The Phase in Three–Pion Correlations

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Abstract

We discuss the complex phase generated in three pion correlation functions. The lowest order contribution to the phase is of order $q^2 R/K$, where $q$ is a typical relative momentum, $K$ is a typical center of mass momentum and $R$ is a typical radius parameter. This contribution is of purely kinematic origin. At next order we find a generic contribution of order $(qR)^3$ which is a result of odd modifications to the source emission function. We argue, that the scale for typical HBT correlations in ultrarelativistic heavy ion collisions is $q/K \ll qR \sim 1$, so that the third order correction actually dominates the phase in the experimentally relevant momentum range. We study in detail such contributions which arise from source asymmetries generated by flow, the source geometry and resonance decays.
I. INTRODUCTION

Recently the first study of three pion correlations ($\pi^+\pi^+\pi^+$) in heavy ion collisions was reported by NA44 [1]. The quality of the data was rather limited due to statistics, but we can expect a steady increase in statistics in forthcoming experiments. This should allow us to explore large parts of the phase space of the three pion correlation function in the foreseeable future.

The theoretical understanding of the three–pion correlation function is still rather limited. In an earlier paper [2] we constructed the three pion correlation function under the assumption that the source was incoherent and only interfered through Bose–Einstein correlations. Thus the three-particle correlation function could be derived from the two-body correlation function except for a complex phase, which will be studied here. Any deviation found experimentally from this prediction would have signalled new physics. We found satisfactory agreement with the available data, but due to the poor statistics could not make any conclusive statements.

Heinz and Zhang [3] explored the structure of the complex phase using an expansion in terms of the relative momentum of the two emitted particles. They found the lowest order contribution to be in the order $q^2 R/K$. Here, the relative momentum is given by $q = (k_1 - k_2)$ the average momentum by $K = (k_1 + k_2)/2$ and $k_i$ is the single particle momentum of particle $i$. This contribution is of purely kinematic origin. Typical heavy ion sources in nuclear collisions are of size $R \sim 5$ fm, so that interference occurs predominantly when $q \lesssim \hbar / R \sim 40$ MeV/c. Since typical particle momenta are $k_i \simeq K \sim 300$ MeV, we find that $q / K \ll 1$ and the lowest order contribution to the phase is very small.

In this paper we will extend the two calculations mentioned above and explicitly investigate the modifications due to generic three pion correlations. In section 2 we review the general derivation of the phase in 3 particle correlations in terms of an expansion up to cubic terms in the relative momenta $q$. In section 3 we estimate the size of the phase for a number of physical effects known to be present in relativistic heavy ion collisions and which produce asymmetric sources leading to a complex phase in the 3 particle correlation function. These are flow, resonance decay and asymmetric source geometry. We find some contributions to be sizable, but the overall effect off such a phase on the measurable correlation function turns out to be rather small and any detection will require exceedingly good resolution in the regime for which $q R \sim \hbar$. Finally, in the conclusions, we summarise our results and discuss some experimental consequences.

II. GENERAL FORMALISM

The three pion correlation function for incoherent sources is given by [2,4–6]

$$C_3(k_1,k_2,k_3) = 1 + F_{12} F_{21} + F_{23} F_{32} + F_{31} F_{13} + F_{12} F_{23} F_{31} + F_{21} F_{32} F_{13}$$
$$= 1 + |F_{12}|^2 + |F_{23}|^2 + |F_{31}|^2 + 2 \text{Re} [F_{12} F_{23} F_{31}] ,$$

(1)

where $F_{ij}$ is the Fourier transform of the source emission function $S(x,K)$ [7]

$$F_{ij} \equiv F(q_{ij},K_{ij}) = \frac{\int d^4x \ S(x,K_{ij}) \exp(ig_{ij}x)}{\sqrt{\int d^4x \ S(x,k_i) \int d^4x \ S(x,k_j)}} = F_{ji}^\star .$$

(2)

We used here the relative momentum $q_{ij} = k_i - k_j$ and the center of momentum variable $K_{ij} = (k_i + k_j)/2$. The emission function $S(x,k)$ is the probability of emission for a pion from space–time point $x$ with momentum $k$. It is related to the experimentally measured pion single particle spectrum

$$E_k \frac{dN}{d^3k} = \int d^4x \ S(x,k) ,$$

(3)
where \( E_k = k_0 = \sqrt{k^2 + m^2} \) is the on–mass–shell energy.

