Chiral Symmetry and the Nucleon Structure Functions

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Abstract : The flavor asymmetry of the sea quark distribution as well as the unexpectedly small quark spin fraction of the nucleon are two outstanding discoveries recently made in the physics of deep-inelastic structure functions. We evaluate here the corresponding quark distribution functions within the framework of the chiral quark soliton model, which is an effective quark model of baryons maximally incorporating the most important feature of low energy QCD, i.e. the chiral symmetry and its spontaneous breakdown. It is shown that the model can explain qualitative features of the above-mentioned nucleon structure functions within a single framework, thereby disclosing the importance of chiral symmetry in the physics of high energy deep-inelastic scatterings.
1 Introduction

Though it is certainly true that high energy deep-inelastic scatterings provide us with the best testing ground for perturbative QCD, the physics behind it is highly nonperturbative. In fact, the perturbative QCD can describe only the evolution of structure functions from one energy scale to the other, so that for determining structure functions themselves one must retreat to a semi-theoretical fitting procedure of empirical cross sections at a certain scale [1]. In recent years, two epoch-making findings have been made concerning the nucleon structure functions. The first concerns the EMC measurement of the spin structure of the proton through deep-inelastic scatterings of polarized muons on polarized proton, which sometimes has been advertised as the so-called “nucleon spin crisis” [2]. Another interesting observation made by the NMC group is the flavor asymmetry of the sea quark distributions in the nucleon [3]. As for this second observation, a widely-accepted explanation is based on the pion cloud effects, which may in turn be interpreted as a manifestation of chiral symmetry of QCD in high energy observables [4-7]. The first observation, i.e. the nucleon spin problem is a little more mysterious than the second. The chiral symmetry may play an important role also here [8], or it may be explained by some other nonperturbative effects like gluon polarization dictated by the $U_A(1)$ anomaly of QCD [9,10].

At any rate, these two remarkable findings have reminded us of a quite plain fact: we absolutely need some theoretical device with which we can evaluate structure functions in a nonperturbative way. Unfortunately, nonperturbative treatment of QCD is quite involved. Then, lattice gauge theory is widely believed to be the most promising tool for it [11]. In the long run, it may be true. Confining to the present status, however, it suffers from quite a few obstacles mainly arising from the limitation of the computer ability. Among others, what seems most serious to us is the use of the so-called “quenched approximation”, the validity of which is strongly suspected when applied to the physics of light quark sector that we are just interested in. At least, it seems very unlikely that a reliable estimation of sea quark distributions is feasible under this approximation. Here is a place where effective theories of QCD can play unique and important roles. Naturally, there exist quite a few effective models of baryons, and some of them were already applied for evaluating nucleon structure functions [12-14]. Here, we emphasize one absolute superiority of the chiral quark soliton model (CQSM) over the other, i.e. its ability to solve the nucleon bound state problem nonperturbatively with full inclusion of the deformed Dirac sea quarks in addition to three valence quarks [15-17]. Undoubtedly, this must be a unique advantage not shared by other models of baryons when discussing the physics of the Gottfried sum [6].

Several group have already attempted to calculate nucleon structure functions within the CQSM or the NJL soliton model. For instance, Weigel et al. evaluated the isovector unpolarized distribution functions as well as some other distribution functions [18]. However, their calculation is incomplete in respect that the effect of afore-mentioned Dirac sea quarks
are totally neglected. On the other hand, both of Diakonov et al. [19,20] and Tanikawa and Saito [21] carried out more consistent calculation including the deformed Dirac sea quarks, but by confining themselves to the isosinglet unpolarized and isovector polarized distribution functions, which have values at the leading order of $1/N_c$ expansion (or the expansion in the collective rotational velocity $\Omega$ of the hedgehog soliton). Unfortunately, the isovector unpolarized and isoscalar polarized distribution functions both vanish at this order. Then, for discussing interesting physics like the Gottfried sum or the quark spin content of the nucleon, one must go ahead to the next-to-leading order contribution in $\Omega$, which turns out to be quite an involved task. The present paper is the first report of such an investigation.

The plan of the paper is as follows. In sect.2, we shall derive general formulas for evaluating quark distribution functions of the nucleon on the basis of the path-integral formulation of the chiral quark soliton model (CQSM). The results of the numerical calculation are given and analyzed in sect.3. We then summarize our results in sect.4.

## 2 Theory of quark distribution functions

We start with the light-cone expression for the quark distribution function [12-14] :

$$q(x) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dz^\perp e^{-ixP^+z^-} \langle N(P) | \bar{\psi}(z) \Gamma \psi(0) | N(P) \rangle |_{z^+=0, z_\perp=0},$$

with the definition of the standard light-cone coordinates $z^\pm = (z^0 \pm z^3)/\sqrt{2}$. The nucleon state is normalized here as

$$\langle N(P') | N(P) \rangle = 2P^0 \delta^3(P' - P),$$

with $P$ the nucleon 4-momentum, while $x = -q^2/(2P \cdot q)$ is the Bjorken variable with $q$ the 4-momentum transfer to the nucleon. We are to take $\Gamma = \gamma^+ \gamma_3$ with $\gamma^+ = (\gamma^0 + \gamma^3)/\sqrt{2}$ for the isovector unpolarized distribution function $u(x) - d(x)$, while $\Gamma = \gamma^+ \gamma_5$ for the isosinglet polarized one $\Delta u(x) + \Delta d(x)$. We recall the fact that, extending the definition of distribution function $q(x)$ to the interval $-1 \leq x \leq 1$, the relevant antiquark distributions are given as [19]

$$\bar{u}(x) - \bar{d}(x) = -[u(-x) - d(-x)],$$

$$\Delta \bar{u}(x) + \Delta \bar{d}(x) = \Delta u(-x) + \Delta d(-x),$$

with $0 \leq x \leq 1$. Taking the nucleon at rest $P^n = (M_N, 0)$, we have $P^+ = M_N/\sqrt{2}$ and then

$$q(x) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dz^0 e^{-ixM_Nz^0} \langle N(P) | \psi^+(z) \gamma^0 \Gamma \psi(0) | N(P) \rangle |_{z^3=-z^0, z_\perp=0}.$$
\[ \langle N(P) | \psi^\dagger(z) O \psi(0) | N(P) \rangle = \frac{1}{Z} \int d^3x \ d^3y \ e^{-iP \cdot x} e^{iP \cdot y} \int DU \times \int \mathcal{D} \psi \mathcal{D} \psi^\dagger J_N \left( \frac{T}{2}, x \right) \psi^\dagger(z) O \psi(0) J_N^\dagger \left( -\frac{T}{2}, y \right) \exp \left[ i \int d^4x \bar{\psi} \left( i \not\partial - MU^{75} \right) \psi \right], \tag{6} \]

