Lower Bound on the Pseudoscalar Mass in the Minimal Supersymmetric Standard Model

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Abstract

In the Higgs sector of the Minimal Supersymmetric Standard Model, the mass of the pseudoscalar $A$ is an independent parameter together with $\tan \beta \equiv v_2/v_1$. If $m_A$ is small, then the process $e^+e^- \to h + A$ is kinematically allowed and is suppressed only if $\tan \beta$ is small. On the other hand, the mass of the charged Higgs boson is now near $M_W$, and the decay $t \to b + h^+$ is enhanced if $\tan \beta$ is small. Since the former has not been observed, and the branching fraction of $t \to b + W$ cannot be too small (by comparing the experimentally derived $t\bar{t}$ cross section from the leptonic channels with the theoretical prediction), we can infer a phenomenological lower bound on $m_A$ of at least 60 GeV for all values of $\tan \beta$. 
The most studied extension of the standard $SU(2) \times U(1)$ electroweak gauge model is that of supersymmetry with the smallest necessary particle content. In this Minimal Supersymmetric Standard Model (MSSM), there are two scalar doublets $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$, with Yukawa interactions $(u, d)_L d_R \Phi_1$ and $(u, d)_L u_R \Phi_2$, respectively, where $\Phi_2 = i \sigma_2^0 \Phi_2^* = (\phi_2^0, -\phi_2^-)$. The Higgs sector of the MSSM has been studied in great detail[1] and it is a current topic of intensive experimental and theoretical scrutiny.[2] There are five physical Higgs bosons in the MSSM: two neutral scalars ($h$ and $H$), one neutral pseudoscalar ($A$), and two charged ones ($h^\pm$). Their masses and couplings to other particles are completely determined up to two unknown parameters which are often taken to be $m_A$ and $\tan \beta \equiv v_2/v_1$, where $v_i$ is the vacuum expectation value of $\phi_i^0$.

In the following, we will show that $m_A > 60$ GeV for all values of $\tan \beta$. Our conclusion is based on a combination of theoretical and experimental inputs from a number of different observations which have become available recently.

In the MSSM, the pseudoscalar Higgs boson $A$ and the charged Higgs bosons $h^\pm$ are given by analogous expressions, namely

\[
A = \sqrt{2} (\sin \beta \text{Im} \phi_1^0 - \cos \beta \text{Im} \phi_2^0),
\]

\[
h^\pm = \sin \beta \phi_1^\pm - \cos \beta \phi_2^\pm.
\]

At tree level, their masses are related by $m_{h^\pm}^2 = m_A^2 + M_W^2$. The mass-squared matrix spanning the two neutral scalar Higgs bosons $\sqrt{2} \text{Re} \phi_{1,2}^0$ is given by

\[
M^2 = \begin{pmatrix}
    m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & -(m_A^2 + M_Z^2) \sin \beta \cos \beta \\
    -(m_A^2 + M_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta + \epsilon/\sin^2 \beta
\end{pmatrix}.
\]

In the above, $\epsilon$ is the leading radiative correction[6] due to the $t$ quark:

\[
\epsilon = \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2} \ln \left(1 + \frac{\tilde{m}^2}{m_t^2}\right),
\]

where $\tilde{m}$ is the mass parameter for the supersymmetric scalar quarks.
Let us take $m_A = 0$ and rotate $\mathcal{M}^2$ to the basis spanned by

$$h_1 = \sqrt{2}(\sin \beta \text{Re}\phi_1^0 - \cos \beta \text{Re}\phi_2^0), \quad h_2 = \sqrt{2}(\cos \beta \text{Re}\phi_1^0 + \sin \beta \text{Re}\phi_2^0).$$  \hspace{1cm} (5)

We get$\text{[3]}$

$$\mathcal{M}^2 = \begin{pmatrix} M_Z^2 \sin^2 2\beta + \epsilon \cot^2 \beta & -M_Z^2 \sin 2\beta \cos 2\beta + \epsilon \cot \beta \\ -M_Z^2 \sin 2\beta \cos 2\beta + \epsilon \cot \beta & M_Z^2 \cos^2 2\beta + \epsilon \end{pmatrix}. \hspace{1cm} (6)$$

It is well-known that in this basis, the $h_1 ZZ$ and $h_2 AZ$ couplings are absent, hence the nonobservation of $e^+ e^- \rightarrow h + A$ does not rule out any value of $m_A$ if $\tan \beta$ is small enough$\text{[4]}$. In this limit, the eigenstates of $\mathcal{M}^2$ are essentially $h_1$ and $h_2$. If $h \simeq h_1$, then it is too heavy to be produced. If $h \simeq h_2$, then its coupling to $A$ is too small to have a measurable branching fraction. Note that $\epsilon \simeq M_Z^2$, i.e. (91 GeV)$^2$, for $m_t = 175$ GeV and $\tilde{m} = 1$ TeV.

