Superstrings with new supersymmetry in (9,2) and (10,2) dimensions

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We construct superstring theories that obey the new supersymmetry algebra $\{Q_\alpha, Q_\beta\} = \gamma_{\alpha\beta}^{\mu\nu} P_{1\mu} P_{2\nu}$ in a Green-Schwarz formalism, with kappa supersymmetry also of the new type. The superstring is in a system with a superparticle so that their total momenta are $P_{2\mu}, P_{1\nu}$, respectively. The system is covariant and critical in (10,2) dimensions if the particle is massless and in (9,2) dimensions if the particle is massive. Both the superstring and superparticle have coordinates with two timelike dimensions but each behaves effectively as if they have a single timelike dimension. This is due to gauge symmetries and associated constraints. We show how to generalize the gauge principle to more intricate systems containing two parts, 1 and 2. Each part contains interacting constituents, such as p -branes, and each part behaves effectively as if they have one timelike coordinate, although the full system has two timelike coordinates. The examples of two superparticles, and of a superparticle and a superstring, discussed in more detail are special cases of such a generalized interacting system. [S0556-2821(97)07822-3]

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I. INTRODUCTION

The duality properties of superstring and super p -brane theories have led to hints that there may be hidden timelike dimensions in the underlying theory [1–8]. Unifying dualities, supersymmetry, and Lorentz invariance suggests a structure in 14 dimensions with signature (11,3) including three timelike dimensions [9–12]. For simplicity one may concentrate on two timelike dimensions since the conceptual jump from one to two timelike dimensions can be easily extended to three. At issue is how to make sense of the extra timelike dimensions?

A main feature of this setting is also that the supersymmetry deviates substantially from the standard one, while its structure is intimately related to the answers we seek. By constructing explicit models with the new supersymmetry, and with two timelike dimensions, progress was made in relating these concepts to the familiar one-time world with standard supersymmetry.

In the present paper more progress is reported by constructing superparticle and superstring models in $(d-2,2)$ dimensions. In the first part of the paper we will discuss a general setup for discussing the two time issues for two groups of particles or p -branes, with interactions within each group. In the second and main part of the paper we will construct actions for the individual superparticles or superstrings as a member of either group.

In recent papers it has been shown that it is possible to formulate an action principle for particles [13], extended objects [14], and field theories [15], with two timelike dimensions, without the traditional problems. The main new ingredient is a gauge symmetry that permits the removal of one timelike dimension for each particle (or point in an extended object), while maintaining covariance in the higher space including the extra timelike dimension.

The timelike dimension that is removed is not the same one for the entire system. In the simplest case there are two particles, labeled No. 1 and No. 2, with timelike

(or lightlike) momenta p_1^μ, p_2^μ , respectively, where $\mu=0', 0, 1, 2, \dots, (d-2)$, is an index in $(d-2,2)$ dimensions. The position coordinates are x_1^μ, x_2^μ . Each coordinate has two timelike dimensions labeled by $\mu=0', 0$. The gauge invariance gives the constraint $p_1 \cdot p_2 = 0$, and removes the timelike dimensions $p_1 \cdot x_2$ and $p_2 \cdot x_1$ in a way that does not break the $SO(d-2,2)$ symmetry of the total system. Then particle No. 1 moves effectively with one timelike dimension, and similarly particle No. 2 moves effectively with another orthogonal timelike dimension.

The definition of canonical variables, and quantization, is straightforward because the formulation involves a single worldline parameter τ that appears in $x_1^\mu(\tau), x_2^\mu(\tau)$. One should not be confused about trying to introduce two Hamiltonians. It was shown in [13] that $SO(d-2,2)$ covariant quantization is carried out by simply applying the constraints on states.

It was suggested in [13], that a cosmological scenario is possible such that, starting at the Big Bang, the particles of type No. 2 remain in compactified dimensions, including one timelike dimension, while particles of type No. 1 move in four dimensions in the observed expanding universe described by an effective theory with a single timelike dimension.

So far the two-times mechanism has been discussed in simple examples without interactions. In Sec. II we expand the scope of this approach by showing that it also applies to two systems, called No. 1 and No. 2, each containing any number of interacting particles or p -branes, with two timelike dimensions. The *total* momenta of each system are $P_{1\mu}, P_{2\mu}$, respectively. We show that these replace the momenta p_1^μ, p_2^μ of individual free particles or p -branes in the mechanism that removes the extra timelike dimensions. Furthermore, previous discussions had given the unintended impression that the approach required a new timelike dimension for each particle or p -brane in the total system. In the present version, it becomes evident that we can discuss many

particles or p -branes with two timelike dimensions, without needing to add a new timelike dimension for each new particle or p -brane.

In Secs. III–V we will discuss superparticle and superstring actions that are supersymmetric under the “new superalgebra”

$$\{Q_\alpha, Q_\beta\} = \gamma_{\alpha\beta}^{\mu\nu} P_{1\mu} P_{2\nu}, \quad P_1 \cdot P_2 = 0. \quad (1)$$

This is the simplest form for a superalgebra in 12 dimensions (12D), but it could also be realized in lower dimensions.¹ There is also a new local kappa supersymmetry, in a Green-Schwarz formulation, that parallels the global supersymmetry of this type. We will show that there are *classical* superstring models in $(d-2,2)$ dimensions with type IIA, type IIB, or heterotic supersymmetry, that provide realizations of the new supersymmetry.

The combined superparticle plus superstring system has $SO(d-2,2)$ invariance² and a number of gauge invariances. After fixing a subset of gauges and using a subset of equations of motion, *if the particle is frozen* to a fixed momentum p_1^μ , which is either timelike or lightlike depending on the dimension, then it can be shown that the string reduces to the standard *classical* supersymmetric string in 3,4,6,10 dimensions. If the particle is not frozen, the system is more general and maintains the $SO(d-2,2)$ invariance.

In the *quantum* theory of the combined superparticle and superstring system, the critical dimension is (10,2) when the superparticle is massless, and (9,2) when the superparticle is massive (assuming there are no spectator degrees of freedom).

¹The dimensions d in which it can be realized are determined by whether $\gamma_{\alpha\beta}^{\mu\nu}$ is symmetric in $(\alpha\beta)$. These are $d = (3,4,5) \bmod(8)$ assuming a single Weyl spinor Q_α in the even $4 \bmod(8)$ dimensions. If the Dirac spinor is taken in even dimensions (i.e., type IIA) then one can also have $d = 6,10 \bmod(8)$. For $SO(d-2,2)$ the spinors above are real in these dimensions. Finally, if there is more than one Weyl spinor (i.e., $Q_{\alpha a}$ $a=1,2,\dots$) then $\gamma_{\alpha\beta}^{\mu\nu}$ can be antisymmetric in $(\alpha\beta)$. This allows $d=7,8,9 \bmod(8)$. The Weyl spinors are complex in these dimensions, so their complex conjugates must also be added. We will discuss only the case of symmetric $\gamma_{\alpha\beta}^{\mu\nu}$ in this paper. The antisymmetric case can be discussed with similar methods. See footnote 5 for a choice of a basis for the gamma matrices.

²The superalgebra (1) first emerged in a Bogomol’nyi-Prasad-Sommerfield (BPS) sector of S theory [4], where it was suggested as a basis for a (10,2) covariant supergravity, with 128 bosonic and 128 fermionic bilocal fields $\phi(x_1^\mu, x_2^\mu)$. It was later realized in 12D super Yang-Mills [6], 12D supergravity [7], and 12D (2,1) heterotic strings [3], but only for a frozen constant lightlike momentum P_2^μ which breaks the (10,2) covariance. These realizations can be interpreted to be a Kaluza-Klein reduction in the coordinate x_2^μ of a bilocal theory. In contrast, an operator version of both P_1^μ, P_2^μ in the form of derivatives was realized in bilocal supersymmetric field theory [15], but in a compactified version such that P_1^μ, P_2^μ point in orthogonal directions. In the present paper both P_1^μ, P_2^μ are quantum operators, and the full $SO(d-2,2)$ covariance is maintained.