The relative momenta of three particles satisfy the relation

\[
q_{12} + q_{23} + q_{31} = 0,
\]

(4)
i.e., they span a triangle. This automatically assures translational invariance of the 3-body correlation function. Any translation in space–time by a distance \( x_0 \) will lead to an extra phase factor \( \exp(i q_{ij} \cdot x_0) \) in \( F_{ij} \) but Eq. (4) insures that the triple product of the three phase factors cancel in the 3-body correlation function (1).

The source emission function in the numerator of Eq. (2) is not evaluated for a momentum \( k_i \), but rather for the center of mass momentum \( K_{ij} \). Since all particles are detected on shell, we will have to evaluate the center of mass momenta in (2) slightly off–shell

\[
K^0_{ij} = E_K \left( 1 + \frac{q^2}{8 E_K^2} + \mathcal{O}\left(\frac{q^4}{E_K^4}\right) \right).
\]

(5)
This on–shell constraint for the two detected particles pushes the source emission function slightly off–shell by an amount

\[
S(x, K^0_{ij}, K_{ij}) = S(x, E_K, K_{ij}) + \frac{q_{ij}^2}{8 E_K} \frac{\partial S}{\partial E_K}(x, E_K, K_{ij}) + \mathcal{O}(q^4_{ij}).
\]

(6)
This correction is of order \((q/K)^2\) and is also present in the 2–particle correlation function. Since the off–shell structure of the source emission function is not accessible to us we will neglect this contribution, like in the 2–particle case, with the remark that a possible signal could be due to this correction.

With this in mind, we can rewrite Eq. (1) using on shell variables \( F_{ij} = F(q_{ij}, K_{ij}) \) and explicitly depict the phase \( \phi_{ij} = \phi(q_{ij}, K_{ij}) \)

\[
C_3(k_1, k_2, k_3) = 1 + |F_{12}|^2 + |F_{23}|^2 + |F_{31}|^2 + 2 |F_{12}| |F_{23}| |F_{31}| \cos(\phi_{12} + \phi_{23} + \phi_{31}).
\]

(7)
All information about possible imaginary contributions to \( F \) are now contained in the cosine of the 3 phases, \( \phi_{ij} \), defined as

\[
\tan \phi_{ij} = \frac{\text{Im} [F_{ij}]}{\text{Re} [F_{ij}]}. \tag{8}
\]
The phases arise from the triple product of Fourier transforms in Eq. (1). For the following discussion it is important to realize, that the three particle correlations as given above are defined over a 9 dimensional momentum space. This space can be either described by the 3 vectors \( k_1, k_2, k_3 \), or preferably by the center of momentum of the three emitted particles \( K = (k_1 + k_2 + k_3)/3 \) and two relative momenta, like \( q_{12} \) and \( q_{23} \). The kinematic transformation into center of momentum and relative momenta will actually fix the lowest order contribution in our expansion below.

To determine the phases we have to extract both the real and imaginary pieces of the Fourier transforms of the source emission function. The source emission function itself is real. The only source for an imaginary contribution is thus the Fourier transform via the exponential of the relative momentum. We introduce the symmetric and antisymmetric part, \( S_s \) and \( S_a \), of the source emission function by

\[
S_s(x - \langle x \rangle, K) = \frac{1}{2} [S(x - \langle x \rangle, K) + S(-(x - \langle x \rangle), K)]
\]
\[
S_a(x - \langle x \rangle, K) = \frac{1}{2} [S(x - \langle x \rangle, K) - S(-(x - \langle x \rangle), K)].
\]

(9)
The average of an operator $\xi$ is defined as [8]

$$
\langle \hat{\xi} \rangle = \frac{\int d^4x \, \hat{\xi} \, S(x, K)}{\int d^4x \, S(x, K)}.
$$

The space–time integral over the asymmetric part is identical zero, while the space–time integral over the symmetric part provides the normalisation of the source emission function.

