Factorization and Non-factorization in Diffractive Hard Scattering

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Abstract. Factorization, in the sense defined for inclusive hard scattering, is discussed for diffractive hard scattering. A factorization theorem similar to its inclusive counterpart is presented for diffractive DIS. For hadron-hadron diffractive hard scattering, in contrast to its inclusive counterpart, the expected breakdown of factorization is discussed. Cross section estimates are given from a simple field theory model for non-factorizing double-pomeron-exchange (DPE) dijet production with and without account for Sudakov suppression.

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In the general phenomena of diffractive hard scattering, the initial proton in DIS or both protons at hadron collider participate in a hard process involving a very large momentum transfer, but one or at hadron colliders one or both hadrons is diffractionally scattered, emerging with a small transverse momentum and the loss of a rather small fraction of longitudinal momentum. In this talk I will discuss the factorization theorem of diffractive DIS and its expected breakdown in hadron-hadron initiated diffractive hard processes.

Following [1], I will formulate the hypothesis of diffractive factorization in two step. In the first stage we hypothesize that the diffractive structure function $F_2^{\text{diff}}$ can be written in terms of a diffractive parton distribution:

$$
\frac{dF_2^{\text{diff}}(\beta x_{IP}, Q^2; x_{IP}, t)}{dx_{IP} dt} = x_{IP} \sum_a \int_\beta^1 d\beta' \frac{df_{a/A}^{\text{diff}}(\beta' x_{IP}, \mu; x_{IP}, t)}{dx_{IP} dt} \hat{F}_{2,a}(\beta/\beta', Q^2; \mu),
$$

(1)

where $\hat{F}_2$ is the same function which is convoluted with the inclusive parton densities to compute $F_2$ of inclusive DIS. If for simplicity, we ignore $Z$ exchange, then $\hat{F}_{2,a}(\beta/\beta', Q^2; \mu) = e_a^2 \delta(1 - \beta/\beta') + O(\alpha_s)$. 

In the second stage, we hypothesize that the diffractive parton distribution function has a particular form:

\[
\frac{df^{\text{diff}}_{a/A}(\beta x_{IP}, \mu; x_{IP}, t)}{dx_{IP} \, dt} = \frac{1}{8\pi^2} |\beta_A(t)|^2 x_{IP}^{-2\alpha(t)} f_{a/IP}(\beta, t, \mu). \tag{2}
\]

Here \(\beta_A(t)\) is the pomeron coupling to hadron A and \(\alpha(t)\) is the pomeron trajectory. The function \(f_{a/IP}(\beta, t, \mu)\) defined above is the “distribution of partons in the pomeron”. I distinguish the “diffractive factorization” of Eq. (1) from the “Regge factorization” of Eq. (2). The latter is a special case of the former. The Ingelman-Schlein model \([2]\) is synonymous with “Regge factorization”.

The structure function \(F_2^{\text{diff}}(\beta x_{IP}, Q^2; x_{IP}, t)\) for the IS-model is obtained by inserting Eq. (2) into (1). An inconsistency of data to the IS-model does not also imply an inconsistency to diffractive factorization.

I now give operator definitions of the diffractive parton distribution. The diffractive distribution of a quark of type \(j\) in terms of field operators \(\tilde{\psi}(y^+, y^-, y)\) evaluated at \(y^+ = 0, y^- = 0\) is:

\[
\frac{df^{\text{diff}}_{a/A}(\beta x_{IP}, \mu; x_{IP}, t)}{dx_{IP} \, dt} = \frac{1}{64\pi^3} \frac{1}{2} \sum_{s_A} \int dy^- e^{-i\beta x_{IP} P_{A}\cdot y^-} \sum_{X,s_{A'}} \langle P_A, s_A | \tilde{\psi}_j(0, y^-, 0) | P_{A'}, s_{A'}; X \rangle \gamma^+ \langle P_{A'}, s_{A'}; X | \tilde{\psi}_j(0) | P_A, s_A \rangle. \tag{3}
\]

We sum over the spin \(s_{A'}\) of the final state proton and over the states \(X\) of any other particles that may accompany it. Similarly, the diffractive distribution of gluons in a hadron is

\[
\frac{df^{\text{diff}}_{a/A}(\beta x_{IP}, \mu; x_{IP}, t)}{dx_{IP} \, dt} = \frac{1}{32\pi^3} \frac{1}{2} \sum_{s_A} \int dy^- e^{-i\beta x_{IP} P_{A}\cdot y^-} \sum_{X,s_{A'}} \langle P_A, s_A | \tilde{F}_a(0, y^-, 0)^{+\nu} | P_{A'}, s_{A'}; X \rangle \langle P_{A'}, s_{A'}; X | \tilde{F}_a(0)^{+\nu} | P_A, s_A \rangle. \tag{4}
\]