II. TWO SYSTEMS AND TWO TIMES

In [14] the general scheme for constructing the action with two or more timelike dimensions was given. The discussion involved two or more particles (or p -branes) such that each additional particle (or p -brane) introduced an additional timelike dimension. In this section we extend this construction to any number of interacting particles (or p -branes) in system No. 1 plus any number of interacting particles (or p' -branes) in system No. 2, but without needing to introduce additional timelike dimensions beyond two. It is straightforward to add system No. 3 with a third timelike dimension, but here we will limit ourselves to only two timelike dimensions.

The generalized multiparticle prescription for the action with two timelike coordinates is similar to the previous discussion

$$S = S_1(\lambda_2) + S_2(\lambda_1) + \lambda_1 \cdot \lambda_2. \quad (2)$$

However, now $S_1(\lambda_2)$ describes many particles or p -branes in system No. 1, moving in a background provided by system No. 2, where the background is represented by the moduli λ_2^μ . This is a constant vector constructed from canonical variables of system No. 2, and is independent of world sheet or world line parameters. Similarly, $S_2(\lambda_1)$ describes particles or p' -branes in system No. 2, moving in a background provided by system No. 1, represented by the moduli λ_1^μ . The constituents of system No. 1, labeled by $i=1,2,\dots,N$, may be different “beasts” than the constituents of system No. 2 labeled by $j=1,2,\dots,M$, and each system may include any set of interacting p -branes.

The vectors $\lambda_1^\mu, \lambda_2^\mu$ are directions in which timelike coordinates will be removed by a gauge principle that is analogous to the gauged WZW mechanism discussed in [13,14]. This implies covariant derivatives on the branes $x_{1i}^\mu(\tau, \vec{\sigma}), x_{2j}^\mu(\tau, \vec{\sigma}')$:

$$\begin{aligned} D_\alpha x_{1i}^\mu &= \partial_\alpha x_{1i}^\mu - \lambda_2^\mu A_{1\alpha i}, \\ D_\alpha x_{2j}^\mu &= \partial_\alpha x_{2j}^\mu - \lambda_1^\mu A_{2\alpha j}, \end{aligned} \quad (3)$$

where $\partial_\alpha, A_{1\alpha}, A_{2\alpha}$ are one-forms on the brane world volumes. The gauge invariance for system No. 1 is

$$\delta x_{1i}^\mu = \lambda_2^\mu \Lambda_{1i}, \quad \delta A_{1\alpha i} = \partial_\alpha \Lambda_{1i}, \quad (4)$$

and similarly for system No. 2. Note that there is a single λ_2^μ independent of i , and a single λ_1^μ independent of j .

There may be unspecified interactions within each system independently of the other, but the only interaction considered so far between system No. 1 and system No. 2 is the term $\lambda_1 \cdot \lambda_2$. The moduli $\lambda_1^\mu, \lambda_2^\mu$ (which are independent of $\tau, \vec{\sigma}$) are integrated in the path integral, like other dynamical variables. This implies that the classical action is minimized with respect to the moduli, resulting in the equations of motion for $\lambda_1^\mu, \lambda_2^\mu$:

$$\lambda_1^\mu = - \frac{\partial S_1(\lambda_2)}{\partial \lambda_{2\mu}}, \quad \lambda_2^\mu = - \frac{\partial S_2(\lambda_1)}{\partial \lambda_{1\mu}}. \quad (5)$$

It will now be shown that λ_1^μ (λ_2^μ) is proportional to the *total* momentum P_1^μ (P_2^μ) of system No. 1 (No. 2), for any interactions among the constituents within each system, and that

$$P_1 \cdot P_2 = 0, \quad (6)$$

as in the single constituent cases discussed in [13,14]. To do this use the chain rule³

$$-\frac{\partial S_1(\lambda_2)}{\partial \lambda_{2\mu}} = \sum_i \int d\tau d\vec{\sigma} \frac{\partial S_1(\lambda_2)}{\partial (D_\alpha x_{1i}^\mu)} A_{1i\alpha}(\tau, \vec{\sigma}). \quad (7)$$

Then use the equation of motion for $A_{1i\alpha}(\tau, \vec{\sigma})$,

$$\lambda_2^\mu \frac{\partial S_1(\lambda_2)}{\partial (D_\alpha x_{1i}^\mu)} = 0 \rightarrow \lambda_2 \cdot D^\alpha x_{1i} = 0, \quad (8)$$

and combine it with Eqs. (5) and (7) to show

$$\lambda_1 \cdot \lambda_2 = 0. \quad (9)$$

From Eqs. (3) and (8) one obtains

$$\lambda_2^\mu A_{1i\alpha}(\tau, \vec{\sigma}) = \lambda_2 \cdot \partial_\alpha x_{1i}. \quad (10)$$

Using the gauge freedom $\Lambda_{1i}(\tau, \vec{\sigma})$ one may choose the gauge

$$\lambda_2 \cdot x_{1i}(\tau, \vec{\sigma}) = a_1 \lambda_2^\mu \tau, \quad (11)$$

where a_1 is a constant independent of the particle i . In this gauge $A_{1i\alpha}(\tau, \vec{\sigma})$ simplifies

$$A_{1i\alpha}(\tau, \vec{\sigma}) = a_1 \partial_\alpha \tau = a_1 \delta_{\alpha\tau}. \quad (12)$$

Inserting this form in Eq. (7) shows that only the $\alpha = \tau$ term survives and λ_1^μ becomes⁴

$$\begin{aligned} \lambda_1^\mu &= a_1 \sum_i \int d\tau d\vec{\sigma} p_{1i}^\mu(\tau, \vec{\sigma}) = a_1 \int d\tau \sum_i p_{1i}^\mu(\tau) \\ &= a_1 P_1^\mu \int_0^T d\tau = a_1 T P_1^\mu, \end{aligned} \quad (13)$$

where the definitions of canonical momenta $p_{1i}^\mu(\tau, \vec{\sigma}) = \partial S_1 / \partial (D_\alpha x_{1i}^\mu)$, center of mass momentum $p_{1i}^\mu(\tau) = \int d\vec{\sigma} p_{1i}^\mu(\tau, \vec{\sigma})$, and total conserved momentum $P_1^\mu = \sum_i p_{1i}^\mu(\tau)$ have been used. Note that even if there are interactions within system No. 1, the total P_1^μ is conserved due to translation invariance, although the center of mass momenta of the various p -branes $p_{1i}^\mu(\tau)$ may be time depen-

dent. Thus λ_1^μ is proportional to the total P_1^μ , and similarly λ_2^μ is proportional to P_2^μ . These are orthogonal to each other $P_1 \cdot P_2 = 0$ because of Eq. (9), as promised above.

There is an analogue of the positive energy condition for the combined system. Note that the sign of

$$P_1^0 P_2^{0'} - P_2^0 P_1^{0'} \quad (14)$$

cannot change under $SO(d-2,2)$ transformations. Combined with the condition $P_1 \cdot P_2 = 0$, this implies that the signs of $P_1^0, P_2^{0'}$ are correlated with each other in definite representations of supersymmetry and $SO(d-2,2)$. Therefore, if $\lambda_1^0 \lambda_2^{0'} - \lambda_2^0 \lambda_1^{0'}$ is taken positive in the original action (2) then the combined system is considered to be in the ‘‘particle-particle sector’’ (as opposed to particle-antiparticle sector) as explained in [13].