From the nonobservation of $e^+ e^- \rightarrow h + Z$ where the $Z$ boson may be either real or virtual and the nonobservation of $e^+ e^- \rightarrow h + A$, where $h$ is an arbitrary linear combination of $h_1$ and $h_2$, it is possible to obtain the MSSM exclusion region in the $m_A - \tan \beta$ plane. One such detailed analysis$\text{[5]}$ using only LEP1 data collected at the $Z$ resonance shows that $m_A$ has to be greater than about $M_Z/2$ for $\tan \beta > 1$. With the higher energies available at LEP2 since then, this bound is expected to be at least 60 GeV.

To obtain a lower bound on $m_A$ for $\tan \beta < 1$, we propose to use the MSSM relationship$\text{[6]}$

$$m_{h^\pm}^2 = m_A^2 + M_W^2 - \frac{\epsilon}{4 \sin^2 \beta} \frac{M_W^2}{m_t^2}, \hspace{1cm} (7)$$

where the last term is the leading radiative correction for $\tan \beta < 1$. We then derive bounds on $m_A$ from the bounds on $m_{h^\pm}$ by considering $t$ decay. Taking $m_t = 175$ GeV, we see that $t \rightarrow b + h^+$ is allowed for values of $m_{h^\pm}$ up to 170 GeV, corresponding to $m_A$ up to about 150 GeV. The nonobservation of the above process would then translate into lower bounds on $m_A$ as a function of $\tan \beta$.  

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In the MSSM, the charged-Higgs-boson couplings to the quarks and leptons are given by
\[ \mathcal{H}_{int} = -\frac{g_2}{\sqrt{2} M_W} h^+[\cot \beta m_u \bar{u}_i d_i + \tan \beta m_d \bar{u}_i d_i + \tan \beta m_t \bar{t}_i l_i] + h.c., \] (8)
where the subscript \( i \) represents the generation index, and we have used the diagonal KM matrix approximation\[7\]. The leading-logarithm QCD (quantum chromodynamics) correction is taken into account by substituting the quark mass parameters by their running masses evaluated at the \( h^\pm \) mass scale. The resulting decay widths are
\[ \Gamma(t \rightarrow bh^+) = \frac{g_2^2 \lambda^{1/2}(1, m_b^2/m_t^2, m_{h^+}^2/m_t^2)}{64\pi M_W^2 m_t} [(m_t^2 \cot^2 \beta + m_h^2 \tan^2 \beta)(m_t^2 + m_b^2 - m_{h^+}^2) - 4m_t^2 m_h^2], \] (9)
where \( \lambda \) denotes the usual Kallen function and \( \lambda^{1/2} \) is equal to the magnitude of the momentum of either decay product divided by \( m_t/2 \), and
\[ \Gamma(h^+ \rightarrow \tau^+ \nu) = \frac{g_2^2 m_{h^+}}{32\pi M_W^2} m_t^2 \tan^2 \beta, \] (10)
\[ \Gamma(h^+ \rightarrow cs) = \frac{3g_2^2 m_{h^+}}{32\pi M_W^2} (m_c^2 \cot^2 \beta + m_s^2 \tan^2 \beta). \] (11)
Assuming that the only other competing channel is the standard-model decay \( t \rightarrow bW^+ \), the \( t \rightarrow bh^+ \) branching fraction is then
\[ B = \frac{\Gamma(t \rightarrow bh^+)}{\Gamma(t \rightarrow bh^+) + \Gamma(t \rightarrow bW^+)}, \] (12)
where
\[ \Gamma(t \rightarrow bW^+) = \frac{g_2^2 \lambda^{1/2}(1, m_b^2/m_t^2, M_W^2/m_t^2)}{64\pi M_W^2 m_t} [M_W^2 (m_t^2 + m_b^2) + (m_t^2 - m_h^2)^2 - 2M_W^4]. \] (13)
It is clear from Eq. (9) that \( B \) has a minimum at \( \tan \beta = (m_t/m_b)^{1/2} \simeq 6 \), but it becomes large for \( \tan \beta < 1 \) and \( \tan \beta > m_t/m_b \). Thus we expect to see a sizeable \( t \rightarrow bh^+ \) signal in these two regions if \( m_{h^+} < m_t \).

