SUPERSYMMETRIC LOOP EFFECTS

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We review the loop effects of the low energy supersymmetry. The global success of the Standard Model rises two related questions: how strongly the mass scales of the superpartners are constrained and can they be, nevertheless, indirectly seen in precision measurements. The bulk of the electroweak data is well screened from supersymmetric loop effects, due to the structure of the theory, even with superpartners generically light, $O(M_Z)$. The only exception are the left-handed squarks of the third generation which have to be $\gtrsim O(300\text{ GeV})$ to maintain the success of the SM. The other superpartners can still be light, at their present experimental mass limits, and would manifest themselves through virtual corrections to the small number of observables such as $R_b$, $b \to s\gamma$, $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mixing and a few more for large $\tan\beta$. Those effects require still higher experimental precision to be detectable.

1 Introduction

The masses of the weak bosons, $W^\pm$ and $Z^0$, as well as fermion masses originate from the mechanism of spontaneous symmetry breaking. This mechanism requires the presence of elementary or composite scalar modes, the so-called Higgs modes which develop nonvanishing vacuum expectation values. In the Standard Model (SM), viewed as an effective low energy theory, the Higgs potential looks very unnatural and the theory faces the well known hierarchy problem. In brief, scalar potential is generically unstable with respect to quantum corrections from any new physics (the mass squared parameter of the potential receives loop corrections proportional to masses squared of the heavy
new particles). Thus, the structure of the vacuum in the SM is strongly sug-
gestive of the existence of new scale of fundamental interactions, the physical
cut-off to the SM, close to the electroweak scale.

Supersymmetry offers an interesting solution to the hierarchy puzzle and,
moreover, has several other theoretical and phenomenological (gauge coupling
unification) virtues. The new scale, the mentioned earlier cut-off to the SM,
is the scale of soft supersymmetry breaking. In other words, this is the scale
(often it can be defined only in some average sense) of the mass spectrum
of the superpartners to the particles of the SM. Two immediate and most
important remarks about the superpartner spectrum are the following ones: if
supersymmetry is to cure the hierarchy problem that scale is expected to be
not much above the electroweak scale. On the other hand, it is totally unknown
in detail, as we do not have at present any realistic model of supersymmetry
breaking. Therefore, the minimal supersymmetric extension of the SM, the so-
called Minimal Supersymmetric Standard Model (MSSM) is a very well defined
theoretical framework but contains many free parameters: superpartner soft
masses and their dimensionful couplings.

Lack of any detailed knowledge about the superpartner masses has obvious
implications for the direct search for superpartners which can only be based
on systematic exploration of the higher and higher energy scales. It is, there-
fore, very interesting to discuss the question to what extent the superpartner
spectrum can manifest itself through virtual (loop) effects on the electroweak
observables. Do very high precision measurements of the electroweak observ-
ables provide us with a tool to see supersymmetric effects indirectly or, at
least, to put stronger limits on its spectrum? We remember the important
rôle played by precision measurements in seeing, indirectly, some evidence for
the top quark long ago its direct discovery and with the mass quite close to
its measured mass. Also, the present level of precision makes the electroweak
measurements to some extent sensitive even to the Higgs boson mass, although
the dependence is only logarithmic. With supersymmetric corrections the sit-
uation is different. The dependence on the top quark (and Higgs) mass in
the SM is due to nondecoupling of heavy particles which get their masses
through the mechanism of spontaneous symmetry breaking. The soft SUSY
breaking is explicit and the Appelquist-Carazzone theorem \(^1\) applies to the su-
perpartner spectrum. Thus, supersymmetric virtual effects dissapear at least
as \(O(1/M_{SUSY})\). Nevertheless, several interesting questions can be discussed
and this is the content of this Chapter. First, we discuss the impact of the
general succes of the SM in describing the precision data on the existence of
new physics and on supersymmetry in particular. The resulting constraints
on the SUSY spectrum are reviewed with emphasize on the existence of the
room for very light, $\mathcal{O}(M_Z)$, particles. Indeed, most of the superpartners effectively decouple from most of the electroweak observables much faster than $\mathcal{O}(1/M_{SUSY})$. This high degree of screening follows from the basic structure of the theory. There are only few exceptions to this general rule: effects of light, $\mathcal{O}(M_Z)$, charginos, stops and the charged Higgs boson can be substantial in some specific observables like $R_b \equiv \Gamma(Z^0 \rightarrow \bar{b}b)/\Gamma(Z^0 \rightarrow \text{hadrons})$ and some flavour changing neutral current (FCNC) processes. In addition, for large $\tan \beta$ sizeable loop corrections to the Yukawa couplings from those particles are also possible. Those effects are discussed in subsequent sections. The Chapter ends with a brief summary of the overall prospects.

2 Supersymmetry and the electroweak precision data

The bulk of the electroweak precision measurements ($M_W$, $Z^0$-pole observables, $\nu e$, $ep$ scattering data, etc.) shows that the global comparison of the SM predictions with the data is impressive. Both, the experiment and the theory have at present similar accuracy, typically $\mathcal{O}(1\%_o)$! The predictions of the SM are usually given in terms of the very precisely known parameters $G_\mu$, $\alpha_{EM}$, $M_Z$ and the other three parameters $\alpha_s(M_Z)$, $m_t$, $M_h$. The top quark mass and the strong coupling constant are now also reported from independent experiments with considerable precision: $m_t = (175.6 \pm 5.5)$ GeV and $\alpha_s(M_Z) = 0.118 \pm 0.003$, but those measurements are difficult and it is safer to take $\alpha_s$, $m_t$, $M_h$ as parameters of an overall fit. Such fits give values of $m_t$ and $\alpha_s$ very well consistent with the above values.

The theoretical uncertainties in the SM predictions (for fixed $m_t$, $M_h$, $\alpha_s$) come mainly from the RG evolution of $\alpha_{EM} \equiv \alpha(0) \rightarrow \alpha(M_Z)$ which depends on the hadronic contribution to the photon vacuum polarization $\alpha(s) = \alpha(0)/(1 - \Delta \alpha(s))$ where $\Delta \alpha(s) = \Delta \alpha_{hadr} + \ldots$ and $\Delta \alpha_{hadr} = 0.0280 \pm 0.0007$. The present error in the hadronic vacuum polarization propagates as $\mathcal{O}(1\%_o)$ errors in the final predictions. The other uncertainties come from the neglected higher order corrections and manifest themselves as renormalization scheme dependence, higher order arbitrariness in resummation formulae etc. Those effects have been estimated to be smaller than $\mathcal{O}(1\%_o)$, hence the conclusion is that the theory and experiment agree with each other at the level of $\mathcal{O}(1\%_o)$ accuracy. In particular, the genuine weak loop corrections are now tested at $\mathcal{O}(5\sigma)$ level and the precision is already high enough to see some sensitivity to the Higgs boson mass.