This result extends the scope of the approach in [14] to interacting systems involving any number of constituents within each system No. 1, No. 2. The essential points made in [13] about the meaning of two timelike dimensions, and the cosmological scenario, remain the same as in the simple two particle case discussed there.

It is straightforward to extend this discussion to three systems, No. 1, No. 2, No. 3, with three timelike dimensions, as in [14], by allowing many interacting constituents in each system. More than three timelike dimensions are *a priori* possible, but are not needed in our program since minimal unification of supersymmetry and duality is possible in 14D with signature (11,3) [9]. Furthermore, more than three timelike dimensions seem to be forbidden in some supersymmetric dynamics [10].

III. SUPERPARTICLES IN $(D-2,2)$ DIMENSIONS

We are interested in constructing models with the new supersymmetry of Eq. (1) in various dimensions. Massless superparticles in more than eleven dimensions have been discussed recently in a formalism involving multiple world line parameters τ_i [10]. To avoid inconsistencies in the transformation laws with multiple τ_i , and also be consistent with our generalized scheme (2), we present the following formulation involving a single world line parameter τ . Although there is considerable overlap with Eq. (2) for the single particle in a background of the other, there are differences in the supersymmetry transformation rules. Furthermore, we also discuss massless as well as massive superparticles, superparticles in lower dimensions, and superparticles with multiple supersymmetries.

A. Massless superparticle in $(d-2,2)$

Consider a massless superparticle No. 1 in the background of system No. 2. The background consists of λ_2^μ that can be shown to be proportional to the center of mass momentum of system No. 2, as in the previous section, and below. The τ reparametrization invariant action is $S_1 = \int d\tau L_1(\tau)$, with

$$L_1(\tau) = \frac{1}{2} e_1^{-1} \pi_1^\mu \pi_1^\nu \eta_{\mu\nu}, \quad (15)$$

³Here we assume that the only $\lambda_{1,2}^\mu$ dependence in $S_{1,2}(\lambda_{2,1})$ occurs through the covariant derivatives in the purely bosonic theory. This is not the case in the supersymmetric theory, as seen in the next section. However, the result is still the same as it will be shown there [see discussion following Eq. (24)].

⁴A gauge independent way of proving the same result uses the equations of motion in any gauge, and combines it with the periodicity of the closed p -branes in the world sheet variables $\vec{\sigma}$.

where

$$\pi_1^\mu = \dot{x}_1^\mu - \lambda_2^\mu A_1 + \bar{\theta}_1 \gamma^{\mu\nu} \dot{\theta}_1 \lambda_{2\nu}. \quad (16)$$

The signature is $(d-2, 2)$, $\eta_{\mu\nu} = (-, -, +, \dots, +, +)$, and labeled by the index $\mu = 0', 0, 1, \dots, d-2$, on $x_1^\mu(\tau)$. The spinor representation for $SO(d-2, 2)$ is labeled by the index α on the fermion $\theta_1^\alpha(\tau)$. We consider the dimensions d and the spinor representations in which $\gamma_{\alpha\beta}^{\mu\nu}$ is symmetric in $(\alpha\beta)$, since this is needed for the new supersymmetry (1). The following discussion is valid for all dimensions d that satisfy these criteria.⁵ The main interest in this paper is in (10,2) and (9,2) dimensions since these will turn out to be the critical dimensions for the string (but not for the particle by itself). Note in particular that for (10,2) the Majorana-Weyl spinor with 32 real components satisfies this property. The same real spinor is also the Dirac spinor in (9,2) dimensions.

The global supersymmetry transformations that are consistent with the new superalgebra (1) have the form

$$\delta_{\varepsilon_1} \theta_1 = \varepsilon_1, \quad \delta_{\varepsilon_1} x_1^\mu = -\bar{\varepsilon} \gamma^{\mu\nu} \theta_1 \lambda_{2\nu}, \quad \delta_{\varepsilon_1} A_1 = 0. \quad (17)$$

Under these

$$\delta_{\varepsilon_1} \pi_1^\mu = 0, \quad \delta_{\varepsilon_1} e_1 = 0, \quad \delta_{\varepsilon_1} L_1 = 0. \quad (18)$$

There is also local kappa supersymmetry given by

$$\delta_{\kappa_1} \theta_1 = \gamma^{\mu\nu} \kappa_1 \pi_{1\mu} \lambda_{2\nu}, \quad \delta_{\kappa_1} x_1^\mu = \delta_{\kappa_1} \bar{\theta}_1 \gamma^{\mu\nu} \theta_1 \lambda_{2\nu} \\ \delta_{\kappa_1} A_1 = 2\lambda_2 \cdot \pi_1 \bar{\kappa}_1 \dot{\theta}_1, \quad \delta_{\kappa_1} e_1^{-1} = 4\lambda_2^2 \bar{\kappa}_1 \dot{\theta}_1 e_1^{-1}. \quad (19)$$

Under the local $\kappa_1(\tau)$ supersymmetry the Lagrangian is invariant.

Note that we have used $\delta_{\varepsilon_1} \lambda_2^\mu = \delta_{\kappa_1} \lambda_2^\mu = 0$ in proving that $S_1(\lambda_2)$ is supersymmetric. Similarly, in verifying the supersymmetry of system No. 2, $S_2(\lambda_1)$, we need to use $\delta_{\varepsilon_1} \lambda_1^\mu = \delta_{\kappa_1} \lambda_1^\mu = 0$. Since we expect that $\lambda_1 \sim p_1^\mu$ after using the equations of motion, we need to check if $\delta_{\varepsilon_1} p_1^\mu = \delta_{\kappa_1} p_1^\mu = 0$ on shell, for consistency. This will be verified following Eq. (25).

⁵As in footnote 1, in $d = (3, 4, 5) \bmod(8)$ dimensions, with $\mu = 0, 1, 2, \dots, 0'$, the real (Weyl) spinor has dimension $2[(d-1)/2]$, and the real gamma matrices are given by $\gamma^\mu = (\gamma^m, \gamma^{0'})$, where the last one is proportional to the identity $(\gamma^{0'})^\alpha_\beta = \delta^\alpha_\beta$, while the other γ^m are real and traceless. For $d = 6, 10 \bmod(8)$ the real Dirac spinor has dimension $2^{d/2}$ and the real gamma matrices in these dimensions can again be taken as above, with $\gamma^{0'} = 1$. Then $\gamma^{\mu\nu} = (\gamma^{mn}, \gamma^{m0'})$ are defined by $\gamma^{m0'} = \gamma^m$ and $\gamma^{mn} = \frac{1}{2}[\gamma^m, \gamma^n]$. These form the algebra of $SO(d-2, 2)$. Explicitly, for $d = 3$: $(\gamma^\mu)^\alpha_\beta = (i\sigma_2, \sigma_3, 1)$; for $d = 4$: $(\gamma^\mu)^\alpha_\beta = (i\sigma_2, \sigma_1, \sigma_3, 1)$; for $d = 5$: $(\gamma^\mu)^\alpha_\beta = (i\sigma_2 \otimes \sigma_1, \sigma_1 \otimes \sigma_1, \sigma_3 \otimes \sigma_1, 1_2 \otimes 1_2)$. When multiplied by the antisymmetric charge conjugation matrix $C = \gamma^0$ to lower the indices, $C\gamma^m$ and $C\gamma^{mn}$ are symmetric. These symmetric $\gamma_{\alpha\beta}^{\mu\nu}$ are the ones that appear in the superalgebra, as well as in expressions such as $\bar{\varepsilon} \gamma^{\mu\nu} \theta$, etc. The structure is similar for the other dimensions.