where

\[ \mathcal{L} = \bar{\psi} \left( i \not\partial - MU^{75}(x) \right) \psi, \tag{7} \]

with \( U^{75}(x) = \exp \left[ i \gamma_5 \mathbf{r} \cdot \mathbf{\pi}(x)/f_\pi \right] \) is the basic lagrangian of the CQSM, and

\[ J_N(x) = \frac{1}{N_c!} \epsilon_{\alpha_1 \ldots \alpha_{N_c}} \Gamma_{J J_3 T T_3}^{f_1 \ldots f_{N_c}} \psi_{\alpha_1 f_1(x)} \ldots \psi_{\alpha_{N_c} f_{N_c}(x)}, \tag{8} \]

is a composite operator carrying the quantum numbers \( J J_3, T T_3 \) (spin, isospin) of the nucleon, where \( \alpha_i \) is the color index, while \( \Gamma_{J J_3 T T_3}^{f_1 \ldots f_{N_c}} \) is a symmetric matrix in spin-flavor indices \( f_i \). By starting with a stationary pion field configuration of hedgehog shape \( U^{75}_0(x) = \exp \left[ i \gamma_5 \mathbf{r} \cdot \mathbf{F}(r) \right] \), the path integral over the pion fields \( U \) can be done in a saddle point approximation. Next, we consider two important fluctuations around the static configuration, i.e. the translational and rotational zero-modes. To treat the translational zero-modes, we use an approximate momentum projection procedure of the nucleon state, which amounts to integrating over all shift \( R \) of the soliton center-of-mass coordinates [24,19] :

\[ \langle N(P') | \psi^\dagger(z) O \psi(0) | N(P) \rangle \rightarrow \int d^3R \ \langle N(P') | \psi^\dagger(z_0, z - R) O \psi(0, -R) | N(P) \rangle. \tag{9} \]

The rotational zero-modes can be treated by introducing a rotating meson field of the form :

\[ U^{75}(x,t) = A(t) U^{75}_0(x) A^\dagger(t), \tag{10} \]

where \( A(t) \) is a time-dependent \( SU(2) \) matrix in the isospin space. Note first the identity

\[ \bar{\psi} \left( i \not\partial - MA(t) U^{75}_0(x) A^\dagger(t) \right) \psi = \psi^\dagger_A \left( i \not\partial - H - \Omega \right) \psi_A \tag{11} \]

with

\[ \psi_A = A^\dagger(t) \psi, \quad H = \frac{\alpha \cdot \nabla}{i} + M \beta U^{75}_0(x), \quad \Omega = -i A^\dagger(t) \dot{A}(t). \tag{12} \]

Here \( H \) is a static Dirac Hamiltonian with the background pion fields \( U^{75}_0(x) \), playing the role of a mean field for quarks, while \( \Omega = \frac{i}{2} \Omega_a \tau_a \) is the \( SU(2) \)-valued angular velocity matrix later to be quantized as \( \Omega_a \rightarrow \hat{J}_a/I \) with \( I \) the moment of inertia of the soliton and \( \hat{J}_a \) the angular momentum operator. We then introduce a change of quark field variables \( \psi \rightarrow \psi_A \), which
amounts to getting on a body-fixed rotating frame. Denoting $\psi_A$ anew as $\psi$ for notational simplicity, the nucleon matrix element (6) can then be written as

$$\langle N(P) | \psi(z) O \psi(0) | N(P) \rangle = \frac{1}{Z} \Gamma^{(f)} \Gamma^{(g)*} \int d^3x \ d^3y \ e^{-i P \cdot x + i P \cdot y} \int d^3R \times \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp \left[ i \int d^3x \ \psi^\dagger (i\partial_t - H - \Omega) \psi \right] \times \prod_{i=1}^{N_c} \left[ A(T \frac{1}{2}) \psi_i(T \frac{1}{2}, x) \right] \psi^\dagger(z_0, z - R) \tilde{O} \psi(0, -R) \prod_{j=1}^{N_c} \left[ \psi^\dagger_j(-T \frac{1}{2}, y) A^\dagger(-T \frac{1}{2}) \right], \ (13)$$

with the definition $\tilde{O} \equiv A^\dagger(z_0) O A(0)$. Now performing the path integral over the quark fields, we obtain

$$\langle N(P) | \psi(z) O \psi(0) | N(P) \rangle = \frac{1}{Z} \tilde{\Gamma}^{(f)} \tilde{\Gamma}^{(g)} \prod_{i=1}^{N_c} \left( \frac{T}{2} \right) \times \left[ f_1 \left( \frac{T}{2}, x \right) \frac{i}{i\partial_t - H - \Omega} \left| z_0, z - R \right\rangle_\gamma \cdot (\tilde{O})_{\gamma\delta} \cdot \delta(0, -R) \frac{i}{i\partial_t - H - \Omega} \left| -T \frac{1}{2}, y \right\rangle_{g_1} \right] - \frac{1}{2} \int \frac{\Omega^2_a}{4} \, dt . \tag{14}$$

The second term here is essentially the action of a rigid rotor, which plays the role of the evolution operator in the space of collective coordinates. Using the expansion of the single quark propagator as

$$\langle z_0, z - R \rangle_\gamma = f_1 \left( \frac{T}{2}, x \right) \frac{i}{i\partial_t - H} \left| z_0, z - R \right\rangle_\gamma - \int d\tilde{z}_0' \int d\tilde{z}' \ f_1 \left( \frac{T}{2}, x \right) \frac{i}{i\partial_t - H} \left| \tilde{z}_0', \tilde{z}' \right\rangle_\alpha \cdot \Omega_{\alpha\beta}(z_0) \cdot \beta(\tilde{z}_0', \tilde{z}') \frac{i}{i\partial_t - H} \left| z_0, z - R \right\rangle_\gamma + \ldots, \tag{16}$$

we can separate the zeroth order ($\sim N^0_c$) and the first order ($\sim 1/N_c$) corrections in $\Omega$ for nucleon observables. Here we are interested in the linear order term in $\Omega$, because the isovector unpolarized and isoscalar polarized distribution functions both vanish at the lowest order.
Since necessary manipulations were explained in the previous papers [22,23], we shall skip the detail except what seems absolutely necessary. The only but important difference with the previous case is that we are here handling a nucleon matrix element of a quark bilinear operator which is nonlocal both in space and time coordinates. This is only natural, because we are investigating here a quark-quark correlation function with a light-cone separation. First by neglecting the time-ordering of two operators \( \Omega \) and \( \tilde{O} \), the \( O(\Omega^1) \) contribution to (14) would become

