Cosmological Constraint on the String Dilaton in Gauge-mediated Supersymmetry Breaking Theories

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Abstract

The dilaton field in string theories (if exists) is expected to have a mass of the order of the gravitino mass $m_{3/2}$ which is in a range of $10^{-2}$keV–1GeV in gauge-mediated supersymmetry breaking models. If it is the case, the cosmic energy density of coherent dilaton oscillation easily exceeds the critical density of the present universe. We show that even if this problem is solved by a late-time entropy production (thermal inflation) a stringent constraint on the energy density of the dilaton oscillation is derived from experimental upperbounds on the cosmic X($\gamma$)-ray backgrounds. This excludes an interesting mass region, $100$keV $\lesssim m_{3/2} \lesssim 1$GeV, in gauge-mediated supersymmetry breaking models.

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The existence of a light dilaton field $\phi$ is one of generic predictions in a large class of superstring theories [1]. It is expected to acquire its mass of the order of the gravitino mass $m_{3/2}$ from some non-perturbative dynamics [2]. In hidden sector models for supersymmetry (SUSY) breaking the gravitino has a mass at the electroweak scale. On the other hand, in gauge-mediated SUSY breaking models [3, 4, 5, 6, 7], the gravitino mass is predicted in a range of $10^{-2}$keV–1GeV. If the dilaton has such a small mass $m_\phi \simeq m_{3/2}$, its lifetime is much longer than the age of the present universe and the cosmic energy density of its coherent oscillation easily exceeds the critical density of the universe.

It has been pointed out in Ref. [8] that the above problem may be solved if a late-time thermal inflation [9] takes place. Since the dilaton mass is very small $m_\phi \simeq 10^{-2}$keV–1GeV, the thermal inflation seems only a mechanism to dilute substantially the cosmic energy density of the dilaton oscillation.\(^1\)

In this letter we show that a more stringent constraint [11] on the energy density of the dilaton oscillation is derived from experimental upperbounds on the cosmic X(\(\gamma\))-ray backgrounds. We adopt the thermal inflation as a dilution mechanism of the cosmic dilaton density. The obtained constraint\(^2\) excludes a region of $100$keV $\lesssim m_{3/2} \lesssim 1$GeV which raises a new problem in a large class of gauge-mediated SUSY breaking models observed recently [6, 7].

We briefly review the thermal inflation model proposed by Lyth and Stewart [9]. The potential of the inflaton $\chi$ is given by

$$V = V_0 - m^2|\chi|^2 + \frac{1}{M_{\ast}^{2n}}|\chi|^{2n+4}.$$  \hfill (1)

We suppose that the negative mass squared, $-m^2$, for $\chi$ is generated by SUSY-breaking higher order corrections and assume it of the order of the SUSY-breaking scale $m \simeq O(100)$GeV. The mass $M_{\ast}$ denotes a cut-off scale of this effective theory. We take $M_{\ast}$ as a free parameter in order to make a general analysis.\(^3\) $V_0$ is set so that the cosmological constant vanishes. The vacuum expectation value of $\chi$ is given

\(^1\)If the dilaton has a mass of the order 10TeV as in a class of hidden sector models, we have no cosmological dilaton problem [10].

\(^2\)A similar constraint has been derived without assuming a specific dilution mechanism [11]. The previous constraint is weaker than the present one.

\(^3\)The original thermal inflation model [9] assumes $M_{\ast}$ to be the gravitational scale $M_G = 2.4 \times 10^{18}$GeV.
by
\[ \langle \chi \rangle \equiv M = \left( \frac{1}{n+2} \right)^{\frac{n+1}{n+2}} (mM_*)^{\frac{1}{n+1}}, \tag{2} \]
and
\[ V_0 = m^2 M^2 - \frac{M^{2n+4}}{M_*^{2n}} = \frac{n+1}{n+2} m^2 M^2. \tag{3} \]
We use, hereafter, the vacuum-expectation value \( M \) of \( \chi \) instead of \( M_* \) to characterize the potential of the inflaton.

The inflaton decay rate is important to estimate the reheating temperature \( T_R \). In the SUSY standard model we have only a possible renormalizable interaction of the inflaton \( \chi \) with SUSY-standard model particles:
\[ W = \lambda \chi H \bar{H}, \tag{4} \]
where \( H \) and \( \bar{H} \) are Higgs chiral supermultiplets. The coupling constant \( \lambda \) should be taken very small, \( \lambda \lesssim \mu_H/M \), so that the induced mass \( \lambda \langle \chi \rangle = \lambda M \) is at most the electroweak scale. Here, \( \mu_H \) is the SUSY-invariant mass for the Higgs multiplets.