The momentum is

$$p_1^\mu = e_1^{-1} \pi_1^\mu. \quad (20)$$

In the first order formalism the action may be rewritten as

$$S_1 = \int d\tau \left(p_1 \cdot \dot{x}_1 - p_1 \cdot \lambda_2 A_1 + \bar{\theta}_1 \gamma^{\mu\nu} \dot{\theta}_1 p_{1\mu} \lambda_{2\nu} - \frac{1}{2} e_1 p_1^2 \right). \quad (21)$$

The supersymmetry of this form may be shown by deriving the transformation laws for the momentum from its definition. We find that $\delta_{\varepsilon_1} p_1^\mu = 0$, but $\delta_{\kappa_1} p_1^\mu$ is nonzero off shell (unlike [10])

$$\delta_{\kappa_1} p_1^\mu = 2 \bar{\kappa}_1 \left(\lambda_2^2 \gamma^\mu \gamma \cdot p_1 - \gamma \cdot \lambda_2 \gamma \cdot p_1 \lambda_2^\mu + \lambda_2 \cdot p_1 \gamma \cdot \lambda_2 \gamma^\mu \right) \dot{\theta}_1. \quad (22)$$

The constraints and equations of motion that follow from Eq. (21) or Eq. (15) are

$$p_1^2 = 0, \quad \lambda_2 \cdot p_1 = 0,$$

$$\dot{p}_1 = 0, \quad (\gamma^{\mu\nu} \dot{\theta}_1) p_{1\mu} \lambda_{2\nu} = 0, \quad (23)$$

and the equation of motion for λ_2 in Eq. (2) gives

$$\lambda_1^\mu = \int d\tau (A_1 p_1^\mu + \bar{\theta}_1 \gamma^{\mu\nu} \dot{\theta}_1 p_{1\nu}). \quad (24)$$

The first term is proportional to p_1^μ , which is consistent with $\lambda_1^\mu \sim p_1^\mu$ as desired, but the second term is not obviously so (see footnote 3). The general solution of the equation of motion for θ_1 is

$$\dot{\theta}_1 = (\gamma^{\mu\nu} \psi) p_{1\mu} \lambda_{2\nu}, \quad (25)$$

where $\psi(\tau)$ is any spinor. Inserting this in Eq. (24) and using the constraints, it follows that the second term in Eq. (24) is also proportional to p_1^μ , for any $\psi(\tau)$. Since $p_{1\mu}$ is time independent it can be pulled out of the integral to show once again that $\lambda_1^\mu \sim p_1^\mu$, on shell, as expected.

Now we can verify if $\delta_{\kappa_1} \lambda_1^\mu = 0$ is consistent with $\lambda_1^\mu \sim p_1^\mu$, on shell. In the off shell expression $\delta_{\kappa_1} p_1^\mu$ in Eq. (22) we insert the general form (25) and use the constraints. The result is $\delta_{\kappa_1} p_1^\mu = 0$ on shell, consistent with the original assumption $\delta_{\kappa_1} \lambda_1^\mu = 0$.

The superparticles with the new supersymmetry (1), (17), and (19) are consistent both at the classical and quantum levels, in any number of dimensions d for which $\gamma_{\alpha\beta}^{\mu\nu}$ are symmetric.

Furthermore, it is possible to have any number of supersymmetries by appending an index on $\theta_{1\alpha}^A(\tau)$ and on $\varepsilon_1^A, \kappa_{1\alpha}^A(\tau)$, $A = 1, 2, \dots, N$. There are no restrictions on these generalizations, just as for standard supersymmetry.

The action (15) or (21) has one more bosonic local symmetry with parameter $\omega(\tau)$

$$\delta_\omega \theta = \omega \dot{\theta}, \quad \delta_\omega x^\mu = -\omega \bar{\theta} \gamma^{\mu\nu} \dot{\theta} \lambda_\nu,$$

$$\delta_\omega e = 0, \quad \delta_\omega A_1 = 0, \quad (26)$$

which gives $\delta_\omega \pi^\mu = 0$. However this symmetry has no additional implications for the on shell theory other than those of κ and reparametrization invariances.

B. Massive superparticle in $(d-2,2)$

In the discussion above the superparticle was massless according to the constraint $p_1^2 = 0$. It is easy to obtain an action for the massive superparticle by doing Kaluza-Klein reduction. For example if the massless superparticle is in $(10,2)$ we reduce the 11th dimension to arrive to a massive superparticle in $(9,2)$. For this we distinguish one spacelike component, which will be denoted $\mu=11$ in every dimension (instead of calling it $\mu=d-1$). Then replace the corresponding momentum with the constant mass parameter $p_1^{11} \equiv m_1$, while throwing away the corresponding component in $\lambda_2^{11} = 0$. We thus have one less spacelike dimensions as compared to the massless particle case discussed above. The first order action (21) reduces to

$$S_1 = \int d\tau \left(p_1 \cdot \dot{x}_1 - p_1 \cdot \lambda_2 A_1 + \bar{\theta}_1 \gamma^{\mu\nu} \dot{\theta}_1 p_{1\mu} \lambda_{2\nu} - \frac{1}{2} e_1 p_1^2 + \bar{\theta}_1 \gamma^{11,\nu} \dot{\theta}_1 m_1 \lambda_{2\nu} - \frac{1}{2} e_1 m_1^2 \right), \quad (27)$$

where $\mu, \nu = 0', 0, 1, \dots, d-2$. We have dropped $\dot{x}_1^{11} m_1$ since it is a total derivative. Thus the compactified coordinate x_1^{11} drops out. The supersymmetry transformations follow from the ones in Eqs. (17) and (19) by dropping the direction $\mu=11$ in x_1^μ, λ_2^μ and using $\delta m_1 = 0$. Note however that now the κ transformation has one more term proportional to the mass

$$\delta_{\kappa_1} \theta_1 = \gamma^{\mu\nu} \kappa_1 \pi_{1\mu} \lambda_{2\nu} + \gamma^{11,\nu} \kappa_1 \lambda_{2\nu} m_1 e_1. \quad (28)$$

Thus, from the massless superparticle action in $(10,2)$ we derive the massive superparticle action in $(9,2)$, and similarly for other values of d . Using the equations of motion it is seen that the superparticle is massive.

If two massive superparticles are coupled as in the usual scheme (2), and then we keep the components $\lambda_{1,2}^{11} \sim m_{1,2}$ for both particles in doing the reduction (note $\lambda_2^{11} = 0$ above), then the massive Dirac operator for the combined system becomes

$$(\gamma^\mu p_{1\mu} + \gamma^{11} m_1)(\gamma^\mu p_{2\mu} + \gamma^{11} m_2), \quad (29)$$

with the constraint

$$p_1 \cdot p_2 + m_1 m_2 = 0, \quad (30)$$

while the massive Klein Gordon operator that is obtained by squaring the Dirac operator is $(p_1^2 + m_1^2)(p_2^2 + m_2^2)$.

There are two global supersymmetries, since the total action is invariant under two separate parameters $\varepsilon_{1,2}$ as in Eq.