\[
\langle N(P) | \psi^+(z) \Omega \psi(0) | N(P) \rangle^{\Omega^1} = \frac{1}{Z} \tilde{\Gamma}^{(f)} \tilde{\Gamma}^{(g)^\dagger} N_c \int d^3x \, d^3y \, e^{-iP \cdot x} \int d^3z \, dz' \Omega_{\alpha\beta}(z_0') \tilde{O}_{\gamma\delta}(z_0,0)
\times \left\{ \int_{\frac{T}{2}} \left| \frac{i}{i\partial_t - H} \right| \left| z_0', z' \right| \alpha \cdot \beta \langle z_0', z' | \frac{i}{i\partial_t - H} | z_0, z - R \rangle \delta(0) \frac{i}{i\partial_t - H} | -\frac{T}{2}, y \rangle_{g_1} + \int_{\frac{T}{2}} \left| \frac{i}{i\partial_t - H} \right| \left| z_0', z' \right| \alpha \cdot \beta \langle z_0', z' | \frac{i}{i\partial_t - H} | z_0, z - R \rangle \delta(0) - \frac{i}{i\partial_t - H} | -\frac{T}{2}, y \rangle_{g_1} - \int_{\frac{T}{2}} \left| \frac{i}{i\partial_t - H} \right| \left| z_0', z' \right| \alpha \cdot \beta \langle z_0', z' | \frac{i}{i\partial_t - H} | z_0, z - R \rangle \delta(0) - \frac{i}{i\partial_t - H} | -\frac{T}{2}, y \rangle_{g_1} \right\} \chi \left[ \frac{T}{2} \cdot x \right] \cdot \exp \left[ \text{Sp} \log (i\partial_t - H) + i \frac{T}{2} \int \Omega^2 dt \right], \tag{17} \]

As was extensively argued in [22] and [23], one must be careful about the time order of two operators \( \Omega \) and \( \tilde{O} \), which do not generally commute after collective quantization of the rotational zero-energy modes. In the present case, the proper account of this time-ordering can be achieved by the following replacement in the equation above:

\[
\Omega_{\alpha\beta}(z_0') \tilde{O}_{\gamma\delta}(z_0,0) \rightarrow \left[ \theta(z_0', z_0, 0) + \theta(z_0', 0, z_0) \right] \Omega_{\alpha\beta} \tilde{O}_{\gamma\delta} + \left[ \theta(z_0, 0, z_0') + \theta(0, z_0', z_0) \right] \tilde{O}_{\gamma\delta} \Omega_{\alpha\beta}, \tag{18} \]

where \( \theta(a, b, c) \) is a short-hand notation of a step function which is 1 when \( a > b > c \) and 0 otherwise. Here we have intentionally dropped terms containing a factor \( \theta(z_0, z_0', 0) \) or \( \theta(0, z_0', z_0) \). (Note that, if \( z_0' \) is allowed to lie between \( z_0 \) and 0, the above idea of the time order of the two operators \( \Omega \) and \( \tilde{O} \) would lose its definite meaning.) It amounts to excluding diagrams in which the Coriolis force \( \Omega \) operates in the time interval between \( z_0 \) and 0. This is motivated by the physical picture that a deep inelastic scattering process is a short distance phenomenon and its typical time scale is much shorter than that of the collective rotational motion which we assume is much slower than the velocity of the intrinsic quark motion. Then, one should rather treat \( z_0 \) and 0 as nearly degenerate when performing the expansion in \( \Omega \), which dictates the neglect of diagrams containing \( \theta(z_0, z_0', 0) \) or \( \theta(0, z_0', z_0) \). The exclusion of these special time-order diagrams has an important physical consequence as we shall come back later. Another important remark is that we do not give any constraint on the time order of \( z_0 \) and 0 themselves: \( z_0 \) can be earlier or later than 0. It can be verified that inclusion of both...
possibilities is essential for maintaining the charge conjugation symmetry, which denotes that the quark (antiquark) distributions in the nucleon must be the same as the antiquark (quark) distributions in the antinucleon. The last comment we want to make here is on the treatment of the rotated operator \( \tilde{O} = A^\dagger(z_0) \gamma_0 O A(0) \). If the operator contains an isospin factor (or it is an isovector operator), we are to use the identity

\[
A^\dagger(z_0) \tau_a A(0) = \frac{1}{2} \text{Tr}(A^\dagger(z_0) \tau_a A(0) \tau_b) \tau_b .
\]  

A new feature here is that the factor \( \text{Tr}(A^\dagger(z_0) \tau_a A(0) \tau_b) \) contains two time arguments \( z_0 \) and 0. Still, we can use the usual procedure in which it is replaced by the standard Wigner function \( D_{ab}(A) \). Intuitively, the justification follows from our dynamical assumption that \( z_0 \) and 0 can be regarded as nearly degenerate as compared with the time scale of the collective rotational motion. A formally more rigorous justification for this replacement may be obtained by remembering that the time evolution of the collective space operator \( A \) is given by

\[
A(z_0) = \exp(i z_0 J^2 / I) A(0) \exp(-i z_0 J^2 / I) ,
\]  

with \( J \) the angular momentum operator working in the collective coordinate space and \( I \) the moment of inertia of the soliton. Since the moment of inertia \( I \) is an \( O(N_c) \) quantity, this gives

\[
\frac{1}{2} \text{Tr} ( A^\dagger(z_0) \tau_a A(0) \tau_b ) = D_{ab}(A) [ 1 + O(1/N_c) ] ,
\]  

convincing that the error of the above replacement is a higher order correction in the \( 1/N_c \) expansion.

After stating all the delicacies inherent in the structure function problem, we can now proceed in the same way as [22] and [23]. Using the spectral representation of the single quark Green function

\[
a(x, t) \frac{i}{i\partial_t - H} | x', t' \rangle_{\beta} = \theta(t - t') \sum_{m > 0} e^{-iE_m(t - t')} a(x) | m \rangle \langle m | x' \rangle_{\beta} \\
- \theta(t' - t) \sum_{m < 0} e^{-iE_m(t - t')} a(x) | m \rangle \langle m | x' \rangle_{\beta} ,
\]  

together with the relation

\[
\langle z - R | = \langle -R | e^{iP \cdot z}
\]  

we can perform the integration over \( R, z', \) and \( z'_0 \). The resultant expression is then put into (5) to carry out the integration over \( z_0 \). We then arrive at the formula, which gives the theoretical basis for evaluating the first order contributions in \( \Omega \) to quark distribution functions of the nucleon:

\[
q(x; \Omega^1) = \int dA \, \Psi_{J_3T_3}^{(J)}[A] \, O^{(1)}[A] \, \Psi_{J_3T_3}^{(J)}[A] .
\]  