The electroweak observables depend only logarithmically on the Higgs boson mass (whereas the dependence on the top quark mass is quadratic). Global fits to the present data give $M_h \approx 130^{+130}_{-70}$ GeV and the 95% C.L. upper bound
is around 470 GeV. Thus, the data give some indication for a light Higgs boson. (It is worth noting that $M_h = 1$ TeV is more than $\gtrsim 3\sigma$ away from the best fit). The direct experimental lower limit on the SM Higgs boson mass $M_h$ is $\sim 70$ GeV.

The overall global success of the SM is a bit overshadowed by a couple of (quite relevant!) scattered clouds. The results for $R_b$ are still preliminary and differ by about $2\sigma$ between different experiments. The world average is only $1.8\sigma$ away from the SM prediction but with the error which is only slightly smaller than the maximal possible enhancement of $R_b$ in the MSSM (see later). The effective Weinberg angle is now reported with a very high precision. However, this result comes from averaging over the SLD and LEP results which are more then $3\sigma$ apart. Hoping for further experimental clarification of those few points it is interesting to discuss already now the impact of the general success of the SM on the existence of new physics. The simplest interpretation of the success of the SM within the MSSM is that the superpartners are heavy enough to decouple from the electroweak observables. Explicit calculations (with the same precision as in the SM) show that this happens if the common supersymmetry breaking scale is $\geq O(300 - 400)$ GeV. This is very important as such a scale of supersymmetry breaking is still low enough for supersymmetry to cure the hierarchy problem. However, in this case the only supersymmetric signature at the electroweak scale and just above it is the Higgs sector with a light, $M_h \leq O(150)$ GeV, Higgs boson. This prediction is consistent with the SM fits discussed earlier. We can, therefore, conclude at this point that the supersymmetric extension of the SM, with all superpartners $\geq O(300)$ GeV, is phenomenologically as succesful as the SM itself and has the virtue of solving the hierarchy problem. Discovery of a light Higgs boson is the crucial test for such an extension.

The relatively heavy superpartners discussed in the previous paragraph are sufficient for explaining the success of the SM. But is it necessary that all of them are that heavy? Is there a room for some light superpartners with masses $O(M_Z)$ or even below? This question is of great importance for LEP2. Indeed, a closer look at the electroweak observables shows that the answer to this question is positive. The dominant quantum corrections to the electroweak observables are the so-called "oblique" corrections to the gauge boson self-energies. They are economically summarized in terms of the $S,T,U$ parameters

$$\alpha S \sim \Pi_{3Y}^\prime(0) = \Pi_{L3,R3}^\prime + \Pi_{L3,B-L}^\prime$$

(1)

(the last decomposition is labelled by the $SU_L(2) \times SU_R(2) \times U_{B-L}(1)$ quantum
\[ \alpha T \equiv \Delta \rho \sim \Pi_{11}(0) - \Pi_{33}(0) \]  
(2)

\[ \alpha U \sim \Pi'_{11}(0) - \Pi'_{33}(0) \]  
(3)

where \( \Pi_{ij}(0) (\Pi'_{ij}(0)) \) are the (i,j) left-handed gauge boson self-energies at the zero momentum (their derivatives) and the self-energy correction to the \( S \) parameter mixes \( W_3^\mu \) and \( B_\mu \) gauge bosons. It is clear from their definitions that the parameters \( S, T, U \) have important symmetry properties: \( T \) and \( U \) vanish when quantum corrections to the left-handed gauge boson self-energies leave unbroken "custodial" \( SU_2V(2) \) symmetry. The parameter \( S \) vanishes if \( SU_2L(2) \) remains an exact symmetry (notice that, since \( 3^L_L \otimes 3^R_R = 1^L \oplus 5^L \) under \( SU_2V(2) \), exact \( SU_2V(2) \) is not sufficient for vanishing of \( S \)).

In terms of the parameters \( S, T, U \) the "new physics" contribution to the basic electroweak observables can be approximately written as:

\[ \delta M_W = \frac{M_W}{2} \frac{\alpha}{c_W^2 - s_W^2} \left( c_W^2 T^{new} - \frac{1}{2} S^{new} + \frac{s_W^2 - s_W^4}{4 s_W^2 c_W^2} U^{new} \right) \]  
(4)

\[ \delta \sin^2 \theta_{\text{eff}}^{\text{lept}} = - \frac{s_W^2 c_W}{c_W^2 - s_W^2} \left( \alpha T^{new} - \frac{\alpha}{4 s_W^2 c_W^2} S^{new} \right) \]  
(5)

\[ \delta \Gamma(Z^0 \to f f') = \frac{\alpha M_Z}{12 s_W^2 c_W} \left[ (g_V^2 + g_A^2) (\alpha T^{new}) \right. \right. \]  
(6)

\[ - 4 g_V Q_f s_W^2 c_W \left. \left( - \alpha T^{new} - \frac{\alpha}{4 s_W^2 c_W^2} S^{new} \right) \right] \]

where the parameters \( M_W, c_W \equiv M_W/M_Z, s_W, g_V = -I_3^f + 2 Q_f \sin^2 \theta_{\text{eff}}^{\text{lept}} \) and \( g_A = -I_3^f \) are computed in the SM (taking into account loop corrections) and with some reference values of \( m_t \) and \( M_d \). \( S^{new}, T^{new}, U^{new} \) contain only the contributions from physics beyond the SM.

The success of the SM means that it has just the right amount of the \( SU_2V(2) \) breaking (and of the \( SU_2L(2) \) breaking), encoded mainly in the top

\[ ^c \text{We assume that the supersymmetric contributions of order } M^2_{3} / M^2_{SU SY} \text{ to those observables are small and can be neglected. If it is not the case, the parameters } S, T, U \text{ should be defined at non-zero momentum transfer}^{14} \text{ and one should take into account also the "new physics" contributions through additional parameters like } \Delta \alpha = \Pi'_{\gamma \gamma}(M_Z) - \Pi'_{\gamma}(0) \text{ and } e_5 = M^2_Z F'_{ZZ}(M^2_Z)^{15}. \]
quark-bottom quark mass splitting. Any extension of the SM, to be consistent with the precision data, should not introduce additional sources of large $SU_V(2)$ breaking in sectors which couple to the left-handed gauge bosons. In the MSSM, the main potential origin of new $SU_V(2)$ breaking effects in the left-handed sector is the splitting between the left-handed stop and sbottom masses:

$$M_{\tilde{t}_L}^2 = m_Q^2 + m_t^2 - \frac{1}{6} \cos 2\beta (M_Z^2 - 4M_W^2)$$

$$M_{\tilde{b}_L}^2 = m_Q^2 + m_b^2 - \frac{1}{6} \cos 2\beta (M_Z^2 + 2M_W^2)$$

The $SU_V(2)$ breaking is small if the common soft mass $m_Q^2$ is large enough. So, from the bulk of the precision data one gets a lower bound on the masses of the left-handed squarks of the third generation. However, the right-handed top and bottom squarks can be very light, at their experimental lower bounds $\sim 70$ and $\sim 150$ GeV, respectively.