We see from Eqs. (10) and (11) that \( \tau^+ \nu \) is the dominant decay mode of \( h^+ \) if \( \tan \beta >> 1 \). Thus an excess of \( t \bar{t} \) events in the \( \tau \) channel compared to the standard-model prediction
constitutes a viable $h^\pm$ signal in the large tan $\beta$ region. A recent analysis\cite{8} of the CDF $t\bar{t}$ data in the $\tau l$ channel ($l = e, \mu$) has led to a mass bound of $m_{h^\pm} > 100$ GeV for tan $\beta > 40$. A similar bound has also been obtained from the same $t\bar{t}$ data in the inclusive $\tau$ channel\cite{9}.

The above method is not applicable in the small tan $\beta$ region, where $h^+$ is expected to decay mainly into $c\bar{s}$, i.e. two jets. On the other hand, we can use the so-called disappearance method to look for the presence of $t \rightarrow bh^+$ decay in both the small and large tan $\beta$ regions\cite{7} as described below. The key observation is that $h^\pm$ couples negligibly to the light fermions, particularly $e$ and $\mu$, whereas the $W$ boson couples to them with full strength universally. Since the $e$ and $\mu$ decay modes play an important role in the detection of $t\bar{t}$ events at the Tevatron, the experimentally derived $t\bar{t}$ cross section is sensitive to the branching fraction $B$ of Eq. (12). After all, if $t$ decays into $bh^+$, there would not be any energetic $e$ or $\mu$ in the final state, as would be possible with the $W$ boson.

The experimental $t\bar{t}$ cross sections obtained by the CDF and D0 collaborations\cite{10, 11} are weighted averages of their measured cross sections in the (I) dilepton ($ll$) and (II) lepton plus multijet ($lj$) channels, using the standard formula

$$\sigma = \frac{\Sigma(\sigma_i/\delta_i^2)}{\Sigma(1/\delta_i^2)}. \quad (14)$$

They are summarized below.

CDF : $\sigma_{ll} = 8.5 \pm 3.4$ pb, $\sigma_{lj} = 7.2 \pm 1.7$ pb $\Rightarrow \sigma_{CDF} = 7.5 \pm 1.6$ pb. \quad (15)

D0 : $\sigma_{ll} = 6.3 \pm 3.3$ pb, $\sigma_{lj} = 5.1 \pm 1.9$ pb $\Rightarrow \sigma_{D0} = 5.5 \pm 1.8$ pb. \quad (16)

The $\sigma_{lj}$ of CDF is a weighted average of the measured cross sections using the SVX and SLT $b$-tagging methods; that of D0 is a weighted average of those using kinematic cuts and SLT $b$-tagging. In both cases, the weight of the SLT method is rather low. From Eqs. (15) and (16), we see that for both CDF and D0, $\delta_{lj} \simeq \delta_{ll}/2$, hence

$$\sigma \simeq \frac{\sigma_{ll} + 4\sigma_{lj}}{5}. \quad (17)$$
Furthermore, since the CDF and D0 cross sections have essentially identical errors, we can take a simple average of the two:

$$\sigma_{\text{CDF+D0}} = 6.5^{+1.3}_{-1.2} \text{ pb.} \quad (18)$$

Here we have combined the two errors using $\delta^{-2} = \delta_1^{-2} + \delta_2^{-2}$, since they are largely statistical.