Here

\[
\Psi_{J_3T_3}^{(J)} = \sqrt{\frac{2J + 1}{8\pi^2}} (-1)^{T_3+T_3} D_{-T_3,T_3}^{(J)}(A) ,
\]
are wave functions, describing the collective rotational motion of the hedgehog soliton, while

\[ O^{(1)}[A] = O^{(1)}_{\text{val}}[A] + O^{(1)}_{\text{v.p.}}[A], \]

where

\[
O^{(1)}_{\text{val}} = M_N \cdot N_c \sum_{m > 0} \frac{1}{E_m - E_0} \langle 0 | \hat{O} \frac{\delta(xM_N - E_0 - p^3) + \delta(xM_N - E_m - p^3)}{2} | m \rangle \langle m | \Omega | 0 \rangle \\
+ \sum_{m < 0} \frac{1}{E_m - E_0} \langle m | \hat{O} \frac{\delta(xM_N - E_0 - p^3) + \delta(xM_N - E_m - p^3)}{2} | 0 \rangle \langle 0 | \Omega | m \rangle \\
+ (\hat{O} \leftrightarrow \Omega),
\]

which contains transitions from occupied single quark levels to unoccupied ones in consistent with the Pauli principle [23]. It is important to recognize that this nice feature would not follow, if one would include the afore-mentioned particular time-order diagrams containing \(\theta(z_0, z'_0, 0)\) and \(\theta(0, z'_0, z_0)\). In fact, if we had included these contributions by forgetting about the time order of \(\Omega\) and \(\hat{O}\), we would have obtained

\[
O^{(1)}_{\text{v.p.}} = M_N \cdot N_c \sum_{m > 0, n < 0} \frac{1}{E_m - E_n} \langle n | \hat{O} \frac{\delta(xM_N - E_n - p^3) + \delta(xM_N - E_m - p^3)}{2} | m \rangle \langle m | \Omega | n \rangle + (\hat{O} \leftrightarrow \Omega),
\]

with \(p^3\) the z-component of the single-quark momentum operator. In the above formulas, \(|m\rangle\) and \(E_m\) denote the eigenstates and associated eigenenergies of the static Dirac Hamiltonian \(H\). In particular, \(|0\rangle\) represents the lowest energy eigenstate of \(H\), which emerges from the positive energy Dirac continuum. This particular state is referred to as the valence quark orbital following the convention of previous studies [16-17]. However, the term “valence quark” here should not be confused with the corresponding term in the language of the quark parton model. The two contributions \(O^{(1)}_{\text{val}}\) and \(O^{(1)}_{\text{v.p.}}\) can actually be put together into the form:

\[
O^{(1)} = M_N \cdot N_c \sum_{m > 0, n \leq 0} \frac{1}{E_m - E_n} \langle n | \hat{O} \frac{\delta(xM_N - E_n - p^3) + \delta(xM_N - E_m - p^3)}{2} | m \rangle \langle m | \Omega | n \rangle + (\hat{O} \leftrightarrow \Omega),
\]

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\[
O^{(1)}_{\text{v.p.}} = M_N \cdot \frac{N_c}{2} \sum_{m, n} \frac{1}{E_m - E_n} \langle n | \hat{O} \frac{\text{sign}(E_m) \delta(xM_N - E_m - p^3) - \text{sign}(E_n) \delta(xM_N - E_n - p^3)}{2} | m \rangle \langle m | \Omega | n \rangle,
\]
instead of (28). This essentially coincides with the equation which is obtained from (4.12) in ref.[19]. Clearly, the formula (30) contains unpleasant Pauli-violating contributions due to transitions between occupied levels themselves and unoccupied ones. It is very important to recognize that this Pauli-violating contributions to a quark distribution function has nothing to do with the regularization problem. (This is clear, since we have not yet introduced any regularization at this stage.) This kind of Pauli-principle violation has never been observed when only the nucleon matrix elements of local quark bilinear operators have been dealt with. In fact, one can confirm it very easily, if one carries out the $x$ integration of (30) from $x = -1$ to $x = 1$, which should give the first moment of the relevant distribution function being expressed as a nucleon matrix element of a local quark bilinear operator. One certainly convinces that terms with $E_m \times E_n > 0$ drop completely.

We also call attention to the fact that the delta-function representing the on-shell energy conservation appears in the symmetrized form with respect to the energies of the single-quark levels $|m\rangle$ and $|n\rangle$. The appearance of these two delta-functions can be traced back to two possible time order of $z_0$ and 0 in $\bar{\psi}(z)\Gamma\psi(0)$, and it plays a favorable role to maintain the charge-conjugation symmetry as already mentioned. We point out that a naive cranking procedure without taking care of the non-local (in time space) nature of the quark bilinear operator would violate this property [18].

Using the general formulae given in (24) $\sim$ (28), we can readily obtain the following expression for the isovector unpolarized distribution function for the proton:

$$u(x) - d(x) = M_N \cdot \frac{1}{I} \cdot \frac{1}{3} \sum_{a=1}^{3} \frac{N_c}{2} \sum_{m>0,n<0} \frac{1}{E_m - E_n} \langle n | \tau_a (1 + \gamma^0 \gamma^3) \frac{\delta(xM_N - E_n - p^3) + \delta(xM_N - E_m - p^3)}{2} | m \rangle \langle m | \tau_3 | n \rangle .$$

(31)

(In the actual numerical calculation, we use the expression in which the contribution of the valence quark level is separated from that of the Dirac continuum according to the general formulas (26) $\sim$ (28), since only the vacuum polarization part is to be regularized in our regularization scheme.)

