a member of \( T[P] \).

We assert that the missing lines of any tree in \( T[P] \) is a contributing pole of the diagram, and there is actually a one-one correspondence between contributing poles and individual trees in \( T[P] \). This is the essence of the *path method*.
This method does not restrict what path \( P \) one chooses, but the longer the path the fewer the number of contributing poles, and the easier the calculations. So in practice we often choose the longest path we can manage, though this is not a requirement of the method. Note that the set of contributing poles for a flow diagram is not unique, for we may always adopt the poles in the opposite half planes instead. However, if we have the half-planes fixed, then they are unique and can be computed in many different ways.

Before proceeding to prove the path method let us see how it can be applied to obtain the contributing poles of Figs. 1 and 2 very simply.

A. Examples

For Fig. 1 let us choose the path \( P \) to be (5347), as shown in Fig. 6 as solid lines. Then \( S[P] = \{(P, 6), (P, 1), (P, 2)\} \), and \( T[P] = \{(P, 6), (P, 2)\} \). The tree \((P, 1)\) violates the directional rule for the following reason so it is not in \( T[P] \). When line 6 is inserted into \((P, 1)\), it runs in the same direction along the loop (6153) as line 3 of \( P \), so it has to be rejected. With this \( T[P] \), the contributing poles are the missing lines so they are (1,2) and (1,6), agreeing with the result obtained previously.

Let us next apply the method to obtain the contributing poles of Fig. 2, taking \( P = (6, 13, 12, 8, 5, 9) \) as the path (Fig. 7). Then
\[ T[P] = \{(P, 7, 2, 11), (P, 7, 2, 10), (P, 7, 1, 11), (P, 7, 3, 10), (P, 7, 1, 3)\}, \]
and five more with 7 replaced by 4. The contributing poles are therefore (7,3,1,10), (7,3,1,11), (7,3,10,2), (7,11,2,1), and (7,11,2,10), and another five with 7 replaced by 4, the same 10 terms as before. The trees in \( S[P] / T[P] \) are \( \{(P, 7, 1, 2), (P, 7, 2, 3), (P, 7, 10, 11)\} \), and three more with 7 replaced by 4. \((P, 7, 1, 2)\) violates the directional rule when the line 11 is inserted; \((P, 7, 2, 4)\) violates the directional rule when line 10 is inserted; and \((P, 7, 10, 11)\) violates the directional rule when 2 is inserted.

![Diagram](image)

**FIG. 7.** The solid line is the path \( P \) used to obtain the contributing poles of Fig. 2.

**B. Proof**

A tree \( t \in S[P] \) defines a set of independent loops \( \mathcal{N}[t] \) of the original diagram by filling in the missing lines one at a time. The special feature of \( \mathcal{N}[t] \) is that the missing lines are never on the boundary of two loops.

Now we proceed to the proof of the path method. We fix the half-planes by adopting the convention that the poles in the lower-half planes shall always be chosen to consist of lines running in the opposite direction to those on \( P \).

The proof makes use of the simple fact that the same set of contributing poles can be computed using any independent loops and any order of integration.
Removing the pole lines of a contributing pole from the original diagram gives rise to a tree in $S[P]$. We shall denote the set of all such trees as $T'[P]$. Our task is to show that $T[P] = T'[P]$.

Take any $t' \in T'[P]$. The removed pole lines clearly satisfy the directional rule when they are inserted back, because poles are always taken from those lines running in the opposite direction as $P$. Hence $t' \in T[P]$ and $T'[P] \subset T[P]$.

Conversely, take a $t \in T[P]$, and use the independent loops $\mathcal{N}[t]$ to compute the contributing poles. The missing lines of $t$ are obviously one of the pole lines, for according to the directional rule they all run opposite to the path direction. Hence $t \in T'[P]$ and $T[P] \subset T'[P]$.

Putting the two together, we get $T[P] = T'[P]$, as desired.

V. NONABELIAN CUT DIAGRAMS

General methods found in the literature to compute high energy limits of Feynman diagrams [?,?] are usually valid only in the leading-log approximation. They become insufficient if these leading-log contributions cancel when the Feynman diagrams are summed, a situation which unfortunately occurs quite frequently. A method was developed recently to bypass this difficulty, by allowing the cancellations to occur before high energy limit is taken. The cancellations are incorporated into the individual nonabelian cut diagrams [?,?], whose spacetime amplitudes (for onshell diagrams) turn out to differ from the corresponding Feynman diagram only by having the denominators $(q_i^2 - m^2 + i\epsilon)^{-1}$ of certain propagators replaced by the corresponding Cutkosky propagators $-2\pi i \delta(q_i^2 - m^2)$. It can be shown that the sum of Feynman diagrams is the same as the sum of nonabelian cut diagrams [?,?], but the beauty of the latter is that their leading-log contributions will survive the sum.

For high-energy elastic (e.g., quark-quark) scattering which we will speak on exclusively from now on, the Cutkosky propagators occur only on the quark lines. In the high energy limit, it can be shown that the combination $q_i^2 - m^2$ is actually proportional to the ‘$-$’
momentum on that line, so a $\delta$-function is that variable is a $\delta$-function of the ‘$-$’ momentum $[?,?]$. This has the effect of stopping the ‘$-$’ momentum from flowing through this line, so as far as the flow diagram is concerned we may think of these lines as being absent. For the rest of the nonabelian cut diagram the flows are constructed in exactly the same way as in a Feynman diagram, and contributing poles can be located the same way just as well.

As an example, consider the nonabelian-cut (flow) diagram of Fig. 8, where the Cutkosky propagator is located at line 8, indicated there by a vertical bar ($\mid$). Hence the ‘$-$’ momentum is absent from lines 8, and also from line 4 by continuity. We may therefore ignore these two lines in the rest of the discussions.

To obtain the contributing poles from the path method, we can choose the path to be $P = (1, 5, 10, 6, 3)$, then $T[P] = \{(P, 9), (P, 7)\}$, giving rise to the contributing poles $(2,7)$ and $(2,9)$.

![FIG. 8. A 3-loop (non-abelian cut) flow diagram. The solid line represents the path $P$.](image)

**VI. ACKNOWLEDGEMENT**

This research was supported in part by the Natural Science and Engineering Research Council of Canada and by the Québec Department of Education. Y.J.F. acknowledges the support of the Carl Reinhardt Major Foundation, and C.S.L. wants to thank Hung Cheng for a stimulating discussion.
REFERENCES

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