(17). After using $\lambda_{1,2}^\mu \sim p_{1,2}^\mu$ one finds the $N=2$ superalgebra by applying Eq. (17) twice, i.e., $[\delta_{\varepsilon_i}, \delta_{\varepsilon_j}] x_{1,2}^\mu$, for $i, j = 1, 2$ gives

$$\{Q_{i\alpha}, Q_{j\beta}\} = \delta_{ij} \gamma_{\alpha\beta}^{\mu\nu} p_{1\mu} p_{2\nu}. \quad (31)$$

When particle No. 2 is frozen, and rotated to a standard frame (massless or massive), the superalgebra of particle No. 1 reduces to the standard $N=1$ supersymmetry. When particle No. 2 is allowed to perform all allowed motions the combined system has two supersymmetries. Similar comments apply to the superparticle and superstring system discussed in the following sections.

In constructing field theories that describe the combined system No. 1 and No. 2 (see, e.g., [15]) these structures need to be taken into account.

IV. SUPERSTRINGS IN $(9,2)$ DIMENSIONS

We will discuss the system of one superparticle and one superstring as an example of the more general scheme (2). Since the superparticle is discussed in the previous section we will concentrate on the superstring action. To keep the notation simple we will omit the index No. 2 on the string supercoordinates $x_2^\mu(\tau, \sigma), \theta_{2\alpha}(\tau, \sigma)$, and the index No. 1 on λ_1^μ . We first discuss the string in the background of a *massive* superparticle, thus $\lambda^\mu \sim p_1^\mu$, and therefore λ^2 is timelike. The following discussion applies to $d=4, 5, 11$ with signature $(d-2, 2)$, but we are mainly interested in $(9, 2)$. The space-time index takes the values $\mu=0, 1, \dots, d-2, 0'$. The dimension of the spinor is $2[(d-1)/2]$ and the gamma matrices γ^μ live in this space as described in footnote 5. There is an additional gamma matrix $\gamma^{(d-1)}$ which will be denoted γ^{11} in every dimension for convenience of notation, as in the previous section on massive superparticles. Note that γ^{11} is a scalar under $SO(d-2, 2)$. Furthermore, while γ^{11} is traceless for the cases $d=5, 11$, it is equal to 1 for the case $d=4$ since the spinor is already Weyl projected.

The action we propose is

$$S_2(\lambda) = \frac{1}{2} \int d^2\sigma \sqrt{-g} g^{ij} \pi_i^\mu \pi_j^\nu \eta_{\mu\nu} - \int d^2\sigma \varepsilon^{ij} \partial_i x^\mu \bar{\theta} \gamma_{\mu\nu} \gamma^{11} \partial_j \theta \lambda^\nu + \frac{\sqrt{-\lambda^2}}{2} \int d^2\sigma \varepsilon^{ij} \bar{\theta} \gamma^\mu \gamma^{11} \partial_i \theta \bar{\gamma}_{\mu\nu} \partial_j \theta \lambda^\nu, \quad (32)$$

where

$$\pi_i^\mu = \partial_i x^\mu - \lambda^\mu A_i + \bar{\theta} \gamma^{\mu\nu} \partial_i \theta \lambda_\nu. \quad (33)$$

Evidently there is reparametrization invariance, and the following gauge invariance:

$$\delta x^\mu = \lambda^\mu \Lambda_2(\tau, \sigma), \quad \delta A_i = \partial_i \Lambda_2(\tau, \sigma). \quad (34)$$

Under the global supersymmetry transformations with the new superalgebra (1) one has

$$\delta_\varepsilon \theta = \varepsilon, \quad \delta_\varepsilon x^\mu = -\bar{\varepsilon} \gamma^{\mu\nu} \theta \lambda_\nu, \quad \delta_\varepsilon A_i = 0, \quad \delta_\varepsilon g_{ij} = 0. \quad (35)$$

This gives

$$\delta_\varepsilon \pi_i^\mu = 0, \quad (36)$$

showing that first term of S_2 is supersymmetric. The remainder of $S_2(\lambda)$ gives

$$\begin{aligned} \delta_\varepsilon S_2 = & \int d^2 \sigma \varepsilon^{ij} (\bar{\varepsilon} \gamma_{\mu\rho} \partial_i \theta \bar{\theta} \gamma^{\mu\nu} \gamma^{11} \partial_j \theta) \lambda_\nu \lambda^\rho \\ & + \frac{\sqrt{-\lambda^2}}{2} \int d^2 \sigma \varepsilon^{ij} \left(\begin{aligned} & \bar{\varepsilon} \gamma^\mu \gamma^{11} \partial_i \theta \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \\ & - \bar{\varepsilon} \gamma_{\mu\nu} \partial_i \theta \bar{\theta} \gamma^\mu \gamma^{11} \partial_j \theta \end{aligned} \right) \lambda^\nu. \end{aligned} \quad (37)$$

The integrand of the first term can be rewritten as

$$\begin{aligned} & \frac{1}{3} \varepsilon^{ij} \left(\begin{aligned} & 2 \bar{\varepsilon} \gamma_{\mu\rho} \partial_i \theta \bar{\theta} \gamma^{\mu\nu} \gamma^{11} \partial_j \theta \\ & - \bar{\varepsilon} \gamma_{\mu\rho} \partial_i \theta \bar{\theta} \gamma^{\mu\nu} \gamma^{11} \partial_j \theta \end{aligned} \right) \lambda_\nu \lambda^\rho \\ & + \frac{1}{3} \varepsilon^{ij} \partial_i (\bar{\varepsilon} \gamma_{\mu\rho} \theta \bar{\theta} \gamma^{\mu\nu} \gamma^{11} \partial_j \theta) \lambda_\nu \lambda^\rho. \end{aligned} \quad (38)$$

Since the last term is a total derivative it is dropped in $\delta_\varepsilon S_2$. The second term in $\delta_\varepsilon S_2$ can be written similarly by dropping total derivatives. Then one gets the integrand for the second term

$$\frac{\sqrt{-\lambda^2} \varepsilon^{ij}}{6} \left(\begin{aligned} & 2 \bar{\varepsilon} \gamma^\mu \gamma^{11} \partial_i \theta \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \\ & - \bar{\varepsilon} \gamma^\mu \gamma^{11} \partial_i \theta \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \\ & - 2 \bar{\varepsilon} \gamma_{\mu\nu} \partial_i \theta \bar{\theta} \gamma^\mu \gamma^{11} \partial_j \theta \\ & + \bar{\varepsilon} \gamma_{\mu\nu} \partial_i \theta \bar{\theta} \gamma^\mu \gamma^{11} \partial_j \theta \end{aligned} \right) \lambda^\nu. \quad (39)$$

One can use (2,2), (3,2), and (9,2) gamma matrix identities to show that the two integrands combined gives zero for any timelike λ^μ . To prove it we take advantage of the $SO(d-2,2)$ symmetry of the combined string and particle system (2), and choose a Lorentz frame for the particle by taking $\lambda^\mu = (0, \vec{0}, \lambda^{0'})$. The little group is $SO(d-2,1)$. It is useful to define chiral spinors of the little group in the remaining (2,1), (3,1), or (9,1) dimensions by $\theta_\pm = \frac{1}{2}(1 \pm \gamma^{11})\theta$ [note that for (2,1) case $\theta_- = 0$ since $\gamma^{11} = 1$ for $d=4$ in the Weyl sector, as explained above]. In this frame the total integrand becomes

$$\frac{1}{3} (\lambda^{0'})^2 \varepsilon^{ij} \sum_{\pm} \left(\begin{aligned} & 2 \bar{\varepsilon}_\pm \gamma^m \partial_i \theta_\pm \bar{\theta}_\pm \gamma_m \partial_j \theta_\pm \\ & - \bar{\varepsilon}_\pm \gamma^m \partial_i \theta_\pm \bar{\theta}_\pm \gamma_m \partial_j \theta_\pm \end{aligned} \right), \quad (40)$$

where m is an index in the remaining $(d-2,1)$ little group dimensions. Now we can use the well known gamma matrix identities for symmetrized indices $(\alpha\beta\gamma\delta)$, $(\gamma^m)_{(\alpha\beta}(\gamma_m)_{\gamma\delta)} = 0$, valid in (2,1), (3,1), and (9,1) dimensions, to show that the integrand (40) is a total derivative, as in the usual Green-Schwarz superstring in these little group dimensions. We have thus shown that Eq. (37) vanishes in

any frame only for $d=4,5,11$. So, the action is supersymmetric for any timelike λ^μ (with appropriate positive energy conditions⁶) in these dimensions.