By integrating $u(x) - d(x)$ over $x$ between $-1$ and $1$, the isospin sum rule follows;

$$\int_{-1}^{1} [u(x) - d(x)] \, dx = \int_{0}^{1} \{ [u(x) - d(x)] - [\bar{u}(x) - \bar{d}(x)] \} \, dx = 1 ,$$

(32)

provided that the moment of inertia is given by

$$I = \frac{N_c}{2} \sum_{m>0,n<0} \frac{1}{E_m - E_n} \langle n | \tau_3 | m \rangle \langle m | \tau_3 | n \rangle .$$

(33)

On the other hand, the Gottfried sum is given as

$$S_G = \frac{1}{3} \int_{0}^{1} [u(x) - d(x) + \bar{u}(x) - \bar{d}(x)] \, dx$$

$$= \frac{1}{3} + \frac{2}{3} \int_{0}^{1} [\bar{u}(x) - \bar{d}(x)] \, dx .$$

(34)
The formula for the isosinglet polarized distribution function can similarly be written down as

\[ \Delta u(x) + \Delta d(x) = -M_N \cdot \frac{1}{I} \cdot \frac{N_c}{2} \sum_{m>0,n\leq 0} \frac{1}{E_m - E_n} \times \langle n | (1 + \gamma^0 \gamma^3) \gamma_5 \frac{\delta(xM_N - E_n - p^3) + \delta(xM_N - E_m - p^3)}{2} | m \rangle \langle m | \tau_3 | n \rangle. \]  

Integrating over \( x \), we get

\[ \int_{-1}^{1} [\Delta u(x) + \Delta d(x)] \, dx = \int_{0}^{1} [\Delta u(x) + \Delta d(x) + \Delta \bar{u}(x) + \Delta \bar{d}(x)] \, dx = -\frac{1}{I} \cdot \frac{N_c}{2} \sum_{m>0,n\leq 0} \frac{1}{E_m - E_n} \langle n | \gamma^0 \gamma^3 \gamma_5 | m \rangle \langle m | \tau_3 | n \rangle, \]  

which is noting but the isosinglet axial-vector charge (or the quark spin fraction) of the nucleon first investigated in [16] in the context of the CQSM.

3 Numerical results and discussion

Now we turn to the discussion on the method of numerical calculation. For evaluating isovector unpolarized and isoscalar polarized distribution functions with inclusion of the Dirac-sea quark degrees of freedom, we must perform infinite double sums over all the single-quark orbitals which are eigenstates of the static Hamiltonian \( H \). A numerical technique for carrying out such double sums has been established in the case of static nucleon observables [16]. In the nucleon structure function problem, however, we encounter a new bothering feature. The relevant matrix elements contain delta functions depending on the single-quark momentum \( p^3 \). Kahana and Ripka’s plane wave basis [25] is useful also for treating this feature. (We recall that it is a set of eigenstates of the free Hamiltonian \( H_0 = \alpha \cdot \nabla + \beta M \) discretized by imposing an appropriate boundary condition for the radial wave functions at the radius \( D \) chosen to be sufficiently larger than the soliton size.) For evaluating necessary matrix elements, we find it convenient to expand the eigenstates \( | m \rangle \) and \( | n \rangle \) of \( H \) in terms of this discretized plane wave basis in the momentum representation. We can then make use of the fact that \( p^3 \) is diagonal in this basis. This however does not get rid of all the trouble. Since the delta function in question now depends on the Bjorken variable \( x \), \( E_m \) (or \( E_n \)), \( p^3 = |p|_i \cos \theta_p \), where \( |p|_i \) are the momenta of the discretized basis states, the resultant distribution would be a discontinuous function of \( x \) [20]. To remedy it, Diakonov et al. proposed a smearing procedure in which a calculated distribution function is convoluted with a Gaussian distribution with an appropriate width [20]. This smearing procedure works well as a whole, but it turns out to have some unpleasant features. First, it requires us a special care near \( x \approx 0 \), where a distribution function (extended to the range \(-1 \leq x \leq 1\)) or its derivative may have a
discontinuity. Secondly, the smearing causes unpleasant flattening when the bare distribution
has a sharp peak as is the case for the valence quark contribution. Then, we decided to use
here a conceptually simpler least square fitting procedure. To this end, we first calculate
distribution functions at many points of \( x \) with a small interval, say \( \Delta x = 0.01 \). This gives a
set of data, which are rapidly fluctuating with a small amplitude. A common variance of the
order of this fluctuation amplitude is then given to all the data points, so that the least-square
fit can be carried out. In the fitting of the distribution functions, we find it convenient to use
a set of locally peaked Gaussian functions, the centers of which are distributed homogeneously
in the \( x \) space.

Actually, the expression for the isovector unpolarized distribution as well as that for the
moment of inertia contain divergences, so that they need some regularization. In the case of
static nucleon observables, there is a phenomenologically established regularization method,
based on Schwinger’s proper-time scheme [16,17]. Unfortunately, how to generalize this reg-
ularization scheme in the evaluation of nucleon structure functions has not been known yet.
In the recent investigations of the leading order quark distribution functions [19,20], Diakonov
et al. advocated to use the Pauli-Villars regularization. In this scheme, one subtracts from
divergent sums a multiple of the corresponding sums over eigenstates of the Hamiltonian in
which the quark mass \( M \) is replaced by the regulator mass \( M_{PV} \), uniquely determined from
the condition,

\[
f_{\pi}^2 = \frac{N_c M^2}{4\pi^2} \ln \frac{M_{PV}^2}{M^2},
\]

which gives \( M_{PV} \approx 571 \text{ MeV} \) for \( M = 400 \text{ MeV} \). In the present study, we shall also try this
regularization scheme but somewhat in a different way. That is, although they regularized
not only the Dirac continuum contribution but also the discrete (valence) level one, it is not
an absolute demand of this regularization scheme. Since the valence level contribution is
convergent itself, one can also define and use another regularization scheme, in which only the
Dirac sea part is regularized by the Pauli-Villars subtraction. In fact, a closer correspondence
with the standardly-used regularization scheme like the proper-time one is held by adopting
this latter scheme of regularization. Either way, since this method of regularization with use
of subtraction method is quite dissimilar to more standard energy-cutoff scheme, one may
suspect that the answers would depend strongly on this particular choice of regularization. To
see how much the predicted distributions depend on the choice of the regularization method,
we have invented another semi-theoretical regularization scheme, which we believe is closer in
spirit to the standard method of introducing regularization. Let us explain it for \( u(x) - d(x) \),
since the vacuum quark part of \( \Delta u(x) + \Delta d(x) \) is convergent without regularization. We start
with the nonregularized form for the vacuum polarization part of \( u(x) - d(x) \)

\[
[u(x) - d(x)]_{v.p.} = M_N \cdot \frac{1}{I} \cdot \frac{1}{3} \sum_{a=1}^{3} \frac{N_c}{2} \sum_{m \geq 0, n < 0} \frac{1}{E_m - E_n}
\]
\begin{equation}
\times \langle n \mid \tau_n \left( 1 + \gamma^0 \gamma^3 \right) \frac{\delta(xM_N - E_n - p^3) + \delta(xM_N - E_m - p^3)}{2} \mid m \rangle < m \mid \tau_3 \mid n \rangle . \tag{38}
\end{equation}