The phase $\phi_{ij}$ in Eq. (8) is now

$$
\tan \phi_{ij} = \frac{1}{i} \frac{\int d^4x \, S_a(x, E_K, K_{ij}) \exp(iq_{ij}x)}{\int d^4x \, S_s(x, E_K, K_{ij}) \exp(iq_{ij}x)} ,
$$

which clearly demonstrates that the odd space–time moments of the source emission function $S$ generates the phase. If $S$ is even, then the Fourier transform in (2) will be real and the phases $\phi_{ij}$ vanish.

If we expand the exponential in Eq. (11) for small values of $q_{ij}$

$$
\exp(iq_{ij}x) = 1 + iq_{ij}x - \frac{1}{2} (q_{ij}x)^2 - \frac{i}{6} (q_{ij}x)^3 + O(q_{ij}^4) ,
$$

we obtain for the phase

$$
\tan \phi_{ij} = \langle (q_{ij}x) \rangle - \frac{1}{6} \langle (q_{ij} (x - \langle x \rangle))^3 \rangle + O(q_{ij}^4) .
$$

The sum of the 3 phases is then

$$
\phi_{12} + \phi_{13} + \phi_{23} = \frac{1}{2} q_{12}^\mu q_{23}^\nu \left[ \frac{\partial (x_\mu)}{\partial K^\nu} - \frac{\partial (x_\nu)}{\partial K^\mu} \right] - \frac{1}{24} \left[ q_{12}^\mu q_{12}^\nu q_{23}^\lambda + q_{23}^\mu q_{23}^\nu q_{12}^\lambda \right] \left[ \frac{\partial^2 (x_\mu)}{\partial K^\nu \partial K^\lambda} + \frac{\partial^2 (x_\nu)}{\partial K^\mu \partial K^\lambda} + \frac{\partial^2 (x_\lambda)}{\partial K^\mu \partial K^\nu} \right] - \frac{1}{2} q_{12}^\mu q_{23}^\nu (\langle 2 x_\mu \rangle - \langle x \rangle)_{\mu} (\langle x - \langle x \rangle \rangle_{\nu} (x - \langle x \rangle)_{\lambda} + O(q_{ij}^4) .
$$

This result was also obtained by Heinz and Zhang [3]. To obtain this formula we made use of the triangle relation (4) which assures that the three linear terms sum up to zero. The on mass shell constraint in (5) fixes the time components of the four vectors. These redundant components are not explicitly eliminated.

The last term in Eq. (14) is of order $(qR)^3$ and is generic. In contrast the first two terms are a result of choosing the three momenta $K, q_{12}$ and $q_{23}$ to span our 9 dimensional coordinate space and are of order $q^2R/K$ and $q^3R/K^2$ respectively. Since typical particle momenta are $k_i \simeq K \sim 300$ MeV and typical heavy ion sources in nuclear collisions have a size $R \sim 5$ fm, we find that $q/K \ll qR \sim \hbar$, so that the generic $(qR)^3$-contribution can actually dominate in the experimentally relevant momentum regime.

In a simple but realistic model for flow, we will demonstrate in the next section why contributions of order $q^2R/K$ are so small. Afterwards we check in two further models for resonance and source geometry if we actually can produce a significant phase effect at scales of order $qR \sim \hbar$.

### III. ASYMMETRIC SOURCES

From Eq. (13) and (14) we see that there are contributions to the phase due to odd space–time moments of the source emission function of order $q^2R/K$ and $(qR)^3$. To evaluate these contributions we need a model for the source. In the following we will study a number physical effects, which are known to be present in relativistic heavy ion collisions, and which result in asymmetric sources. The examples we will study in the next four subsections are flow, resonances, moving and bursting sources respectively. We will estimate their quantitative influence on the phases $\phi_{ij}$. 