The proton state \(|P_A, s_A\rangle\) has spin \(s_A\) and momentum \(P_A^\mu = (P_A^+, M_A^2/[2P_A^+], 0)\). We average over the spin. Our states are normalized to \(|k|p\rangle = (2\pi)^3 2p^+ \delta(p^+ - k^+) \delta^3(p - k)\). The tilde on the fields \(\tilde{\psi}_j(0, y^-, 0)\) and \(\tilde{F}_a(0, y^-, 0)^{+\nu}\) is to imply that they are multiplied by an exponential of a line integral of the vector potential as shown in [1].

The diffractive parton distributions are ultraviolet divergent and require renormalization. It is convenient to perform the renormalization using the \(\overline{\text{MS}}\) prescription, as discussed in [3,4]. This introduces a renormalization scale \(\mu\) into the functions. In applications, one sets \(\mu\) to be the same order of magnitude as the hard scale of the physical process.

The renormalization involves ultraviolet divergent subgraphs. Subgraphs with more than two external parton legs carrying physical polarization do not
have an overall divergence. Thus the divergent subgraphs are the same as for the ordinary parton distributions. We conclude that the renormalization group equation for the diffractive parton distributions is

\[
\frac{d}{d\mu} \frac{df_{a/A}^{\text{diff}}(\beta x, \mu; x, t)}{dx dt} = \sum_b \int_{\beta x}^1 \frac{dz}{z} P_{a/b}(\beta x, \mu; z, \alpha_s(\mu)) \frac{df_{b/A}^{\text{diff}}(z, \mu; x, t)}{dx dt}
\]

(5)

with the same DGLAP kernel [5], \( P_{a/b}(\beta x, \mu; z, \alpha_s(\mu)) \), as one uses for the evolution of ordinary parton distribution functions.

The diffractive parton distribution \( df_{a/A}^{\text{diff}}(\beta x, \mu; x, t)/dx dt \), like the ordinary parton distribution, is essentially not calculable using perturbative methods. Recall, however, that it is possible to derive “constituent counting rules” that give predictions for ordinary parton distributions \( f_{a/A}(x, \mu) \) in the limit \( x \to 1 \) for not too large values of the scale parameter \( \mu \) in the sense of the analysis by Brodsky and Farrar [6]. In the same spirit, in [1] we have considered the diffractive parton distributions in the limit \( \beta \to 1 \).

We find that the diffractive gluon distribution behaves as \((1 - \beta)^p\) for \( \beta \to 1 \) at moderate values of the scale \( \mu \), say 2 GeV, with \( 0 \leq p \leq 1 \). The choice \( p \approx 0 \) corresponds to an effectively massless final state gluon, while \( p \approx 1 \) corresponds to an effective gluon mass. For the diffractive quark distribution we find they behave as \((1 - \beta)^2\). However, suppose that we interpret the calculation as saying that the diffractive distribution of gluons is proportional to \((1 - \beta)^0\) for \( \beta \) near 1 when the scale \( \mu \) is not too large. Then the evolution equation for the diffractive parton distributions will give a quark distribution that behaves like

\[
\frac{df_{q/A}^{\text{diff}}(\beta x, \mu; x, t)}{dx dt} \propto (1 - \beta)^1,
\]

(6)

when the scale \( \mu \) is large enough that some gluon to quark evolution has occurred, but not so large that effective power \( p \) in \((1 - \beta)^p\) for the gluon distribution has evolved substantially from \( p = 0 \). A signature of this phenomenon is that the diffractive quark distribution will be growing as \( \mu \) increases at large \( \beta \), rather than shrinking. Perhaps this is seen in the data [7].

I will now turn to diffractive hard scattering in hadron-hadron collisions. There is an especially important difference in these processes to their counterparts in the inclusive case. For the latter the leading twist cross section can be expressed as a product of parton distribution functions, one for each hadron, times the hard partonic cross section. Furthermore the parton distribution functions for the hadrons are the same as those for inclusive DIS. This is what we have understood as factorization in inclusive hard processes. In the diffractive case factorization is expected to breakdown [8,10,11] when both impinging particles can interact strongly. It is not expected that the diffractive parton
distributions of diffractive DIS should correctly predict diffractive hard cross sections at hadron collider nor does that appear to be true [9]. Understanding the origin of non-factorization challenges our theoretical knowledge of strong interactions and in a fruitful direction since the effect is experimentally measurable.