One of the simplest choices for introducing a regularization, which does not violate the charge-conjugation symmetry, would be given by the replacement,

\begin{equation}
\frac{\delta(xM_N - E_n - p^3) + \delta(xM_N - E_m - p^3)}{2} \rightarrow \frac{g(E_n; E_{\text{max}}) \delta(xM_N - E_n - p^3) + g(E_m; E_{\text{max}}) \delta(xM_N - E_m - p^3)}{2} , \tag{39}
\end{equation}

where \( g(E_n; E_{\text{max}}) \) is an appropriate energy cutoff which may, for instance, be taken as

\begin{equation}
g(E_n; E_{\text{max}}) = \text{erfc}\left(\frac{|E_n|}{E_{\text{max}}}\right) , \tag{40}
\end{equation}

or

\begin{equation}
g(E_n; E_{\text{max}}) = \exp\left(-\frac{E_n^2}{E_{\text{max}}^2}\right) . \tag{41}
\end{equation}

For definiteness, we will take the second choice in the following explanation. Integrating over \( x \) from \(-1\) to \( 1 \), this leads to the Dirac sea contribution to the moment of inertia of the form:

\begin{equation}
I_{v.p.} = N_c \sum_{m \geq 0, n < 0} \frac{1}{E_m - E_n} \cdot \frac{e^{-E_n^2/E_{\text{max}}^2} + e^{-E_m^2/E_{\text{max}}^2}}{2} \langle n \mid \tau_3 \mid m \rangle \langle m \mid \tau_3 \mid n \rangle , \tag{42}
\end{equation}

where use has been made of the symmetry of the matrix element of \( \tau_3 \). One notices that this expression is pretty different from that of the standardly-used proper-time regularization, which gives

\begin{equation}
I_{v.p.} = N_c \sum_{m,n} f(E_m, E_n; \Lambda) < n \mid \tau_3 \mid m > < m \mid \tau_3 \mid n > , \tag{43}
\end{equation}

with

\begin{equation}
f(E_m, E_n; \Lambda) = \frac{1}{4} \cdot \left\{ \frac{\text{sign}(E_m) \text{erfc}\left(\frac{|E_m|}{\Lambda}\right) - \text{sign}(E_n) \text{erfc}\left(\frac{|E_n|}{\Lambda}\right)}{E_m - E_n} - \frac{2}{\sqrt{\pi}} \frac{E_m^2 - E_n^2}{x^2 - E_n^2} \right\} , \tag{44}
\end{equation}

though both are reduced to the following same expression in the infinite cutoff limit:

\begin{equation}
I_{v.p.} = N_c \sum_{m \geq 0, n < 0} \frac{1}{E_m - E_n} < n \mid \tau_3 \mid m > < m \mid \tau_3 \mid n > . \tag{45}
\end{equation}

A noticeable difference is that the formulas based on the proper-time regularization contains transitions between occupied levels themselves and unoccupied ones, which clearly violates Pauli exclusion principle. However, this physically unacceptable Pauli-principle violation of the proper-time regularization is a mere appearance. Due to the favorable function of the second term of the cutoff function \( f(E_m, E_n; \Lambda) \), it turns out that the contribution of the
Pauli-violating processes with $E_m \times E_n > 0$ turns out to be greatly suppressed. On the other hand, the regularized expression (42) is free from the Pauli-principle violation by construction. However, since the behavior of the cutoff functions in (42) and (43) are pretty different, there is no reason to expect that the cutoff parameter $E_{\text{max}}$ in (41) is close to $\Lambda$ in (44). That means that we need some criterion to determine the parameter $E_{\text{max}}$ in (40) or (41). In view of the fundamental importance of the moment of inertia in the collective quantization approach, a natural procedure would be to fix the cutoff parameter so as to reproduce this key quantity. One may then fix $E_{\text{max}}$ so that the new regularization scheme gives the same moment of inertia as the standard proper-time scheme. Here, for the sake of convenience, we would rather fix it so that the new regularization scheme gives the same moment of inertia as the Pauli-Villars regularization scheme. The reason is as follows. In the previous studies of static nucleon observables, all the observables can be evaluated within the single regularization scheme, which also enables us to get a self-consistent soliton solution [16,17]. In the present semi-theoretical treatment of the regularization problem, we must give up this complete self-consistency. Under this circumstance, the best starting choice would be to use a phenomenologically tested soliton profile, which has been obtained in the proper-time regularization scheme with use of the dynamical quark mass $M = 400$ MeV and the physical pion mass $m_\pi = 138$ MeV. Using this soliton profile, we obtain $I_{\text{total}} = 1.16$ fm in the proper-time regularization scheme, while the Pauli-Villars regularization scheme with the same profile function gives $I_{\text{total}} = 1.14$ fm. This means that the above two criterions for fixing $E_{\text{max}}$ can be thought of as practically the same.

Now by setting

$$E_{\text{max}} = \begin{cases} 728 \text{ MeV} & \text{for } g(E_n; E_{\text{max}}) = \exp\left(-\frac{E_n^2}{E_{\text{max}}^2}\right), \\ 1080 \text{ MeV} & \text{for } g(E_n; E_{\text{max}}) = \text{erfc}\left(\frac{|E_n|}{E_{\text{max}}}\right), \end{cases}$$

the new regularization schemes give

$$I_{\text{v.p.}}(\text{Gaussian-function}) \simeq 0.254 \text{ fm},$$
$$I_{\text{v.p.}}(\text{error-function}) \simeq 0.254 \text{ fm} \quad (47)$$

which approximately reproduce the number

$$I_{\text{v.p.}}(\text{Pauli-Villars}) \simeq 0.255 \text{ fm},\quad (48)$$

obtained with the Pauli-Villars regularization scheme. Thus, at least once an appropriate soliton profile is given, either of these three regularization schemes thought expected to be a reasonable candidate for regularizing divergences in our problem. In the following, we shall calculate the vacuum polarization contributions to $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$ by using these three different schemes of regularization, to see the regularization dependence of distribution functions.
Now we are in a position to show the results of our numerical calculation. Shown in Fig. 1 are the contributions of the (discrete) valence level to the distribution functions $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$ in the proton. One clearly sees that $\bar{u}(x) - \bar{d}(x)$ is negative in the range of $x$ where it has dominant support. This means that the excess of the $\bar{d}$ sea over the $\bar{u}$ in the proton comes out very naturally even at the level of valence quark approximation. This feature, which was also observed in ref. [18], is an interesting consequence of the CQSM, which assumes the symmetry breaking mean-field of hedgehog form. One may be interested in what conclusion one would obtain if one evaluates the net flavor asymmetry of sea quarks under this valence quark approximation. One gets

$$
\int_0^1 [\bar{u}(x) - \bar{d}(x)]_{\text{val}} \, dx \simeq -0.017 .
$$

(49)