The other possible source of $SU_V(2)$ violation is in the slepton sector. For small values of $m_L^2$ the splitting between the left-handed slepton and sneutrino masses

$$M_{\tilde{\nu}_L}^2 = m_L^2 + \frac{1}{2} \cos 2\beta M_Z^2$$

$$M_{\tilde{l}_L}^2 = m_L^2 + m_l^2 + \frac{1}{2} \cos 2\beta (M_Z^2 - 2M_W^2)$$

becomes non-negligible. Since this mass splitting may be of similar magnitude for all three generations of sleptons this effect should also be considered. It is also worth noting different behaviour of the slepton and squark contributions to the $SU_V(2)$ breaking: the former vanishes in the limit $\tan \beta \to 1$ whereas the latter is maximal in this limit and slightly decreases as $\tan \beta \to \infty$.

Additional source of the $SU_V(2)$ breaking is also in the $A$-terms. In principle, there can be cancellations between the soft mass term and the $A$-term contribution, such that another solution with small $SU_V(2)$ breaking exists with a large inverse hierarchy $m_{\tilde{t}_L}^2 \gg m_Q^2$. This is very unnatural from the point of view of the GUT boundary conditions and here we assume $m_Q^2 \geq m_{\tilde{t}_L}^2$.

It is also worth mentioning that the $SU_V(2)$ breaking in the sector of the first two generation left-handed squarks is similar to that in the slepton sector (but enhanced by the colour factor of 3) i.e. it is determined by the “electroweak” terms in their mass formulae (the mass squared splittings of the up- and down-type left-handed squarks are proportional to $\approx \cos 2\beta M_W^2$). Such effects can be used to constrain from below the soft SUSY breaking terms in the case of broken $R$–parity or if the gluino is heavier than $O(350)$ GeV (i.e. when the direct Tevatron bounds, $M_{\tilde{g}} > \sim 150$ GeV, for $\tilde{q} = \tilde{t}$, do not apply).
Figure 1: Lower bounds on the heavier stop mass $M_{\tilde{t}_2}$, as a function of $M_{\tilde{t}_1}$ for $\tan \beta = 1.6$ (solid line) and $\tan \beta = 10$ (dashed line). A scan over the top quark mass and the top squarks mixing angle $\theta_{\tilde{t}}$ has been performed.

In Fig. 1 we show the lower bound on the mass of the heavier top squark as a function of the mass of the lighter stop, which follows from the requirement that a fit in the MSSM is at most by $\Delta \chi^2 = 2$ worse than the analogous fit in the SM. From the analysis of the SUSY contributions to the parameters $T$ and $S$ it follows that in the MSSM $T^{\text{new}}$ is always positive whereas $S^{\text{new}}$ is always negative. Therefore, the full fits to the electroweak observables give more restrictive limits on the MSSM parameter space than do e.g. bounds on the $\Delta \rho(0)$ parameter alone because, as follows from eqs (4-6), the effects in the $T$ and $S$ always add up. In the context of Fig. 1 there is one more interesting observation to be made. In the low $\tan \beta$ region, for a given $M_{\tilde{t}_1}$, an absolute lower bound on $M_{\tilde{t}_2}$ is set by the (conservative) experimental lower bound on the lightest supersymmetric Higgs boson mass, $M_h > \sim 60$ GeV. For low $\tan \beta$, the tree level Higgs boson mass is close to zero and radiative corrections are very important. They depend logarithmically on the product $M_{\tilde{t}_1}M_{\tilde{t}_2}$. Also, since the best fit in the SM requires $M_h \approx 130$ GeV, too small values of $M_{\tilde{t}_2}$ (leading to too small value of $M_h$) are disfavoured by the MSSM fit. The limit shown in Fig. 1 take both effects into account. They explain the difference

\footnote{$S^{\text{new}}$ could be positive only in the small window of the chargino parameter space which is already excluded by the unsuccessful LEP search.}
Figure 2: Predictions for $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ (a) and $M_W$ (b) in the SM (the band bounded by the dashed lines) and in the MSSM (solid lines) as functions of the top-quark mass. The bands are obtained by scanning over the MSSM parameters ($M_h$ in the SM) respecting all available experimental limits. The SLC and the (average) LEP measurements for $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ and 1$\sigma$ experimental range for $M_W$ are marked by horizontal dash-dotted lines. Dotted line in (a) shows the lower limit for $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ in the MSSM if all sparticles are heavier than $Z^0$.

between the limits for small and large tan $\beta$ cases. The absolute limits on the stop masses obtained from the bound on $M_h$ (which apply only for low tan $\beta$) turns out to be slightly weaker than the limits from the fit shown in Fig. 1.

The important rôle played in the fit by the precise result for $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ is illustrated in Fig. 2a. The world average value (used in obtaining the bounds shown in Fig. 1) is obtained in the SM model with $m_t = (175 \pm 6)$ GeV and $M_h \sim (120 - 150)$ GeV, with little room for additional supersymmetric contribution. Hence, the relevant superpartners ($\tilde{t}_L$ and $\tilde{b}_L$) have to be heavy. With lighter superpartners, one obtains the band (solid lines) shown in Fig. 2a. We see that the SLD result for $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ leaves much more room for light superpartners. Thus, settling the SLD/LEP dispute is very relevant for new physics. Similar dependence for $M_W$ is shown in Fig. 2b. The experimental result, $M_W = 80.401 \pm 0.076$ GeV, is the average of $M_W$ measured by UA2, LEP and Tevatron.$^{17}$

All squarks of the first two generations as well as sleptons can be still at their present lower experimental limits, and the success of the SM in the description of the precision electroweak data is still maintained. The same applies to the gaugino/higgsino sectors, since they do not give any strong
$SU_3(2)$ breaking effects. In conclusion, most of the superpartners decouple from most of the electroweak observables, even if very light, $O(M_Z)$. This high degree of screening follows from the basic structure of the model. The remarkable exception is the famous $R_b$.

3 $R_b$ in the MSSM

As already mentioned, although the new ALEPH measurement of $R_b$ is in perfect agreement with its value predicted in the SM, the average of all measurements still deviates from the SM value by $\sim 1.8 \sigma$. In view of this fact and because the $R_b$ value in the MSSM is sensitive to different set of parameters than the bulk of the other electroweak observables, it is interesting to discuss this observable in more detail.