As pointed out in Ref. [9], with this small coupling the decays of \( \chi \) into a pair of Higgs fields give rise to the reheating temperature high enough \( (T_R \gtrsim 10 \text{MeV}) \) to maintain the success of big bang nucleosynthesis as far as \( M \lesssim 10^{12} \text{GeV} \). To derive a conservative constraint we assume the lowest value of the reheating temperature \( T_R = 10 \text{MeV} \) in the present analysis, since lower \( T_R \) yields weaker constraint as we will see later.

If the mass of \( \chi \) is below the threshold of a pair production of Higgs fields, the above decay processes are, however, not energetically allowed.\(^4\) Fortunately, one-loop diagrams of the Higgs multiplets induce a coupling of \( \chi \) to two photons as\(^5\)
\[ \mathcal{L}_{\text{eff}} = \frac{\alpha_{\text{em}}}{4\pi} \frac{\lambda}{\mu_H} \chi F_{\mu\nu} F^{\mu\nu}. \tag{5} \]
We assume that the SUSY-invariant Higgs mass \( \mu_H \) is dominated by the induced mass \( \lambda M \), i.e. \( \mu_H = \lambda M \). (We have a stronger constraint for \( \lambda/\mu_H < 1/M \), otherwise.) The decay rate is given by
\[ \Gamma(\chi \to 2\gamma) = \frac{1}{4\pi} \left( \frac{\alpha_{\text{em}}}{4\pi} \right)^2 \frac{m_\chi^3}{M^2}. \tag{6} \]

\(^4\)A possible decay into two pairs of bottom and antibottom quarks is strongly suppressed by phase volume effects. We thank K. Hikasa for this point.

\(^5\)In the calculation we have neglected SUSY-breaking effects.
which leads to the reheating temperature

$$T_R \simeq 0.25 \frac{\alpha_{em}}{(4\pi)^{3/2}} \left( \frac{m_\chi}{M} \right) \sqrt{m_\chi M_{pl}}.$$  \hspace{1cm} (7)

Here, $M_{pl}$ is the Planck mass $M_{pl} = \sqrt{8\pi G} = 1.2 \times 10^{19}$ GeV and $m_\chi$ the physical mass of inflaton $\chi$ around the vacuum $\langle \chi \rangle = M$. We find from Eqs. (1) and (2)

$$m_\chi^2 = 2(n + 1)m^2.$$  \hspace{1cm} (8)

In summary, for the case of $m_\chi < 130$ GeV we use the above reheating temperature (7) while for the case of $m_\chi \geq 130$ GeV we use the lowest possible value for $T_R$, $T_R = 10$ MeV, to derive a conservative constraint on the cosmic dilaton density.\(^7\)

Let us now discuss dynamics of the thermal inflation and estimate the cosmic energy density of coherent dilaton oscillation in the present universe.

When the inflaton $\chi$ couples to particles which are in thermal equilibrium,\(^8\) the effective potential of $\chi$ is written as

$$V_{eff} = V_0 - m^2 |\chi|^2 + \frac{1}{M_{\ast}^{2n}} |\chi|^{2n+4} + cT^2 |\chi|^2,$$  \hspace{1cm} (9)

where $T$ is the cosmic temperature and $c$ a constant of the order 1. Then, at high temperature $T > T_c \simeq m$, the effective mass squared of the inflaton $\chi$ is positive and $\chi$ sits near the origin $\langle \chi \rangle \simeq 0$. At this epoch the radiation energy density is given by $\rho_{rad} = g_\ast \pi^2 T^4 / 30$ where $g_\ast$ is the effective number of degrees of freedom. The important fact is that $\rho_{rad}$ becomes less than the vacuum energy density $V_0$ for $T < T_\ast \simeq V_0^{1/4}$. Thus, at temperature $T_\ast \lesssim T \lesssim T_\ast$ the vacuum energy $V_0$ dominates the energy density of the universe and a mini-inflation (i.e. the thermal inflation) occurs [9].\(^9\)

\(^6\)The experimental lower bounds on the masses for Higgs bosons and Higgsinos are about 65 GeV [12].

\(^7\)Since the potential Eq.(1) possesses a global $U(1)$ symmetry, we have a massless Nambu-Goldstone (NG) boson (the angular part of $\chi$). The NG bosons produced by the $\chi$ decay are cosmologically dangerous [9]. Thus, we assume that some explicit breaking of the $U(1)$ symmetry generates a mass of the NG boson large enough to suppress the $\chi$ decay into the NG bosons. The detailed analysis with the explicit $U(1)$-breaking term will be given in Ref. [13].