(Here the upper limit of $x$ integration is actually extended to $\infty$, since the theoretical distribution functions have non-zero support beyond $x = 1$. See the discussion before (52).) The above number may be compared with the empirical estimate

$$
\int_0^1 [\bar{u}(x) - \bar{d}(x)] \, dx \simeq -0.148 ,
$$

(50)

which is obtained from the NMC measurement $S_G = 0.235 \pm 0.026$ [26]. One sees that the valence quark contribution alone is not enough to generate the required magnitude of flavor asymmetry of the sea quark distributions. As a matter of course, a comparison with the experimental data is premature at this stage of calculation. First of all, we have not yet paid any attention to the renormalization scale dependence of the distribution functions. (We shall later come back to this point.) Secondly, the distribution functions of Fig. 1 obtained within the valence quark approximation do not saturate the isospin (or Adler) sum rule. In fact, we obtain

$$
\int_0^1 \{ [u(x) - d(x)] - [\bar{u}(x) - \bar{d}(x)] \}_{\text{val}} \, dx = 0.775 ,
$$

(51)

denoting that the remaining 23% of the nucleon isospin is carried by the Dirac sea quarks. The isospin sum rule can be made to hold within the valence quark approximation if the total moment of inertia $I$ in (31) is replaced by its valence quark contribution $I_{\text{val}}$, as was done in [18]. However, this would also change all the $O(\Omega^1)$ distribution functions by the factor of $I / I_{\text{val}} \simeq 1.3$, which is by no means justifiable.

Anyhow, it is clear that a consistent calculation should also include the Dirac sea contribution. Since this part is generally dependent on the selected regularization scheme, we first investigate in Fig. 2 the regularization dependence of the vacuum polarization contribution to $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$. Here, the solid curves are the results of the Pauli-Villars regularization scheme, while the dashed and dash-dotted curves are respectively those of the energy-cutoff scheme with the Gaussian and error-function type regularization functions. One sees that the regularization scheme with use of the energy cutoff has a tendency to suppress
distribution functions at small $x$ as compared with the Pauli-Villars scheme. However, the differences between the three regularization scheme are not so large. One can rather say that the regularization scheme dependence of the distribution functions is pretty small once the cutoff parameters are determined so as to reproduce a reasonable value of the moment of inertia. Note also that the regularization scheme dependence becomes further insignificant if we see the sum of the valence and the vacuum polarization contributions to the structure functions, since the contribution of the valence level is dominant anyway. Shown in Fig.3 are the sums of the valence and the vacuum polarization contributions to $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$. Here only the distribution functions obtained with the Gaussian-type regularization function is shown for the reason explained above. We have numerically confirmed that the isospin sum rule (32) is satisfied within the precision of 0.2%. One sees that, as compared with the distribution functions obtained with the valence quark approximation (Fig.1), a further enhancement is observed for the excess of the $\bar{d}$ sea over the $\bar{u}$ sea inside the proton.

So far we have postponed the discussion on the energy scale of the model calculation. Roughly speaking, the quark distribution functions evaluated here corresponds to the energy scale of the order of the Pauli-Villars mass $M_{P.V.} \simeq 600$ MeV or the cutoff energy $E_{\text{max}} \simeq 700$ MeV contained in the Gaussian-form regularization function. The $Q^2$-evolution must be taken into account in some way to be compared with the observed nucleon structure functions at large $Q^2$. Fortunately, after elaborate analyses of high $Q^2$ data by taking account of the perturbative $Q^2$ evolution, Glück, Reya and Vogt gave a simple parameterization of quark distribution functions at a normalization point very close to the energy scale of the present model [29]. We may therefore carry out a preliminary comparison of our predictions with their low scale parameterization of quark distribution functions. Fig.4 shows this comparison. Here the solid curves stand for the distribution functions $x [u(x) - d(x)]$ and $x [\bar{u}(x) - \bar{d}(x)]$ obtained from the ones given in Fig.3, whereas the boxes represent the NLO parameterization of ref.[29]. One can say that the qualitative feature of the NLO parameterization is nicely reproduced. The main discrepancy between the theory and the phenomenological fit is that the theoretical distribution functions have nonzero support beyond $x = 1$. This unphysical tail of the theoretical distribution functions comes from an approximate nature of our treatment of the soliton center-of-mass motion (as well as the collective rotational motion), which is essentially nonrelativistic. A simple procedure to remedy this defect was proposed by Jaffe based on the 1 + 1 dimensional bag model [27] and recently reinvestigated by Gamberg et al. within the context of the NJL soliton model [28]. (The latter model is essentially equivalent to the CQSM.) According to the latter authors, the effect of Lorentz contraction can simply be taken into account by first evaluating the distribution functions in the soliton rest frame (as we are doing here) and then by using the simple transformation

$$f_{\text{IMF}}(x) = \frac{\Theta(1 - x)}{1 - x} f_{\text{RF}}(- \ln (1 - x)),$$  

as far as the order $\Omega^0$ contributions to the distribution functions are concerned. We are not sure
whether their proof can be generalized for the order $\Omega^1$ contributions to distribution functions, in which we must treat three dimensional collective rotational motion. Nonetheless, it may be interesting to see the effects of this transformation, especially on $u(x) - d(x)$. The distribution functions obtained after this transformation are shown in Fig.4 by the dashed curves. The crucial influence of this transformation at large $x$ region is obvious from this figure. Since (52) is a normalization-preserving transformation, i.e.

$$\int_0^\infty f_{RF}(x) \, dx = \int_0^1 f_{IMF}(x) \, dx,$$

it follows that the peaks of the transformed distribution functions becomes sharper than the original ones. However, the behavior of the distribution functions at smaller values of $x$ turns out to be rather insensitive to this transformation. In this small $x$ region, there remains a qualitative difference between the phenomenological distribution functions and the theoretical ones (obtained after the transformation (52)) especially for the antiquark distribution $\bar{u}(x) - \bar{d}(x)$. Note however that the enhanced behavior of the GRV parameterization at smaller values of $x$ is connected with the Regge-like behavior ($\sim 1/x^\alpha$ with $0 < \alpha < 1$) of the distribution functions assumed in their fit. It is clear that such hidden dynamics of the distribution functions never enters into the model calculations as carried out here. Probably, within the model calculation as carried out here, what is more reliable than the detailed $x$ dependence would be sum rules obtained after the $x$ integration. Integrating over $x$, we obtain from Fig.4,

$$\delta_G \equiv \int_0^1 [\bar{u}(x) - \bar{d}(x)] \, dx \simeq -0.130,$$

or equivalently

$$S_G = \frac{1}{3} \int_0^1 \{ [u(x) - d(x)] + [\bar{u}(x) - \bar{d}(x)] \} \, dx \simeq 0.247.$$