In the MSSM there are new contributions to the $Z^0\bar{b}b$ vertex, namely, Higgs bosons exchange in the loops, neutralino-sbottom and chargino-stop loops. For low and intermediate values of $\tan\beta$, the first give negative contribution to $R_b$ and to minimize this effect the pseudoscalar mass $M_A$ has to be sufficiently large, say, $M_A > O(300 \text{ GeV})$ whereas the neutralino-sbottom contribution is negligible. The chargino-stop loops can be realized in two ways: with stop coupled to $Z^0$ and with charginos coupled to $Z^0$. In both cases the lighter the stop and chargino the larger is the positive contribution.

The Dirac charginos are defined as

$$C^-_i = \left( \frac{\lambda^-_i}{\chi^-_i} \right) \quad i = 1, 2$$

where $\lambda^-_i$ (Higgsinos) are linear combinations of the negatively (positively) charged $SU(2)$ gauginos and down-(up-)type higgsinos. The $b\tilde{t}_1 C^-$ coupling is enhanced for a right-handed stop (it is then proportional to the top quark Yukawa coupling). Then, however, the stop coupling to $Z^0$ is suppressed (it is proportional to $g \sin^2 \theta_W$). Therefore, significant contribution can only come from the diagrams in which charginos are coupled to $Z^0$. Their actual magnitude depend on the interplay of the couplings in the $C^-_i \tilde{t}_1 b$ vertex and the $Z^0 C^-_i C^-_j$ vertex. The first one is large only for charginos with large up-higgsino component, the second - for charginos with large gaugino component in at least one of its two-component spinors. It has been observed that, the situation in which both couplings are simultaneously large never happens for $\mu > 0$. Large $R_b$ can then only be achieved at the expense of extremely light $C^-_j$ and

\[ \text{Up- and down- type higgsinos are superpartners of the Higgs boson doublets giving masses to the up- and down-type quarks, respectively.} \]
In addition, for fixed $m_{\tilde{C}_1}$ and $M_{\tilde{t}_1}$, $R_B$ is larger for $r \equiv M_2/|\mu| > 1$ i.e. for higgsino-like chargino as the enhancement of the $C_1^{-}\tilde{t}_1b$ coupling is more important than of the $Z^0C_1 C_1^{-}$ coupling.

For $\mu < 0$ the situation is different. In the range $0.5 \lesssim r \lesssim 1.5$ a light chargino can be a strongly mixed state with a large up-higgsino and gaugino components (the higgsino-gaugino mixing comes from the chargino mass matrix). Large couplings in both vertices of the diagram with charginos coupled to $Z^0$ give significant increase in $R_B$ even for the lighter chargino as heavy as $80 - 90$ GeV (similar increase in $R_B$ for $\mu > 0$ requires $m_{C_1} \approx 50$ GeV and $M_{\tilde{t}_1} \approx 50$ GeV). This is illustrated in Fig. 3a where we show the contours of constant $\delta R_B$ in the $(M_2, \mu)$ plane for fixed parameters of the stop sector.

The chargino-stop contributions do not change the value of the left-right asymmetry in $b$ quarks $A_b \equiv (g_L^2 - g_R^2)/(g_L^2 + g_R^2)$ where $g_L$ ($g_R$) are the effective couplings of left-handed (right-handed) $b$ quarks to $Z^0$ (measured at SLD) as they mostly modify only the left-handed effective coupling $g_L$. In this case we get $\delta A_b \approx 5.84 \times (1 - A_0^{SM}) \times \delta R_B$ i.e. a very small, positive shift.

An enhancement of $R_B$ is also possible for large $\tan \beta$ values, $\tan \beta \approx m_t/m_b$. In this case, in addition to the stop-chargino contribution (and neutralino-sbottom contribution enhanced by large $\tan \beta$) there can be even...
larger positive contribution from the $h^0$, $H^0$ and $A^0$ exchanges in the loops, provided those particles are sufficiently light (in this range of tan $\beta$, $M_h \approx M_A$) and non-negligible sbottom-neutralino loop contributions. The main difference with the low tan $\beta$ case is the approximate independence of the results on the sign of $\mu$ (which can be traced back to the approximate symmetry of the chargino masses and mixings under $\mu \rightarrow -\mu$). Also, the effects are always maximal for $M_2/|\mu| \gg 1$ i.e. for higgsino-like chargino. With present experimental constraints ($M_A \gtrsim 60$ GeV) values of $R_b$ up to $\sim 0.2178$ can still be obtained. This is illustrated in Fig. 3b. Significant enhancement of $R_b$ in the large tan $\beta$ regime is, however, rather unlikely as it requires very precise cancellation of the SUSY contributions to obtain acceptable rate for $b \rightarrow s \gamma$ (see later).

We conclude that additional supersymmetric contributions to the $Z^0b\bar{b}$ vertex, from the chargino-right-handed stop loop (and from a light $CP$-odd Higgs boson for large tan $\beta$), can be non-negligible when both are light, $\sim O(M_Z)$. However, even with the chargino and stop at their present experimental mass limit, the prediction for $R_b$ in the MSSM depends strongly on the chargino composition (see Fig. 3) and on the stop mixing angle. The values ranging from 0.2158 (the SM prediction) up to $\sim 0.2178$ for both small and large tan $\beta$ values can be obtained (given all the experimental constraints). (Only marginal modification of the SM result for $R_c$ is possible, though.) Those predictions hold with or without $R$-parity conservation and with or without the GUT relation for the gaugino masses. The upper bound is reachable for chargino masses up to $\sim 90$ GeV provided they are mixed gaugino-higgsino states ($M_2/|\mu| \sim 1$) with $\mu < 0$ for low tan $\beta$ and higgsino-like for large tan $\beta$. In the same chargino mass range $\delta R_b \approx 0$ in the deep higgsino and gaugino regions for low tan $\beta$ and gaugino region for large tan $\beta$.

In conclusion, the new values of $R_b$ and $R_c$ are good news for supersymmetry. At the same time, one should face the fact that, unfortunately, in the MSSM

$$\delta R_b^{max} \sim O(1 \sigma^{exp})$$

so much better experimental precision is needed for a meaningful discussion.

4 $g-2$ and supersymmetry

One of the best measured electroweak observables is the muon anomalous magnetic moment

$$a_{\mu} = (g_{\mu} - 2)/2 = (116592300 \pm 840) \times 10^{-11}$$
The theoretical value for $a_\mu$, $a_\mu = (116591830 \pm 150) \times 10^{-11}$ is dominated by the ordinary QED contribution (known up to $O(\alpha^5)$) $a_\mu^{QED} = (116584706 \pm 2) \times 10^{-11}$ and the hadronic contribution to vacuum polarization $a_\mu^{had} = (7020 \pm 150) \times 10^{-11}$\textsuperscript{h}. Standard electroweak contribution to $a_\mu$ gives $a_\mu^{EW} = (152 \pm 3) \times 10^{-11}$ for the combined one- and two-loop corrections (the weak 2-loop terms calculated recently\textsuperscript{31} are small). Thus, the present experimental accuracy, $\sim 10^{-3}$%, is sufficient to test only the QED sector of the SM\textsuperscript{30}.