3
A. Flow

The most common ansatz for particle production and collision dynamics in ultrarelativistic heavy ion collisions is the Bjorken scenario. One assumes cylindrical symmetry and requires local thermal equilibrium with longitudinal Bjorken flow \((u_z = z/t)\) as well as transverse flow \(v\) through a Boltzmann factor. Thus

\[
S(x, \mathbf{K}) \sim e^{-K \cdot u/T} S_x(x).
\]

where \(u = \gamma(v)(v, \sinh(\eta), \cosh(\eta))\) is the flow four-vector so that

\[
K \cdot u = m_\perp \gamma(v)(\cosh(\eta - Y) - \beta_K \cdot v).
\]

Here, \(\tau = \sqrt{t^2 - z^2}\) is the invariant time, \(\eta = 0.5 \ln(t + z)/(t - z)\) the space-time rapidity. and \(v = (v_x, v_y)\) the transverse flow. The pair velocity is \(\beta_K = \mathbf{K}/K^0\). The transverse flow contribution in Eqs. (15) and (16) provide a strong \(K\)-dependence. Further \(K\)-dependences in the emitting source, \(S_x(x)\), would introduce additional \(K\)-dependences. All these \(K\)-dependences should contribute to the second order kinematic correction in Eq. (14).

When transverse flow is present only a few terms contribute. They all vanish though, if we boost into the so called longitudinal center of momentum frame (LCMS). In this frame we have \(Y = 0\) and the center of momentum of the pair is parallel to the \(x\)-direction. This direction is labeled the outward direction (o), while the beam axis or \(z\)-direction is labeled the longitudinal direction (l). Perpendicular to these is the sideways (s) or \(y\)-direction. With this in mind we can evaluate the \((q/K)^2\) contribution of Eq. (14)

\[
q^\mu_1 q^\nu_3 \left[ \frac{\partial (x_\mu)}{\partial K^\nu} - \frac{\partial (x_\nu)}{\partial K^\mu} \right] = - \frac{1}{T} q^\mu_1 q^\nu_3 \left[ \langle x_\mu u_\nu \rangle - \langle x_\nu u_\mu \rangle \right] = 0.
\]

In the last step we made use of the fact, that \(\langle \gamma \rangle = 0\) due to cylindrical symmetry and reflection symmetry in the \(x - z\) plane. We also boosted to the LCMS in which \(Y = 0\), so that \(\langle z \rangle = \langle u_z \rangle = 0\) and \(v = (v_x, 0)\) points into the outward direction. Furthermore, due to symmetry any choice for \(v_x\) has to be an even function in \(y\) and \(z\), so that \(\langle v_x z \rangle = \langle v_x y \rangle = 0\). Finally, in the LCMS the pair velocity simplifies, so that \(q^\mu_{ij} = \beta_L q^\nu_{ij}\). As a result the \(xt\) and \(tx\) contribution in Eq. (17) cancel each other.

This result demonstrates, that the standard \(K\)-dependent contribution to the source emission function, i.e., flow does not produce sizable contributions to the kinematic correction of second order. This correction actually vanishes in the LCMS. We will see in the following examples that higher order, but generic, contributions like resonance and source geometry can in principal dominate the experimentally accessible momentum range.

B. Resonances

We can study the influence of resonances on the phase within a simple model [9]. Under the assumption of classical propagation, the resonance travels an extra distance \(\Delta x = u_\tau \Delta \tau\) before it decays and produces a pion. \(\Delta \tau\) is hereby the life time of the resonance and \(u_\tau\) its velocity. If, furthermore, the resonance life time is exponentially distributed with a decay width \(\Gamma_\tau = 1/\tau_\tau\), we obtain after averaging over the resonance life time a modified source emission function. Its Fourier transform reads

\[
F_\tau(q_{ij}, \mathbf{K}_{ij}) \sim \int dt_x S(x, E_K, \mathbf{K}_{ij}) \exp(iq_{ij} x) (1 - iq_{ij} \cdot u_\tau \tau)^{-1}.
\]
Every resonance \( r \) will supply such a contribution to the correlation function and \( x \) refers to the space–time production point of the resonance.