Next we consider local κ supersymmetry with

$$\delta_\kappa x^\mu = \delta_\kappa \bar{\theta} \gamma^{\mu\nu} \theta \lambda_\nu, \quad \delta_\kappa \bar{\theta} = \bar{\kappa}^l \pi_l^\mu \gamma_{\mu\nu} \lambda^\nu,$$

$$\delta_\kappa A_j = 2 \bar{\kappa}^l (\pi_l \cdot \lambda) \left(\partial_j \theta - \frac{g_{ji} \epsilon^{ik} \gamma^{11}}{\sqrt{-g}} \partial_k \theta \right), \quad (41)$$

$$\delta_\kappa (\sqrt{-g} g^{ij}) = 2 \lambda^2 \left(\begin{aligned} & \bar{\kappa}^j (\sqrt{-g} g^{il} - \epsilon^{il} \gamma^{11}) \partial_l \theta \\ & + \bar{\kappa}^i (\sqrt{-g} g^{jl} - \epsilon^{jl} \gamma^{11}) \partial_l \theta \end{aligned} \right).$$

They are constructed so that the two Fermi terms in the κ variation of the action vanish. The four Fermi terms in the variation are

$$\begin{aligned} & -\epsilon^{ij} \left(\begin{aligned} & 2 \delta_\kappa \bar{\theta} \gamma_{\mu\nu} \gamma^{11} \partial_i \theta \bar{\theta} \gamma^{\mu\rho} \partial_j \theta \\ & - \delta_\kappa \bar{\theta} \gamma^{\mu\rho} \theta \partial_i \bar{\theta} \gamma_{\mu\nu} \gamma^{11} \partial_j \theta \end{aligned} \right) \lambda^\nu \lambda_\rho \\ & + \frac{\sqrt{-\lambda^2}}{2} \epsilon^{ij} \left(\begin{aligned} & 2 \delta_\kappa \bar{\theta} \gamma^\mu \gamma^{11} \partial_i \theta \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \\ & + \delta_\kappa \bar{\theta} \gamma^\mu \gamma^{11} \theta \partial_i \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \\ & - 2 \delta_\kappa \bar{\theta} \gamma_{\mu\nu} \partial_i \theta \bar{\theta} \gamma^\mu \gamma^{11} \partial_j \theta \\ & - \delta_\kappa \bar{\theta} \gamma_{\mu\nu} \theta \partial_i \bar{\theta} \gamma^\mu \gamma^{11} \partial_j \theta \end{aligned} \right) \lambda^\nu. \end{aligned} \quad (42)$$

The structure of these terms are similar to those of Eqs. (38) and (39) except for the insertions of γ^{11} being in different places. By using gamma matrix identities in the special dimensions (2,2), (3,2), (9,2) it can be shown that this is a total derivative for any timelike λ^μ . To see this, we can use once again the special frame and reduce these four Fermi terms to the same structure as Eq. (40) except for replacing $\delta_\kappa \bar{\theta}$ instead of $\bar{\varepsilon}$. The argument for the vanishing of $\delta_\kappa S_2(\lambda) = 0$ is then similar to $\delta_\varepsilon S_2(\lambda) = 0$. So, the action is κ supersymmetric for any timelike λ^μ (with appropriate positive energy conditions) in dimensions $d=4,5,11$.

Next we wish to analyze the degrees of freedom described by the string action. After using a subset of equations of motion and fixing a subset of gauges, it can easily be shown that the action (32) reduces to the string actions in (2,1), (3,1), and (9,1) dimensions with a single timelike coordinate and standard supersymmetry. To see this, we again choose the special frame for the particle $\lambda^\mu = (0, \vec{0}, \lambda^{0'})$ so that (see footnote 5)

$$\gamma_\mu \lambda^\mu = -\lambda^{0'}, \quad \gamma_{\mu\nu} \lambda^\nu = -\lambda^{0'} \gamma_\mu,$$

⁶Note that in the special frame we have used $\sqrt{-\lambda^2} = |\lambda^{0'}| = \lambda^{0'}$ by recalling the positive energy condition (14) and the fact that we wish to describe a superstring with positive energy $P^0 \geq 0$. More generally in the total action Eq. (2) that includes Eq. (32) one may multiply $\sqrt{-\lambda_1^2}$ with the sign of $\lambda_1^0 \lambda_2^{0'} - \lambda_2^0 \lambda_1^{0'}$ to insure the desired result.

$$\pi_i^{0'} = \partial_i x^{0'} - \lambda^{0'} A_i, \quad \pi_i^m = \partial_i x^m - \lambda^{0'} \bar{\theta} \gamma^m \partial_i \theta, \quad (43)$$

where m is an index in $d-1$ dimensions with signature $(d-2,1)$. Furthermore, by a gauge choice and solving the corresponding constraint we can eliminate $x^{0'}$ and A_i . Then $\pi_i^{0'} = 0$. Defining chiral spinors in $d-1$ dimensions $\theta_{\pm} = \frac{1}{2}(1 \pm \gamma^{11})\theta$, one can write the first term in the action (32) in terms of π_i^m as

$$\pi_i^m = \partial_i x^m - \lambda^{0'} (\bar{\theta}_+ \gamma^m \partial_i \theta_+ + \bar{\theta}_- \gamma^m \partial_i \theta_-) \quad (44)$$

(recall $\theta_- = 0$ in the case of $d=4$). Similarly, the second term in the action (32) becomes

$$\lambda^{0'} \varepsilon^{ij} \partial_i x^m (\bar{\theta}_+ \gamma_m \partial_j \theta_+ - \bar{\theta}_- \gamma_m \partial_j \theta_-). \quad (45)$$

Although not needed, $\lambda^{0'}$ can be absorbed into a renormalization of the spinors. The remaining string action is recognized as the Green-Schwarz action in (2,1), (3,1), (9,1) dimensions (see, e.g., [16]). This gives the full content of the degrees of freedom in the special frame. The $SO(d-2,2)$ covariance of the total system of particle and string is valid with these degrees of freedom, as can be shown by constructing the generators of $SO(d-2,2)$ with methods similar to those given in [14].

V. SUPERSTRINGS IN (10,2) DIMENSIONS

As usual we have in mind the total system described by Eq. (2). To keep the notation simple we will again omit the index No. 2 on the supercoordinates $x_2^\mu(\tau, \sigma)$, $\theta_2^\alpha(\tau, \sigma)$ and the index No. 1 on λ^μ . The following discussion applies to $d=5,12$ with signature $(d-2,2)$, but we will concentrate mainly on (10,2). The spinor is real and has dimension 4,32 for $d=5,12$, respectively.