A renewed interest in the muon anomalous magnetic moment is due to the ongoing Brookhaven National Laboratory experiment, with the anticipated accuracy $\delta a_\mu^{exp} \approx \pm 40 \times 10^{-11}$. Even with this measurement done with the foreseen accuracy, the one-loop weak corrections can be tested only after a substantial reduction of the hadronic vacuum polarization uncertainty. This can only be achieved by new measurements of the cross section for $e^+e^- \rightarrow \text{hadrons}$ in the low energy range. Under the same condition, the precise measurement of $a_\mu$ will be a very important test of new physics, sensitive to mass scales beyond the reach of the present accelerators\textsuperscript{33,34}.

At present, the requirement that the supersymmetric contribution $\delta^{new}a_\mu$ lies within the difference between experimental and theoretical results

$$-900 \times 10^{-11} < \delta^{new}a_\mu < 1900 \times 10^{-11}$$

puts already some constraints, though marginal, on the MSSM parameter space. In particular, for large $\tan \beta$ the dominant supersymmetric contribution due to neutralino-smuon and chargino-sneutrino loops gives approximately\textsuperscript{35,36}

$$\delta^{susy}a_\mu \approx \pm \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{M_{SUSY}^2} \tan \beta \approx \pm 150 \times 10^{-11} \left( \frac{100 \text{GeV}}{M_{SUSY}} \right)^2 \tan \beta \quad (10)$$

($M_{SUSY}$ is the average supersymmetric mass and the sign is correlated with the sign of the $\mu$ parameter) and for $\tan \beta > 10$ eliminates some portion of the chargino-sneutrino mass plane\textsuperscript{35,36}. It is clear that the new Brookhaven experiment if supplemented with the reduction of the hadronic vacuum polarization uncertainty will enhance the sensitivity to the chargino-smuon sector of the MSSM.

5 FCNC with light superpartners

Another important class of processes, where light superpartners could manifest themselves through virtual corrections, are the Flavour Changing Neutral

\textsuperscript{h}Recently the hadronic photon vacuum polarization contribution has been estimated using the ALEPH data for hadronic $\tau$ decays to be $(6950 \pm 150) \times 10^{-11}$\textsuperscript{32}.}
Current (FCNC) transitions. Gauge invariance, renormalizability and particle content of the SM imply the absence (in the lepton sector) or strong suppression (in the quark sector) of such processes. This prediction of the SM is in beautiful agreement with the presently available experimental data. In the MSSM there are two kinds of new contributions to the FCNC transitions. First of all, they may originate from flavour mixing in the sfermion mass matrices. The strong suppression of the FCNC transitions observed in Nature puts severe constraints on flavour changing elements in the sfermion mass matrices. Even in the absence of such effects the other kind of new contributions to FCNC processes arise through the ordinary $K$-$M$ mixings due to additional exchanges of (light) supersymmetric particles in loops.

The present section is devoted to discussing such a scenario. The only extra MSSM contributions to the FCNC processes we consider are the charged Higgs boson-top and chargino-stop loops. The third generation sfermions and chargino(s) are indeed expected to be among the lightest superpartners. It is reasonable to assume (as follows from the analysis of the renormalization group equations for soft supersymmetry breaking parameters) that the first two generations of sfermions are heavier and degenerate in mass. The relevant parameter space is then identical to the one tested in corrections to $R_b$ discussed earlier. Following the results on the precision tests we will assume that the heavier stop is heavy enough to decouple and that the lighter one is dominantly right-handed, i.e. that stop left-right mixing angle $\theta_{t\tilde{t}}$ is relatively small (of order $10^0$).

Within this scenario, sizeable effects can still occur in the neutral meson mixing ($K^0-\bar{K}^0$ and $B^0-\bar{B}^0$). Supersymmetric contributions to other FCNC processes are either small or screened by long-distance QCD effects. A remarkable exception is the inclusive weak radiative B meson decay, $B \rightarrow X_s\gamma$, to which light superpartners can contribute significantly, and where strong interaction effects are under control. Therefore, in the following, we shall focus on neutral meson mixing and the $B \rightarrow X_s\gamma$ decay only and summarize the main points. More extensive discussion is given in ref. 42.

With the assumption that the effects of the off diagonal entries of the sfermion mass matrices can be neglected, the predictions for $\Delta m_{B_d}$ and $\epsilon_K$ can be written as

\begin{align}
\Delta m_{B_d} &= \frac{\alpha_{em} m_t^2}{12 \sin^4 \theta_W M_W^4} f_{B_d}^2 B_{B_d} m_{B_d} | V_{tb} V_{td}^* |^2 \Delta, \\
|\epsilon_K| &= \frac{\sqrt{2} \alpha_{em} m_e^2}{48 \sin^2 \theta_W M_W^4} f_{K}^2 B_{K} m_K \frac{m_K}{\Delta m_K} | \Im \Omega |,
\end{align}

(11, 12)
where $V_{ij}$ are the elements of the $K-M$ matrix,

$$\Omega = \eta_{cc}(V_{cs}V_{cd}^*)^2 + 2\eta_{ct}(V_{cs}V_{cd}^*V_{ts}V_{td}^*)f\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right) + \eta_{tt}(V_{ts}V_{td}^*)^2\frac{m_t^2}{m_c^2}\Delta, \quad (13)$$

and

$$f(x, y) = \log \frac{y}{x} + \frac{3y}{4(y-1)}(1 - \frac{y}{y-1} \log y).$$

The charged Higgs boson and chargino boxes contribute, together with the SM terms, only to the quantity $\Delta$. The QCD correction factors $\eta_{cc}$, $\eta_{ct}$, $\eta_{tt}$ and $\eta_{QCD}$ are known up to the next-to-leading order. The theoretical predictions for $\epsilon_K$ and $\Delta m_{B_d}$ have a well known uncertainty due to non-perturbative parameters $B_K$, $f_{B_s}B_{B_d}$ estimated from lattice calculations. Moreover, the value of the element $V_{td} = A\lambda^3(1 - \rho - i\eta)$ of the $K-M$ matrix (we use the Wolfenstein parametrization), which is not measured directly, is extracted from the fit to the observables (11-13) and obviously changes after inclusion of new, nonstandard, contributions to $\Delta$. Thus, the correct approach is to fit the parameters $A$, $\rho$, $\eta$ and $\Delta$ in a model independent way to the experimental values of $\epsilon_K$ and $\Delta m_{B_d}$ keeping the values of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ within their experimentally allowed ranges.