We note that the dilepton channel (I) corresponds to the leptonic ($e, \mu$) decay of both the $t$ and $\bar{t}$ quarks, whereas the lepton plus multijet channel (II) corresponds to the leptonic decay of one, say $t \to b l^+ \nu$, and the hadronic decay of the other. For the standard-model decay $t \to b W^+$, the respective branching fractions are 2/9 and 2/3, whereas for the postulated decay $t \to b h^+$, they are 0 and a function which rises rapidly to 1 for $\tan \beta < 1$. Thus the relative contributions of different final states to the two channels are $WW : Wh^\pm : h^\pm h^\mp = 1 : 0 : 0$ for (ll) and $1 : 3/4 : 0$ for (lj). [We have used the maximum value of $3/4$ corresponding to very small $\tan \beta$. This is a conservative approach, because any smaller value will give us a better bound on $m_{h^\mp}$ as explained below.] We have then a suppression factor relative to the standard model of

$$f_{ll} = (1 - B)^2 \simeq 0.5 \quad \text{(for } B = 0.3), \quad (19)$$

$$f_{lj} = (1 - B)^2 + 2B(1 - B)(3/4) \simeq 0.8 \quad \text{(for } B = 0.3). \quad \quad (20)$$

Since the relative weights of the (ll) and (lj) channels are 1:4, Eqs. (19) and (20) correspond to an effective suppression factor of

$$f = 0.74 \quad \text{(for } B = 0.3). \quad \quad (21)$$

We note that for large $\tan \beta$, $h^\pm$ decays mainly into $\tau$, hence it would be hard for the $Wh^\pm$ final state to pass the $n_{\text{jet}} \geq 3$ cut required for the (lj) channel. This implies an extra suppression factor of about 1/3 for the $Wh^\pm$ contribution, hence $f$ is about 0.7 already for $B = 0.2$, i.e. our bound is conservative because it assumes $B = 0.3$. 

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Finally the theoretical estimates of the $t\bar{t}$ cross section including higher-order QCD corrections are 4.13 to 5.48 pb[12], and 5.10 to 5.59 pb[13]. These ranges are not identical, but the two estimates are in reasonable agreement as to their upper bounds. We shall thus assume for our purpose that

$$\sigma(t\bar{t}) \leq 5.6 \, \text{pb}. \quad (22)$$

Combining this with the suppression factor of Eq. (21), we obtain an upper bound of

$$\sigma \leq 4.1 \, \text{pb} \quad (23)$$

for the weighted cross section of Eq. (17). This is $2\sigma$ lower than the combined CDF and D0 estimate of Eq. (18), as well as the CDF estimate of Eq. (15). Hence we can take $B = 0.3$ as a $2\sigma$ upper bound for the branching fraction of $t \to bh^+$ decay. In Figure 1 we plot the exclusion regions of $m_{h^\pm}$ as a function of $\tan \beta$ using $B = 0.3$. We also show the exclusion region obtained in Ref. [8], which used the “appearance” method of looking for $\tau$, instead of the “disappearance” method of not finding $e$ or $\mu$ discussed here.

To convert a bound on $m_{h^\pm}$ to one on $m_A$, we use the full expression including all one-loop radiative corrections[6] in place of Eq. (7) which is approximate and valid only for $\tan \beta < 1$. In Figure 2 we plot the exclusion regions of $m_A$ as a function of $\tan \beta$ deduced from $t$ decay and $t\bar{t}$ production corresponding to Fig. 1. We note that the radiative correction is negative for small $\tan \beta$ which increases the $m_A$ bound, and is positive for large $\tan \beta$ which decreases it. We note also that at extreme values of $\tan \beta$, near 0.2 and 100, the Yukawa couplings involved are becoming too large for a perturbative calculation to be reliable. We then add a line at $m_A = 60$ GeV for $\tan \beta > 1$ as a conservative upper limit from the combined LEP data[5, 14]. Our conclusion is simple: in the Minimal Supersymmetric Standard Model, combining what we know from LEP and the Tevatron and using a conservative estimate of the theoretical $t\bar{t}$ cross section, the pseudoscalar mass $m_A$ is now known to be greater than 60 GeV for all values of $\tan \beta$. 

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References


[14] A detailed updated analysis (A. Sopczak, private communications) shows that the $m_A$ bound is actually much higher than 60 GeV.
**Figure Captions**

Fig. 1. Exclusion regions at 95% confidence level in the $m_{h^\pm} - \tan \beta$ plane using $B = 0.3$ (solid lines) for $t \rightarrow bh^+$ as explained in the text. The dashed line corresponds to the method used in Ref. [8].

Fig. 2. Exclusion regions at 95% confidence level in the $m_A - \tan \beta$ plane. Regions I and III correspond to those depicted in Fig. 1 with $m_{h^\pm}$ converted to $m_A$ taking into account the MSSM one-loop radiative corrections. Region II represents a conservative estimate of the expected limit from LEP1 and LEP2 for $\tan \beta > 1$ (dotted line).