A. Heterotic string in $(d-2,2)$

The action we propose is

$$S_2^{\text{het}}(\lambda) = \frac{1}{2} \int d^2 \sigma \sqrt{-g} g^{ij} \pi_i^\mu \pi_j^\nu \eta_{\mu\nu} - \int d^2 \sigma \varepsilon^{ij} \partial_i x^\mu \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \lambda^\nu, \quad (46)$$

where π_i^μ is given by Eq. (33). As before, there is reparametrization invariance, gauge invariance as in Eq. (34), and new supersymmetry under which $\delta_\varepsilon \pi_i^\mu = 0$, as in Eq. (35). The first term of S_2^{het} is supersymmetric. The remainder of S_2 gives

$$\delta_\varepsilon S_2^{\text{het}} = \int d^2 \sigma \varepsilon^{ij} (\bar{\varepsilon} \gamma_{\mu\rho} \partial_i \theta \bar{\theta} \gamma^{\mu\nu} \partial_j \theta) \lambda^\nu \lambda^\rho. \quad (47)$$

The integrand can be written as

$$\frac{1}{3} \varepsilon^{ij} \left(\begin{array}{l} 2 \bar{\varepsilon} \gamma_{\mu\rho} \partial_i \theta \bar{\theta} \gamma^{\mu\nu} \partial_j \theta \\ - \bar{\varepsilon} \gamma_{\mu\rho} \theta \partial_i \bar{\theta} \gamma^{\mu\nu} \partial_j \theta \end{array} \right) \lambda^\nu \lambda^\rho, \quad (48)$$

by dropping a total derivative term. One can use 5D or 12D gamma matrix identities to show that its integral is zero for $\lambda^2=0$. To prove it we will choose a Lorentz frame in 5D or 12D, $\lambda^\mu = (\lambda^{0'}, 0, \vec{0}, \lambda^{11})$. Using footnote 1 one finds

$$\gamma^{\mu\nu} \lambda_\nu = (-\lambda^{11} \gamma^{11}, \gamma^m (\lambda^{11} \gamma^{11} - \lambda^{0'}), -\lambda^{0'} \gamma^{11}), \quad (49)$$

where m is in 3D or 10D. In this frame (48) becomes

$$+ (\lambda_{0'}^2 - \lambda_{11}^2) \varepsilon^{ij} \left(\begin{array}{l} 2 \bar{\varepsilon} \gamma^{11} \partial_i \theta \bar{\theta} \gamma^{11} \partial_j \theta \\ - \bar{\varepsilon} \gamma^{11} \theta \partial_i \bar{\theta} \gamma^{11} \partial_j \theta \end{array} \right) + \varepsilon^{ij} \left(\begin{array}{l} 2 \bar{\varepsilon} \gamma^m (\lambda^{11} \gamma^{11} - \lambda^{0'}) \partial_i \theta \bar{\theta} \gamma_m (\lambda^{11} \gamma^{11} - \lambda^{0'}) \partial_j \theta \\ - \bar{\varepsilon} \gamma^m (\lambda^{11} \gamma^{11} - \lambda^{0'}) \theta \partial_i \bar{\theta} \gamma_m (\lambda^{11} \gamma^{11} - \lambda^{0'}) \partial_j \theta \end{array} \right). \quad (50)$$

The 4 or 32 components of θ in (3,2) or (10,2), can be rewritten in terms of 2 or 16 dimensional (chiral) spinors $\theta_{\pm} = \frac{1}{2}(1 \pm \gamma^{11})\theta$ of 3D or 10D, respectively. Then, $\bar{\theta} \gamma^{11} \partial_j \theta = \partial_j (\bar{\theta}_- \theta_+)$ is a total derivative, and the term proportional to $(\lambda_{0'}^2 - \lambda_{11}^2)$ drops out. Furthermore, $\bar{\theta} \gamma_m \partial_j \theta = \bar{\theta}_+ \gamma_m \partial_j \theta_+ + \bar{\theta}_- \gamma_m \partial_j \theta_-$, and $\bar{\theta} \gamma_m \gamma^{11} \partial_j \theta = \bar{\theta}_+ \gamma_m \partial_j \theta_+ - \bar{\theta}_- \gamma_m \partial_j \theta_-$, and similarly for terms involving $\varepsilon_{\pm}$. The terms with all spinors of same chirality vanish by virtue of 3D or 10D gamma matrix identities for symmetrized indices $(\alpha\beta\gamma\delta)$, $(\gamma^m)_{(\alpha\beta}(\gamma_m)_{\gamma\delta)} = 0$. The remaining cross terms are

$$-(\lambda_{0'}^2 - \lambda_{11}^2) \sum_{\pm} \varepsilon^{ij} \left(\begin{array}{l} 2 \bar{\varepsilon}_{\pm} \gamma^m \partial_i \theta_{\pm} \bar{\theta}_{\mp} \gamma_m \partial_j \theta_{\mp} \\ - \bar{\varepsilon}_{\pm} \gamma^m \theta_{\pm} \partial_i \bar{\theta}_{\mp} \gamma_m \partial_j \theta_{\mp} \end{array} \right). \quad (51)$$

The only solution for $\delta_\varepsilon S(\lambda) = 0$ is $\lambda_{0'}^2 - \lambda_{11}^2 = -\lambda^2 = 0$.

The κ symmetry transformations have the same structure as Eq. (41) except for replacing the insertion γ^{11} by the identity. The two Fermi terms in the κ variation of the action vanish. The four Fermi terms are

$$- \int d^2 \sigma \varepsilon^{ij} \left(\begin{array}{l} 2 \delta_\kappa \bar{\theta} \gamma_{\mu\nu} \partial_i \theta \bar{\theta} \gamma^{\mu\rho} \partial_j \theta \\ - \delta_\kappa \bar{\theta} \gamma^{\mu\rho} \theta \partial_i \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \end{array} \right) \lambda^\nu \lambda^\rho. \quad (52)$$

This has the same structure as Eq. (48) and thus is zero for $\lambda^2=0$. Therefore the four Fermi term vanishes and the action we proposed for the (3,2) or (10,2) string is κ symmetric for $\lambda^2=0$.

To determine the degrees of freedom of the string we will partially choose gauges and use a subset of equations of motion to show that the action (46) reduces to the 3D string and heterotic string in 10D for $d=5,12$, respectively, provided $\lambda^2=0$. For this we choose a Lorentz frame $\lambda^\mu = (\lambda^{0'}, 0, \vec{0}, \lambda^{11})$ with $\lambda^{0'} = |\lambda^{11}|$, and use lightcone components involving the extra timelike coordinate $\lambda^\pm = (\lambda^{0'} \pm \lambda^{11})$ so that all components of λ^μ except for λ^+ are zero, $\lambda^\mu = \lambda^+ \delta_\mu^+$. Then

$$\gamma_\mu \lambda^\mu = \lambda^+ \gamma_+ = \lambda^+ (-1 + \gamma^{11}), \quad \gamma_\pm = \mp 1 + \gamma^{11}, \\ \gamma_{-+} = -\gamma^{11}, \quad \gamma_{m+} = \gamma_m (-1 + \gamma^{11}),$$

$$\pi_i^+ = \partial_i x^+ - \lambda^+ A_i - \lambda^+ \bar{\theta} \gamma^{11} \partial_i \theta, \quad \pi_i^- = \partial_i x^-,$$

$$\pi_i^m = \partial_i x^m + \bar{\theta} \gamma^m (-1 + \gamma^{11}) \partial_i \theta \lambda^+, \quad (53)$$

where m is an index in (2,1) or (9,1) dimensions. The second term in S_2 becomes

$$-\varepsilon^{ij} \partial_i x^\mu \bar{\theta} \gamma_{\mu\nu} \partial_j \theta \lambda^\nu = -\varepsilon^{ij} \partial_i x^m \bar{\theta} \gamma_m (-1 + \gamma^{11}) \partial_j \theta \lambda^+ + \varepsilon^{ij} \partial_i x^- \bar{\theta} \gamma^{11} \partial_j \theta \lambda^+. \quad (54)$$

The 4 or 32 components of θ in (3,2) or (10,2) can be rewritten in terms of 2 or 16 dimensional (chiral) spinors $\theta_\pm = \frac{1}{2}(1 \pm \gamma^{11})\theta$ of 3D or 10D, respectively. Then, $\bar{\theta} \gamma^{11} \partial_i \theta = \partial_i (\bar{\theta}_- \theta_+)$ is a total derivative, and the second term above drops out.