Such a fit with $B_K$ and $f_{B_s}B_{B_d}$ varied in the ranges $0.6 < B_K < 0.9$, $0.160\ \text{GeV} < \sqrt{f_{B_s}^2B_{B_d}} < 0.240\ \text{GeV}$ gives rather liberal “absolute” bounds on $\Delta$: $0.2 \lesssim \Delta \lesssim 2.0$. This should be compared with the theoretical prediction for the parameter $\Delta$ in the SM: $\Delta = 0.53$. Larger values of $\Delta > \Delta_{SM}$ (interesting in the MSSM, as discussed later) prefer $\rho > 0$, small values of $f_{B_s}(B_{B_d})^{1/2}$ and, to a lesser extent, large $B_K$. For instance, $\Delta > 1$ requires $\rho > 0$ and $f_{B_s}(B_{B_d})^{1/2} < 0.19\ \text{GeV}$. This is illustrated in Fig. 4. Values of $\Delta$ smaller than $\Delta_{SM}$ prefer negative $\rho$ and large $\eta$. $\Delta \sim \Delta_{SM}$ gives the biggest allowed range for $\rho$ and $\eta$ with both $\rho < 0$ and $\rho > 0$ possible.

The model independent bounds for $\Delta$ can be compared with the predictions for this quantity in the MSSM. In Fig. 5, we plot contour lines of constant $\Delta$ computed in the MSSM with light spectrum, for which SUSY effects are most visible. As seen from Fig. 5, the values of $\Delta$ in the MSSM are always bigger than in the SM, i.e. the new contributions to $\Delta$ from the Higgs and chargino sectors have the same sign as $\Delta_{SM} \approx 0.53$ (for $m_t = 175\ \text{GeV}$). This is a general conclusion, always true for the Higgs contribution and valid

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The values of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ are known from tree level processes and are practically unaffected by new physics which contributes only at one and more loops.
Figure 4: Allowed regions in the $(\rho, \eta)$ plane for two different values of $\Delta$. Regions (A) are allowed by $\epsilon_K$, regions (B) - by $\Delta m_{B_d}$. Regions (C) are allowed by both measurements simultaneously.

also for the chargino-stop contribution when SUSY parameters are chosen as specified at the beginning of this section. The charged Higgs boson contribution increases $\Delta$ by about 0.12 for $M_{H^\pm} = 100$ GeV and $\tan \beta = 1.8$ as used in Fig. 5. The value of the genuine supersymmetric contribution to $\Delta$ depends strongly on the ratio $r \equiv M_2/|\mu|$. For small values of $r$, when the lighter chargino is predominantly gaugino-like, the chargino-stop contribution to $\Delta$ is very small (of order $10^{-2}$) and weakly dependent on the lighter stop mass. This can be easily understood: In this case, the lighter stop is coupled to the lighter chargino mostly through the left-right mixing in the stop sector, and the appropriate contribution is suppressed by $\sin^4 \theta_{\tilde{t}}$. For larger values of $r$, $r \sim 1$, this contribution is bigger and, due to the interference between the diagrams with and without the left-right mixing, may reach its maximal value for $\theta_{\tilde{t}} \neq 0$, depending on the sign of $\mu$. Chargino-lighter stop contribution increases further with $M_2/|\mu|$, when lighter chargino is predominantly up-type Higgsino, and become again independent on the sign of $\mu$.

Increasing the charged Higgs mass to $M_{H^\pm} \approx 500$ GeV and chargino mass to $m_{C_1^\pm} = 300$ GeV suppresses the magnitude of each contribution by a factor of 3 approximately, but does not change the character of its dependence on $\theta_{\tilde{t}}$. The results illustrated in Fig. 5 are also weakly dependent on the mass of the
left stop: Increasing the heavier stop mass, $M_{\tilde{t}_2}$, from 250 to 500 GeV modifies $\Delta$ only marginally.

We conclude that in the $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ mixing there is a room for important supersymmetric contributions. In the presence of such contributions the values of the CP violation parameters $\rho$ and $\eta$ are different from their SM values and can be tested in the study of the CP violation in $B$-factories.

We now turn to the discussion of $B \to X_s \gamma$ decay rate which in the first approximation is given by simple one-loop graphs. However, the strong interaction corrections to these one-loop diagrams are enhanced by the large logarithms $\ln(M_W^2/m_b^2)$ and, in the SM, they increase the decay rate by a factor of order 2. Thus, resumming these large QCD logarithms up to (at least) next-to-leading order (NLO) is necessary to acquire sufficient accuracy. This is conveniently done in three steps of which only the first one depends on the presence of “new physics” (supersymmetry) close to the electroweak scale.

In the first step one integrates out at the scale $Q = M_W$ all heavy fields and introduces the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^8 C_i(Q) \mathcal{O}_i(Q)$$

(14)

Figure 5: Contour lines of $\Delta$ as a function of right stop mass and stop mixing angle for $\tan \beta = 1.8$, $M_{H^\pm} = 100$ GeV, $M_{\tilde{t}_2} = 250$ GeV, $m_{C^\pm} = 90$ GeV and $m_t = 175$ GeV. a) $M_2/\mu = -1$, b) $M_2/\mu = 0.1$. 
where $\mathcal{O}_i$ are the operators and $C_i(Q)$ are their Wilson coefficients. The relevant for $B \to X_s \gamma$ operators are

\begin{align}
\mathcal{O}_7 &= \frac{e}{16\pi^2} m_b (\bar{s} L \sigma^{\mu\nu} b_R) F_{\mu\nu} \\
\mathcal{O}_8 &= \frac{g_s}{16\pi^2} m_b (\bar{s} L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}
\end{align}

where $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the photonic and gluonic field strength tensors, respectively. The leading-order SM $^{45}$ and MSSM $^{46}$ contributions to the coefficients $C_7(M_W)$ and $C_8(M_W)$ are well known. The next-to-leading corrections to $C_7(M_W)$ have been computed fully only in the case case of the SM $^{47}$. In the supersymmetric case, only contributions proportional to logarithms of superpartner masses are known $^{48}$.

In the next step, resummation of large logarithms $\ln(M_W^2/m_b^2)$ is achieved by evolving the coefficients $C_i(Q)$ from $Q \sim M_W$ to $Q \sim m_b$ according to the renormalization group equations. The necessary for the complete NLO evolution coefficients of the RGE have been computed only recently $^{49}$.

Finally, the Feynman rules derived from the effective Hamiltonian at the scale $Q \sim m_b$ are used to calculate the $b$-quark decay rate $\Gamma(b \to X_s \gamma)$ which is a good approximation to the corresponding $B$-meson decay rate $^{52}$. Radiative corrections to this computation, necessary to achieve NLO precision of the whole procedure, have also been computed recently $^{50}$.

We stress again that all these calculations are identical in the SM and MSSM except for the initial numerical values of the Wilson coefficients $C_7$ and $C_8$ at $Q \sim M_W$ which contain the information about “new physics”. Another important remark is that even in the NLO computation, the theoretical prediction for $\Gamma(b \to X_s \gamma)$ still has an uncertainty (shown in Fig 6a) of order 15% $^{49}$ which is always taken into account in the bounds on sparticle masses presented below.