On the other hand, using the NLO parameterization of ref.[29], we find that

$$\delta_G \simeq -0.168,$$

or

$$S_G \simeq 0.221.$$

We can say that the agreement between the theory and the phenomenological fit is pretty good in view of the extreme simplicity of the CQSM. Needless to say, the flavor asymmetric polarization of the sea quark distributions obtained above would never arise if there is no flavor asymmetry in the valence quark numbers of the nucleon. This then denotes that the NMC observation is nothing mysterious: it is explained very naturally as a combined effects of two ingredients, i.e. the apparently existing flavor asymmetry of the valence quark numbers in the nucleon and the spontaneous chiral symmetry breaking of the QCD vacuum [30].
Next we turn to the discussion on the flavor-singlet (isoscalar) polarized distribution function. As has been already pointed out, the vacuum polarization contribution to this quantity is convergent without regularization. (This feature comes from the fact that the flavor-singlet axial-charge is related to the imaginary part of the Euclidean effective meson action.) What is more, we find that the Dirac sea contributions to $\Delta u(x) + \Delta d(x)$ as well as $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ are consistent with zero within the numerical accuracy of the model calculation. This means that the valence quark approximation is a good approximation for this special quantity. However, the valence quark approximation here should be distinguished from the corresponding one in ref.[18] where the total moment of inertia $I$ had to be replaced by its valence quark part $I_{val}$ so as to maintain the isospin (or Adler) sum rule. In making this replacement, one would necessarily overestimate $O(\Omega^1)$ distribution functions like the isoscalar polarized one, since they are inversely proportional to $I$. In any case, since the vacuum polarization contribution to $\Delta u(x) + \Delta d(x)$ as well as $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ are almost negligible, we will show below only the total contributions. The solid curves in Fig.5 stand for the isoscalar polarized distribution functions $\Delta u(x) + \Delta d(x)$ and $\Delta \bar{u}(x) + \bar{d}(x)$ directly obtained from the formula (35), whereas the dashed curves here take account of the effect of Lorentz contraction by making use of the transformation (52). Again, the sizable effect of Lorentz boost is self-explanatory especially at larger values of $x$. Since the low energy scale parameterization of the polarized distribution functions is not yet available, we shall postpone the detailed comparison with the observed structure functions for future studies. A qualitatively interesting feature observed in Fig.5 is that the isoscalar combination of $\bar{u}$ and $\bar{d}$ seas is slightly polarized (in most range of $x$) against the nucleon spin direction. Although this combined polarization of $\bar{u}$ and $\bar{d}$ seas is rather small in magnitude, this does not mean that $\Delta \bar{u}(x)$ and $\Delta \bar{d}(x)$ are both small. The leading order calculation of Diakonov et al. for the isovector polarized distribution functions tells us that $\Delta \bar{u}(x) - \Delta \bar{d}(x)$ is likely to be positive and fairly large [19,20]. This together with the above result for $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ indicates that the seas of $\bar{u}$ and $\bar{d}$ are sizably polarized in such a way that their polarizations cancel each other. Now using the distribution functions given in Fig.5, we can calculate the first moment of the flavor singlet (longitudinally) polarized quark distribution functions. We find that

$$\int_0^1 [ \Delta u(x) + \Delta d(x) + \Delta \bar{u}(x) + \Delta \bar{d}(x) ] \, dx = \ 0.494 .$$  \hspace{1cm} (58)$$

On the other hand, the direct calculation of the flavor-singlet axial charge with use of (36) gives

$$\langle g_A^{(0)} \rangle = \ 0.492 .$$  \hspace{1cm} (59)$$

The agreement between the above two numbers can be interpreted as showing the accuracy of our numerical procedure for evaluating quark distribution functions. In any case, from the above value of the sum rule, one reconfirms that the spin fraction of the nucleon carried by
quarks comes out to be quite small (less than half) in the CQSM. As has been repeatedly emphasized by one of the authors [16], the smallness of the quark spin fraction (or the largeness of the orbital angular momentum) is inseparably connected with the basic dynamical assumption of this unique model, i.e. the identification of a rotating hedgehog with the physical nucleon.

4 Summary

In summary, we have evaluated the isovector unpolarized and isoscalar polarized distribution functions of the nucleon within the framework of the CQSM with full inclusion of the Dirac sea quarks. It has been shown that the characteristic features of the observed distribution functions, i.e. the excess of the $\bar{d}$ sea over the $\bar{u}$ sea in the proton as well as the very small quark spin fraction of the nucleon, are reproduced at least qualitatively within a single theoretical framework of the CQSM, which is the simplest effective quark model of QCD taking maximal account of chiral symmetry. We then interpret this success as revealing the crucial importance of chiral symmetry in the physics of high energy deep-inelastic structure functions.

Acknowledgement

Numerical calculation was performed by using the workstation “miho” at the Research Center for Nuclear Physics, Osaka University.

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Figure caption

Fig. 1. The contributions of the valence level to the distribution functions $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$.

Fig. 2. The contributions of the Dirac continuum to the distribution functions $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$. The solid curves represent the results of the Pauli-Villars regularization scheme, whereas the dashed and dash-dotted curves are those of the energy-cutoff scheme with the Gaussian and error-function type regularization functions.

Fig. 3. The sum of the valence and Dirac sea contributions to $u(x) - d(x)$ and $\bar{u}(x) - \bar{d}(x)$ obtained with the regularization function of Gaussian form.

Fig. 4. The theoretical distribution functions in comparison with the low energy scale parameterization (NLO fitting) of ref.[29]. The solid curves just correspond to the distribution functions of Fig.3 multiplied by $x$, while the dashed curves are obtained by using the transformation (52).

Fig. 5. The isoscalar polarized distribution functions $\Delta u(x) + \Delta d(x)$ and $\Delta \bar{u}(x) + \Delta \bar{d}(x)$. The solid curves are directly obtained from (35), while the dashed curves are obtained with the transformation (52).
Fig. 1

\[ u(x) - d(x) \]

\[ \bar{u}(x) - \bar{d}(x) \]
Fig. 2

$u(x) - d(x)$

$\bar{u}(x) - \bar{d}(x)$

- Pauli–Villars
- Gauss
- Error Func.
Fig. 3

\[ u(x) - d(x) \]

\[ \tilde{u}(x) - \tilde{d}(x) \]
Fig. 4

$\mathbf{x \left[ u(x) - d(x) \right]}$

- **RF**
- **IMF**
- **GRV Parametrization**
Fig. 5

\[ \Delta u(x) + \Delta d(x) \]

\[ \Delta \bar{u}(x) + \Delta \bar{d}(x) \]