We can evaluate the phase in (8) for a source consisting of one resonance only. For a resonance velocity independent of the space–time production point \( x \) we find

\[
\tan \phi_{ij} = u_r \cdot q_{ij} \tau_r. \tag{19}
\]

We would like to investigate the significance of this result and compare it with current experiments. Due to the limited statistics in the experiments one reduces the 9 dimensional momentum space, on which the cosine defined in Eq. (7) depends, down to one invariant momentum

\[
Q^2_a = q_{12}^2 + q_{31}^2 + q_{23}^2, \tag{20}
\]

which is used to analyse the data.

In a previous paper we evaluated the radial part of the Fourier transform \( F \), neglecting any phase contribution and found that the correlation function in Eq. (7) can be rewritten as

\[
C_3(Q_3) = 1 + 3 \lambda_2 \exp\left(-\frac{x^2}{3}\right) (1 + \mathcal{O}(x^4))
+ 2 \lambda_2^{1.5} \exp\left(-\frac{x^2}{2}\right) (1 + \mathcal{O}(x^4)) \cos(\phi_{12} + \phi_{23} + \phi_{31}), \tag{21}
\]

where \( x = Q_3 R_{av} \) and \( R_{av} \) is an average source emission size parameter. \( R_{av} \) is obtained by forming the average mean square of the experimentally measured radii from two particle correlations. The parameter \( \lambda_2 \) accounts phenomenologically for a number of effects like a partially coherent sources, long lived resonances, final state interactions and Coulomb screening effects. These effects tend to reduce the value of this parameter form its ideal value 1. It’s value is also taken from the experimentally determined two particle correlation functions. The corrections of order \( x^4 \) are due to the non–sphericity of the source emission function. These corrections are found to be rather small experimentally, in the order of a percent, \( i.e. \) the source is close to spherical.

In calculating the phase we consider for simplicity only the momentum component in detector or \( K_{ij} \)-direction, \( i.e. \) the outward component \( q_{ij,o} \), and set all other momentum contributions to zero. With this simplifications we can evaluate the cosine in terms of only the outwards momenta

\[
\langle \cos (\phi_{12} + \phi_{23} + \phi_{31}) \rangle_{Q_{3,o}} = \frac{\int dq_{12,o} dq_{31,o} dq_{23,o} \cos (\phi_{12} + \phi_{23} + \phi_{31}) \delta(Q_{3,o}^2 - q_{12,o}^2 - q_{31,o}^2 - q_{23,o}^2) \delta(q_{12,o} + q_{31,o} + q_{23,o})}{\int dq_{12,o} dq_{31,o} dq_{23,o} \delta(Q_{3,o}^2 - q_{12,o}^2 - q_{31,o}^2 - q_{23,o}^2) \delta(q_{12,o} + q_{31,o} + q_{23,o})}. \tag{22}
\]

The \( \delta \)-function of the sum of the relative outward momenta assures that the 3 particles span a triangle.