Fig. 6a shows the lower limits on the mass of the $CP$-odd MSSM Higgs boson mass $^{1}$, $M_A$, as a function of $\tan \beta$. Solid lines correspond to the case when all the superpartner masses are very large (above 1 TeV). In this case, the MSSM results are the same as in the Two Higgs Doublet Model (2HDM). Dashed (dotted) lines in Fig. 6a show the same limits in the presence of chargino and stop with masses $m_{C_1} = M_{\tilde{t}_1} = 500$ (250) GeV (with all other particles heavy) obtained by scanning over the values of $r = M_A/\mu$ and $\theta_{\tilde{t}}$. In the presence of light stop and chargino limits on $M_A$ are significantly weaker and totally disappear for large values of $\tan \beta$ for which the chargino-stop

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$^1$The charged Higgs boson mass is in one-to-one correspondence with $M_A$: $M_{H^\pm}^2 = M_A^2 + M_W^2$ (up to small radiative corrections $^{54}$).
contribution can be very large. The price however is a high degree of fine tuning (see the next Section).

Figure 6: a) Lower limits on $M_A$ from $b \to s\gamma$ as a function of $\tan\beta$. Solid line correspond to very heavy, $>\mathcal{O}(1\text{ TeV})$ sparticles. Dashed (dotted) line show the limit for $m_{C_1} = M_{\tilde{t}_1} = 250$ (500) GeV. b) Lower limits on $M_A$ as a function of $M_2/|\mu|$, based on CLEO $BR(B \to X_s \gamma)$ measurement. Thick lines show limits for $\mu > 0$, thin lines for $\mu < 0$. Solid, dashed and dotted lines show limits for lighter stop and chargino masses $M_{\tilde{t}_1} = m_{C_1^\pm} = 90, 150$ and 300 GeV, respectively.

The existing measurement of $BR(b \to s\gamma)$ imposes already significant constraints on the MSSM parameter space. To understand them, it is important to remember that the charged Higgs contribution to the $b \to s\gamma$ amplitude has always the same sign as the SM one whereas the chargino-stop contribution to this amplitude may have opposite sign. Since the actually measured value of $BR(b \to s\gamma)$ is close to the SM prediction, SUSY and charged Higgs contributions must either be small by themselves or cancel each other to a large extent. There exists, however, a third possibility where negative chargino-stop contribution overcomes the SM and charged Higgs ones yielding the correct absolute magnitude of the total amplitude but with the opposite sign compared to the SM case. In particular, it is worth stressing that the supersymmetric contribution, coming from a light chargino and stop, can provide a natural mechanism for lowering the $b \to s\gamma$ rate compared to the SM value, in agreement with the trend seen in the present data.\textsuperscript{46,23}

Another important observation is that, large chargino-stop contribution to $b \to s\gamma$ amplitude arise when the chargino is higgsino-like rather then gaugino-
like i.e. when $M_2/|\mu| > 1$. In addition, the size of the chargino-stop contribution can be modified by changing the stop mixing angle $\theta_t$.

![Graph](image)

Figure 7: Bounds on $(\tilde{M}_1, m_{C^+})$ plane for $\tan \beta = 1.8$ and $M_A = 100$ and 200 GeV. Thick lines show limits for $\mu > 0$, thin lines for $\mu < 0$. Dotted, dashed and solid lines show limits for $M_2/|\mu| = 0.1, 1$ and 10, respectively.

Fig. 6b (taken from ref. 42) shows the lower limit on the allowed pseudoscalar Higgs boson mass $M_A$ as a function of $r = M_2/|\mu|$ for three different values of the lighter chargino and lighter stop masses. In Fig. 7, the limits on lighter chargino and lighter stop mass for chosen $M_A$ and $M_2/\mu$ values is plotted. In both plots a scan over $\theta_t$ in the range $-60^\circ < \theta_t < 60^\circ$ has been performed.

Fig. 6b shows that for small $M_2/|\mu|$, i.e. for gaugino-like lighter chargino (when the chargino-stop contribution to $BR(b \rightarrow s\gamma)$ is suppressed) the resulting limits on $M_A$ are quite strong even for very light chargino and stop e.g. $M_A \geq \mathcal{O}(200 \text{ GeV})$ for $M_{\tilde{t}} = m_{C^+} = 90 \text{ GeV}$ (we take 95% errors of CLEO measurement). The limits decrease when $M_2/|\mu|$ increases and approximately saturate for $M_2/|\mu| \geq 1$. Similar effects are visible in Fig. 7 where upper bounds on $M_{\tilde{t}}$ are shown as a function of the lighter chargino mass $m_{C^+}$). For small $M_2/|\mu|$, very light stop and chargino are necessary to cancel the charged Higgs contribution. Thus, the corresponding upper limits on their masses are very strong. For large $M_2/|\mu|$, chargino and stop even 2-3 times heavier than the charged Higgs are allowed.
Large effects for $\tan \beta \sim m_t/m_b$

Large values of $\tan \beta$, $\tan \beta \sim m_t/m_b$, have been frequently advocated in the literature as a possible dynamical explanation of the large top to bottom quark mass ratio. In this scenario the Yukawa couplings of the down-type quarks and leptons are enhanced leading in some cases to large loop effects. One example of this kind is the already discussed large contribution of the chargino-sneutrino loop to $g - 2$ of the muon.

Particularly large in this regime are the chargino-stop corrections to the $b \to s\gamma$ amplitude. Indeed, in the limit of higgsino-like lighter chargino we get

$$C_{\tilde{C}_i \tilde{t}_1} (M_W) \approx -\frac{m_i^2}{2m_{\tilde{C}_1}} \cos^2 \theta_{\tilde{t}_i} f_\gamma^{(1)} (x) \pm \tan \beta \frac{m_t}{2m_{\tilde{C}_1}} \sin \theta_{\tilde{t}_i} f_\gamma^{(3)} (x)$$  \hfill (17)

where $x \equiv (M_{\tilde{t}_i}/m_{\tilde{C}_1})^2$, functions $f_\gamma^{(i)} (x)$ are defined in the second paper of ref. 46 and the sign in the second term is the same as the sign of the $\mu$ parameter.