We have been studying non-factorization in a particular model for double pomeron exchange (DPE) dijet production [11]. For the remainder of this talk, I will report on that work. The reaction of interest is

\[ A + B \rightarrow A' + B' + 2 \text{jets}, \]

(7)

where hadrons \( A \) and \( B \) lose tiny fractions \( x_a \) and \( x_b \) of their respective longitudinal momenta, and they acquire transverse momenta \( Q_1 \) and \( Q_2 \). Such events are called hard double-pomeron exchange (DPE) events because both incoming hadrons survive unscathed with a hard process in the central region of final-state rapidity. The process (7) has a quite dramatic signature: the final state consists of the two diffracted hadrons, two high-\( E_T \) jets, and nothing else.

I present results from our work [11], that applied the CFS mechanism [8] in a simple field theory model to compute the DPE jet cross section with the lowest order Feynman graphs that are appropriate. Although numerical estimates from our model in its base form are crude, the results of our calculation establish that the exclusive processes of DPE to jets is leading twist and non-factorizing. This is quite non-trivial, since firstly, there are in fact several two-jet emission graphs of which only certain survive and secondly, some of our graphs are a power law larger than the final answer. The proof of the necessary cancelation relies on Ward identities and power counting [11]. To show that general principles do not imply some other cancelation, it is important to have a complete, consistent and gauge-invariant model, which ours provides.

One modification to our base model in [11] is to treat Sudakov suppression [12,13]. This effect arises from soft and collinear gluon emission before the parton-parton hard vertex. In an inclusive hard process, this effect is not present due to cancelation of appropriate real and virtual emissions. However the diffractive constraint prohibits real emissions along the direction of the diffractive final state hadron. I have computed the DPE dijet cross section with and without the Sudakov suppression factor and the results are in table 1. The kinematic limits I used were \( E_T > 5 \text{GeV}, 0 < x_a, x_b < 0.05 \) and I have integrated over jet rapidities \( y_-, y_+ \) consistent with these cuts.

One can compare the results in table 1 to (a) the inclusive two-jet cross section (i.e., without a diffractive requirement: \( A + B \rightarrow \text{jet} + \text{jet} + X \)), and (b) the result of applying the Ingelman-Schlein model to DPE [2,11], which gives a result for the process \( A + B \rightarrow A' + B' + \text{jet} + \text{jet} + X \). This process we call factorized double-pomeron-exchange (FDPE).
\[ \sqrt{s} \quad \sigma^{\text{NDPE}}_{\text{dijet}}(E_T^{\text{min}} = 5.0) \]

<table>
<thead>
<tr>
<th>(GeV)</th>
<th>without Sudakov</th>
<th>with Sudakov</th>
</tr>
</thead>
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<tr>
<td>630</td>
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<td>0.024</td>
</tr>
<tr>
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<td>0.086</td>
</tr>
<tr>
<td>14000</td>
<td>0.65</td>
<td>0.31</td>
</tr>
</tbody>
</table>

**TABLE 1.** Table 1: Non-factorizing Double Pomeron dijet cross section with and without one loop Sudakov suppression with \( E_T^{\text{min}} = 5 \text{GeV} \).

For this I find the total cross sections integrated over \( y_+, y_- \) with the same \( x_a, x_b \) cuts and for \( E_T > 5.0 \text{ GeV} \) are, at \( \sqrt{s} = 1800 \text{ GeV}, \sigma_{\text{incl}}(1800,5) = 2.4 \text{ mb, } \sigma_{\text{FDPE}}(1800,5) = 0.0022 \text{ mb, and at } \sqrt{s} = 630 \text{ GeV, } \sigma_{\text{incl}}(630,5) = 0.31 \text{ mb, } \sigma_{\text{FDPE}}(630,5) = 0.000062 \text{ mb.} \)

There is no experimental cross sections yet reported for the DPE dijet process. However the preliminary CDF/D0 results [14,15] suggest that our cross section estimates are too high perhaps even by a couple of orders of magnitude. We have not as yet treated absorptive corrections in our calculation. It remains to be seen how much of a suppression they will give. It also remains to be seen what will be the experimentalists final results. Thus a test of our model still awaits further developments.

**REFERENCES**

15. A. Brandt (D0 Collaboration), these proceedings.