In figure 1 we plot the cosine defined in Eq. (22) for different resonances. At large relative momentum each phase approach \( \pi/2 \) as seen from Eq. (19). Therefore the cosine of the three phases vanish for large \( Q_{3,o} \)
on a scale set by the decay width \( 1/\tau_r \). In the momentum region where correlation functions are sizeable, \( Q_{ij} \gtrsim 50 \text{ MeV} \), the short lived resonances as the \( K^*, \Delta \) and \( \rho \) do not produce a sizeable phase. The long lived resonances as \( \eta, \eta' \) and \( K_0^S \) cannot be seen at all due to their small form factor \( \sim (1 + u_r \cdot q_{r})^{-2} \) which effectively removes these long lived resonances from Bose-Einstein correlations and leads to an effectively smaller \( \lambda \) [9]. Only the \( \omega \) could have a significant influence on the phase at momenta \( Q_{ij} \sim 50 \text{ MeV} \). However, it would be small due to its own form factor and due to the small fraction of pions that are decay products of \( \omega \)’s.
FIG. 1. The phase contribution for different resonances as a function of the momentum $Q_{3,0}^2 = q_{12,0}^2 + q_{13,0}^2 + q_{23,0}^2$. The fall-off of the curves scales with the resonance life time. The transverse momentum of the pion pair is chosen $\beta_\perp = 0.7$.

The important part to notice is, that the phase contribution is suppressed additionally by a Gaussian form factor as in Eq. (21). Even if the cosine is sizeable, like for example for the $\omega$, the overall modification to the measured correlation function is still rather small due to the form factor suppression. Only an experiment with excellent resolution will be able to resolve the resonance contribution.

C. Moving surfaces

To have a detectable contribution to the phase we need to create an odd modification of the source emission function, which appears at a momentum scale small enough, so that it is not suppressed to drastically by the Gaussian form factor. On the other hand such a contribution should be weighted strong enough to overcome the suppression for small momenta shown in Eq. (13). In this section we try to construct an odd source geometry, which has this properties.

Inspired by hydrodynamical models we assume a source which predominantly emits particles from a thin surface layer. Such a source emission function can be described as

$$S_A(x) \sim \delta(R(\tau) - r_\perp),$$

where $r_\perp$ is the transverse radius and $\tau$ is the proper time. The source is cylindrically symmetric with a
radius \( R(\tau) = R_0 \left(1 - (\tau/\tau_f)^\alpha\right) \). The transverse radius starts out at \( R_0 \) and the source disintegrates at time \( \tau_f \).

It is straightforward to evaluate the phase such an extreme geometry produces. We find

\[
\tan(\phi_\alpha) = \frac{a \cos a - \sin a + \sum_{n=0}^{\infty} \frac{(-1)^n a^{2n+3}}{(2n+1)! (2n+\alpha+3)}}{\cos a + a \sin a - 1 - \sum_{n=0}^{\infty} \frac{(-1)^n a^{2n+2}}{(2n)! (2n+\alpha+2)}}, \tag{24}
\]

if we assume, that the outward component of the momentum is dominant, i.e. if we neglect the other two components. The only remaining variable is \( a = \beta_{\perp} q_{ij,\alpha} \tau_f \), where \( \beta_{\perp} \) is the transverse velocity of the pion pair. For \( \alpha = 1 \) we find \( \phi_1 = -a/2 \) and in the limit of \( a \to 0 \) we obtain

\[
\phi_\alpha = -\frac{2}{3} a \frac{2 + \alpha}{3 + \alpha} + \mathcal{O}(a^3). \tag{25}
\]

The term linear in \( a \) vanishes once the cosine of the sum of the three phases is evaluated due to the triangle constraint, Eq. (4). We plot in figure 2 the difference \( \Delta \phi \) between the phase given in Eq. (24) and the linear term of Eq. (25). This difference is a direct measure for the strength of the cubic correction in Eq. (13).

![FIG. 2](image)

FIG. 2. The difference \( \Delta \phi \) between the phase \( \phi_\alpha \) and its limit for small \( q_0 \) in the case of surface emission.

The contribution for \( \alpha = 1 \) to the phase difference vanishes identically. A surface, moving with constant speed does not produce any phase. In this case the product \( \tau R(\tau) \propto \tau (\tau_f - \tau) \) is symmetric around \( \tau_f/2 \). The additional factor of \( \tau \) in the product comes from the integration measure suitable for the Bjorken scenario. For accelerated surfaces, where \( \alpha \neq 1 \) we find a sizable decrease at values \( \beta_{\perp} \tau_f q_0 \gg 5 \). For typical
values of the transverse momentum $\beta_\perp \sim 0.7$ and typical time scales $\tau_f$ of a few Fermi this corresponds to outward momenta in the GeV region. We have to rule out such a surface process.