It is clear that for large $\tan \beta$, $m_{\tilde{C}_1} \sim M_W$ and the stop mixing angle not too small the second term dominates and is much larger than the SM $W^\pm$-$t$ contribution

$$C_{\tilde{t}_W} (M_W) = \frac{3}{2} \frac{m_t^2}{M_W^2} f_\gamma^{(1)} \left( \frac{m_t^2}{M_W^2} \right)$$  \hfill (18)

Moreover, the higgsino-like chargino-stop contribution vanishes only as $1/m_{\tilde{C}_1}$ and remains non negligible up to relatively large $m_{\tilde{C}_1}$. For the gaugino-like higgsino, instead, we get:

$$C_{\tilde{C}_i \tilde{t}_1} (M_W) \approx -\frac{M_W^2}{m_{\tilde{C}_1}} \cos^2 \theta_{\tilde{t}_i} f_\gamma^{(1)} (x)$$  \hfill (19)

which is much smaller and vanishes as $1/m_{\tilde{C}_1}^2$. Large contribution from light higgsino-like chargino and stop can be cancelled by the charged Higgs boson loop. However, this requires a high degree of fine-tuning. This is illustrated in Fig. 8 where for fixed chargino parameters and charged Higgs boson mass the allowed region in the plane $(\theta_{\tilde{t}_i}, M_{\tilde{t}_i})$ consists of two very narrow bands (corresponding to two different signs of the total amplitude). Outside these bands the generic prediction for the $BR (b \to s\gamma)$ is one-two orders of magnitude larger than the value measured by CLEO.

For similar reasons, the existence of very light, $O(M_Z)$, pseudoscalar and charged Higgs bosons (and consequently significant enhancement of $R_b$) in the large $\tan \beta$ regime is rather unlikely. Indeed, charged Higgs contribution by itself would then give too high a rate for $b \to s\gamma$. It can be compensated
by the chargino-stop loop but, again, at the expense of strong fine-tuning (due to the tan $\beta$ enhancement factor present in eq. (17)) Nothing, of course, prevents the existence of light gaugino-like chargino and stop in the large tan $\beta$ regime. For heavy enough pseudoscalar $A$ and chargino or stop (or both) the contribution of eq. (17) may have, however, interesting consequences. Indeed, for negative values of $A_t \times \mu$ the rate can be easily smaller than the one of the SM in agreement with the trend of the data. (This scenario can be realized in supergravity models with non-universal soft terms and in the gauge mediated models with $B = 0$).

Another class of interesting effects in the large tan $\beta$ regime originate from finite corrections to the down-type quark and lepton Yukawa couplings. For sparticle masses $\gtrsim M_Z$ these effects can be concisely described by the effective lagrangian describing Yukawa interactions arising after integrating out (heavy) sparticles. At the tree level, terms with $Y_{2ab}^d$ and $Y_{1ab}^u$ are absent in the MSSM (and in the SM). They are, however, generated by triangle diagrams (with helicity flips on fermion lines)
with squarks and either gauginos or higgsinos circulating in loops\textsuperscript{k}. Most interesting effects are due to the new term $Y_{2ab}$ which reads

\[
Y_{2ab} = Y_{1ae} \left( \delta_{eb} \frac{2}{3} \frac{\alpha_s}{\pi} \mu m^2 \right) M^2_{q_L} M^2_{d_R} + \frac{1}{(4\pi)^2} \frac{1}{2} \left( Y_{2}^{u} Y_{2}^{u} \right)_{eb} A_{\tilde{g}} I(M^2_{q_L}, M^2_{d_R}, \mu^2) + \text{smaller terms} \tag{21}
\]

where $A_{\tilde{g}}$ are the trilinear soft supersymmetry breaking terms (for simplicity we assume that the soft SUSY breaking matrices $M^2_{q_L}$, $M^2_{d_R}$ and $A_{\tilde{g}}$ are all proportional to the unit matrix) and

\[
I(x, y, z) = \frac{xy \log(x/y) + yz \log(y/z) + zx \log(z/x)}{(x - y)(y - z)(z - x)}
\]

The presence of $Y_{2ab}$ modifies the value of $Y_{1ab}$ and (neglecting nondiagonal terms in Yukawa couplings) we get

\[
Y_b = Y_{1bb} = \frac{e m_b}{\sqrt{2} \sqrt{1 + \tan^2 \beta}} \frac{\sqrt{1 + \tan^2 \beta}}{1 + \tan \beta \Delta_{bb}} \tag{22}
\]

It is clear that sizeable effects appear for large values of $\tan \beta$ and light sparticles involved. Moreover, these effects vanish only as $1/M_{soft}$ and persist therefore even for relatively heavy sparticles. The corrections affect all processes where the bottom quark Yukawa coupling is involved. In particular, they modify the well known limits on the $(\tan \beta, M_{H^\pm})$ plane\textsuperscript{59} derived from the experimental result for $b \to c \tau \bar{\nu}$\textsuperscript{60}. Qualitatively, with these corrections included, $Y_b$ becomes larger for $\mu < 0$ and enhances the contribution of the $H^+$ Higgs boson to the process $b \to c \tau \bar{\nu}$, strengthening thus the limits on the $(\tan \beta, M_{H^\pm})$ plane. For $\mu > 0$, instead, the bound is weakened and can even disappear. For detailed discussion, see the ref.\textsuperscript{60}.

The same effective lagrangian (20) describes also the dominant part of the (large) supersymmetric corrections to the processes $H^+ \to t b$\textsuperscript{61} and $t \to H^+ b$ which in the case of large $\tan \beta$ and sufficiently light $H^+$ (e.g. when $R_b$ is enhanced) competes with the standard decay $t \to W^+ b$\textsuperscript{62}.

These corrections, interpreted as supersymmetric threshold corrections, are also very important in context of the GUT models and unification of the

\textsuperscript{k}These diagrams are finite due to the so-called non-renormalization theorems which in the case of unbroken supersymmetry would also force these corrections to vanish.
Yukawa couplings. In particular, they significantly lower the values of $\tan \beta$ and $m_t$ predicted from the bottom-tau Yukawa coupling unification (for details see ref. 56, 63) and also, when their full generation dependence is taken into account, they significantly modify the naive predictions of the GUT models for CKM mixing angles 66.

7 Summary

There is an apparent contradiction between the hierarchy problem (which suggest new physics to be close to the electroweak scale) and the striking success of the Standard Model in describing the electroweak data. The supersymmetric extension of the SM offers an interesting solution to this puzzle. The bulk of the electroweak data is well screened from supersymmetric loop effects, due to the structure of the theory, even with superpartners generically light, $\mathcal{O}(M_Z)$. The only exception are the left-handed squarks of the third generation which have to be $\lesssim \mathcal{O}(300 \text{ GeV})$ to maintain the success of the SM. The other superpartners can still be light, at their present experimental mass limits, and would manifest themselves through virtual corrections to the small number of observables such as $R_b$, $b \to s\gamma$, $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing and a few more for large $\tan \beta$. Those effects are very interesting but require still higher experimental precision to be detectable.

Our goal here was to study unconstrained minimal supersymmetric model, with arbitrary soft supersymmetry breaking parameters. Under stronger assumption, e.g. of universal soft terms at the GUT scale 67, 56, 63, one can get from virtual effects stronger constraints on the superpartner spectrum (for recent studies see e.g. refs. 68).

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