Now we choose gauges and use equations of motion as follows: let $x^+ = 0$ by a gauge choice, $\pi_i^- = 0$ by equation of motion δA_i , and A_i is determined by the equation of motion of δx^- . The remaining action is the 3D superstring or 10D heterotic superstring, respectively. We have shown that they are embedded covariantly in (3,2) or (10,2) dimensions, respectively, provided the particle to which it is coupled is massless, since $\lambda^2 = 0$.

B. Type IIA and IIB superstrings in (10,2)

Type IIA,B superstrings are constructed by doubling the fermions $\theta^{A\alpha}$, $A=1,2$. The discussion applies in $d=5,12$ so that each spinor is real and has dimension 4,32, respectively. For type-IIB the 12D chirality of the two spinors are the same while for type IIA they are opposite. The covariant momentum is

$$\pi_i^\mu = \partial_i x^\mu - \lambda^\mu A_i + \bar{\theta}^A \gamma^{\mu\nu} \partial_i \theta^A \lambda_\nu, \quad (55)$$

and the action is

$$S^H(\lambda) = \frac{1}{2} \int d^2 \sigma \sqrt{-g} g^{ij} \pi_i^\mu \pi_j^\nu \eta_{\mu\nu} - \int d^2 \sigma \varepsilon^{ij} \partial_i x^\mu (\bar{\theta}^1 \gamma_{\mu\nu} \partial_j \theta^1 - \bar{\theta}^2 \gamma_{\mu\nu} \partial_j \theta^2) \lambda^\nu + \int d^2 \sigma \varepsilon^{ij} \bar{\theta}^1 \gamma^{\mu\rho} \partial_i \theta^1 \bar{\theta}^2 \gamma_{\mu\nu} \partial_j \theta^2 \lambda_\rho \lambda^\nu. \quad (56)$$

The supersymmetry transformation has parameters $\varepsilon^{A\alpha}$, and is given by Eq. (35), except that the fermions should be doubled. As usual $\delta_\varepsilon \pi_i^\mu = 0$, and the first term of the action is invariant. The second term gives

$$\delta_\varepsilon S_2^H = \int d^2 \sigma \varepsilon^{ij} \left(\bar{\varepsilon}^1 \gamma^{\mu\rho} \partial_i \theta^1 \bar{\theta}^1 \gamma_{\mu\nu} \partial_j \theta^1 + \bar{\varepsilon}^2 \gamma^{\mu\rho} \partial_i \theta^2 \bar{\theta}^2 \gamma_{\mu\nu} \partial_j \theta^2 \right) \lambda_\rho \lambda^\nu. \quad (57)$$

This vanishes since it has the same structure as in the heterotic case (47), provided $\lambda^2 = 0$.

The local κ supersymmetry transformations are given by

$$\delta_\kappa x^\mu = \delta_\kappa \bar{\theta}^A \gamma^{\mu\nu} \theta^A \lambda_\nu, \quad \delta_\kappa \bar{\theta}^A = \bar{\kappa}^{A1} \pi_l^\mu \gamma_{\mu\nu} \lambda^\nu,$$

$$\delta_\kappa (\sqrt{-g} g^{ij}) = 4\lambda^2 \sqrt{-g} \left(\frac{P_-^{il} \bar{\kappa}^{1j} \partial_l \theta^1 + P_+^{il} \bar{\kappa}^{2j} \partial_l \theta^2}{P_-^{jl} \bar{\kappa}^{1i} \partial_l \theta^1 + P_+^{jl} \bar{\kappa}^{2i} \partial_l \theta^2} \right), \quad (58)$$

$$\delta_\kappa A_j = 2(\pi_l \cdot \lambda) \left[\bar{\kappa}^{1l} \left(\partial_j \theta^1 - \frac{1}{\sqrt{-g}} g_{ji} \epsilon^{ik} \partial_k \theta^1 \right) + \bar{\kappa}^{2l} \left(\partial_j \theta^2 - \frac{1}{\sqrt{-g}} g_{ji} \epsilon^{ik} \partial_k \theta^2 \right) \right],$$

where we defined the projection tensors as

$$P_\pm^{ij} = \frac{1}{2} \left(g^{ij} \pm \frac{\epsilon^{ij}}{\sqrt{-g}} \right). \quad (59)$$

The κ^A parameters are anti-self-dual for $A=1$ and self-dual for $A=2$ [16]

$$\kappa^{1i} = P_-^{ij} \kappa_j^1, \quad \kappa^{2i} = P_+^{ij} \kappa_j^2. \quad (60)$$

The κ transformations are designed so that two Fermi terms in the variation of the action vanish. The four Fermi terms take the form

$$\epsilon^{ij} \left(\begin{aligned} &-2 \delta_\kappa \bar{\theta}^1 \gamma_{\mu\nu} \partial_i \theta^1 \bar{\theta}^1 \gamma^{\mu\rho} \partial_j \theta^1 \\ &+ \delta_\kappa \bar{\theta}^1 \gamma^{\mu\rho} \partial_i \theta^1 \bar{\theta}^1 \gamma_{\mu\nu} \partial_j \theta^1 \\ &+ 2 \delta_\kappa \bar{\theta}^2 \gamma_{\mu\nu} \partial_i \theta^2 \bar{\theta}^2 \gamma^{\mu\rho} \partial_j \theta^2 \\ &- \delta_\kappa \bar{\theta}^2 \gamma^{\mu\rho} \partial_i \theta^2 \bar{\theta}^2 \gamma_{\mu\nu} \partial_j \theta^2 \end{aligned} \right) \lambda^\nu \lambda_\rho. \quad (61)$$

This has the same structure as Eq. (48) and thus vanishes provided $\lambda^2 = 0$.

To analyze the degrees of freedom we choose gauges and use equations of motion as in the heterotic case. For $\lambda^2 = 0$ in (10,2), this procedure gives the 10D superstrings, but now with the fermions doubled, so that $\theta_\pm^{1,2} = \frac{1}{2}(1 \pm \gamma^{11})\theta^{1,2}$ are 16 dimensional chiral spinors in (9,1). When the 12D chiralities of $\theta^{1,2}$ are the same/opposite then the 10D chiralities are also the same/opposite. Therefore we have obtained the IIA,B superstring theory embedded covariantly in (10,2) provided the particle to which it is coupled is massless, since $\lambda^2 = 0$.

The discussion above was at the classical level. The quantum theory can be analyzed either in $SO(d-2,2)$ covariant quantization or lightcone quantization. The methods are discussed in detail in [14]. However, we can quickly determine the outcome since we have already identified the degrees of freedom and we know the critical dimensions of the sub-theories. On this basis, we can determine that the type IIA,B theories (massless particle plus string) are critical and quantum consistent in (10,2) dimensions. Similarly, the massive particle plus string theory is critical and quantum consistent in (9,2) dimensions.

The remaining theories in lower dimensions and the heterotic case become quantum consistent when additional degrees of freedom are added so that the Virasoro algebra becomes critical.

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