D. Bursts

Another possibility to geometrically obtain odd modification functions to the source emission is asymmetric bursts

$$S_{V:2}(x) \sim \frac{1}{2} [(1 - \epsilon) \delta(t - t_c) + (1 + \epsilon) \delta(t - t_c - \Delta t)] S_s(r). \tag{26}$$

This source corresponds to two bursts of particles emitted at times $t_c$ and $t_c + \Delta t$ from a spatial emission source function, $S_s(r)$; the latter is arbitrary and irrelevant as long as it is symmetric. The two contributions are weighted by $\epsilon$ such that $|\epsilon| \leq 1$.

The phase for such a source is from Eq. (8)

$$\tan (\phi - q_4(t_c + \Delta t/2)) = \epsilon \tan (q_4\Delta t/2), \tag{27}$$

where $q_4 = E_i - E_j \simeq \beta_\perp q_0$. The contribution proportional to the mean time $q_4(t_c + \Delta t/2)$ on the right hand side cancels when the three phases are added due to the triangle constraint (4). Thus the phase effectively vanishes for equal weights $\epsilon = 0$, if one weight vanishes $\epsilon = \pm 1$ or for zero time separation of the two flashes. This is simply because the source is symmetric in all these cases. The scale of variation of the phase is given by the $\Delta t$ and the size of the phase is given by the asymmetry parameter $\epsilon$. 

![Graph showing the phase variation with $\beta_\perp$ and $\Delta t/2$ for different values of $\epsilon$.]
FIG. 3. The phase contribution in the case of volume emission from two instantaneous sources separated by a time interval $\Delta t$. The weight $\epsilon$ is set to .25, .5 and .75.

In figure 3 we plot the cosine of the three phases according to Eq. (22) for different values of $\epsilon$, assuming again that the outward component of the relative momentum is dominant. The phase scales in the variable $\beta_{\perp} Q_{3,o} \Delta t$. For outward momenta close to $\beta_{\perp} Q_{3,o} \Delta t/2 \sim 2$ one of the phases in Eq. (27) changes sign and we see a strong signal. In the experimentally accessible momentum range this would correspond to a temporal separation of the bursts in the order of 10 fm/c. While such a scenario is rather unrealistic it might open in the long run some new ideas and approaches to investigating possible phase signals.

IV. CONCLUSION

We have studied 3-body correlations for incoherent sources which can be calculated from 2-body correlations except for a phase. The phase is due to odd space-time moments of the source emission function and vanish at small momentum transfer to order $(qR)^3$ and $q^2 R/K$, where the former dominates in the experimentally relevant momentum regime in relativistic heavy ion collisions.

Effects of flow, resonances, moving and bursting sources were studied as they are known to be present in relativistic heavy ion collisions, and result in asymmetric sources. Their influence on the phases $\phi_{ij}$ was only significant at large relative momenta where the form factors, $F(q)$, and thus also correlation functions were small or in some extreme scenarios, where all particles were emitted in 2 sudden bursts. In both cases, the experimental extraction of the phases needs very high resolutions. The phase in 3-body correlation functions seems to be very elusive.

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FIGURE CAPTIONS

Figure 1. The phase contribution for different resonances as a function of the momentum $Q_{3,o}^2 = q_{12,o}^2 + q_{13,o}^2 + q_{23,o}^2$. The fall-off of the curves scales with the resonance life time. The transverse momentum of the pion pair is chosen $\beta_{\perp} = 0.7$.

Figure 2. The difference $\Delta \phi$ between the phase $\phi_{\alpha}$ and its limit for small $q_o$ in the case of surface emission.

Figure 3. The phase contribution in the case of volume emission from two instantaneous sources separated by a time interval $\Delta$. The weight $\epsilon$ is set to .25, .5 and .75.