\(^8\)The particles coupled to $\chi$ have large masses of the order $M$ in the true vacuum $\langle \chi \rangle = M$. But in the false vacuum $\langle \chi \rangle \simeq 0$ they are light and could be in thermal bath.

\(^9\)The energy density of the universe is most likely dominated by the coherent dilaton oscillation just before the beginning of the thermal inflation. In this case the thermal inflation starts at $T_\ast \simeq (V_0^2 / (m_\phi M_G))^{1/6}$, which does not, however, affect the present analysis as long as $T_\ast > T_c$. 

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After $T < T_c$ the effective mass squared of the inflaton $\chi$ becomes negative and $\chi$ rolls down toward the true minimum (2) of the potential (1) and the thermal inflation ends. Then, the inflaton $\chi$ oscillates around it. The energy density of the oscillating field is finally transferred to the radiation with temperature $T_R$ through the inflaton decay. Therefore, the thermal inflation increases the entropy of the universe by a factor

$$\Delta \simeq \frac{4V_0/3T_R}{(2\pi^2/45)g_*T_c^3} \simeq \frac{V_0}{70T_RT_c^3}. \quad (10)$$

Here we have used the fact that the energy density of the oscillating field and the entropy density decrease as $a^{-3}$ where $a$ is the scale factor of the universe.

The dilaton $\phi$ starts to oscillate with the initial amplitude $\phi_0$ when its mass $m_\phi$ becomes larger than the Hubble parameter $H$. Hereafter, we call this dilaton as ‘big bang dilaton’. In order for the thermal inflation to dilute this dilaton energy density the dilaton $\phi$ should start to oscillate before the thermal inflation. Thus we assume that the Hubble parameter $H$ during the thermal inflation is less than $m_\phi$. Then, the abundance of the ‘big bang dilaton’ before the thermal inflation is given by \[9\]

$$\left(\frac{n_\phi}{s}\right)_{BB} \simeq \frac{m_\phi^2/2}{8.6m_\phi^3/2} \frac{\phi_0^2}{17m_\phi^{1/2}M_G^{3/2}}. \quad (11)$$

The abundance of ‘big bang dilaton’ after the thermal inflation is given by

$$\left(\frac{n_\phi}{s}\right)_{BB} \simeq 4 \left(\frac{T_c}{m_\chi}\right)^3 \left(\frac{\phi_0}{M_G}\right)^2 \left(\frac{M_G}{m_\phi}\right)^{1/2} \frac{m_\phi^3T_R}{V_0}. \quad (12)$$

In addition to the ‘big bang dilaton’, the dilaton oscillation is reproduced by the thermal inflation since the minimum of the potential is shifted from its true vacuum by an amount of $\delta \phi \sim (V_0/m_\phi^2M_G^2)\phi_0 [9]$, which results in the dilaton density $\sim m_\phi^2\delta \phi^2/2$. The abundance of this ‘thermal inflation dilaton’ is estimated as

$$\left(\frac{n_\phi}{s}\right)_{TI} \simeq \frac{3}{8} \left(\frac{\phi_0}{M_G}\right)^2 \frac{V_0T_R}{m_\phi^3M_G^2}. \quad (13)$$

Let us consider first the case of $m_\chi \geq 130\text{GeV}$. The lower bound of total energy density of the dilaton $\phi$ is given by

$$\frac{\rho_\phi}{s} \simeq m_\phi \max \left[\left(\frac{n_\phi}{s}\right)_{BB}, \left(\frac{n_\phi}{s}\right)_{TI}\right] \simeq m_\phi \sqrt{\left(\frac{n_\phi}{s}\right)_{BB} \left(\frac{n_\phi}{s}\right)_{TI}}$$

\[10\] We have assumed that full reheating after ordinary inflation occurs before the thermal inflation. A more detailed analysis without this assumption will be given by in Ref. [13].
\[
\sum \left( \frac{\phi_0}{M_G} \right)^2 \left( \frac{T_c}{m_\chi} \right)^{3/2} \left( \frac{m_\chi^2}{m_\phi M_G} \right)^{3/4} T_R.
\]

We see that the minimum abundance is given for the lowest possible reheating temperature \( T_R = 10 \text{MeV} \). For \( \phi_0 \approx M_G, T_c \approx m_\chi, T_R \approx 10 \text{MeV} \) and \( m_\chi \geq 130 \text{GeV} \) we obtain the lower bound of \( \rho_\phi/s \) as

\[
\frac{\rho_\phi}{s} \geq 5.3 \times 10^{-11} \text{GeV} \left( \frac{m_\phi}{\text{MeV}} \right)^{-3/4}.
\]

Comparing with the critical density of the present universe, \( \rho_c/s = 3.6 \times 10^{-9} h^2 \text{GeV} \), we find

\[
\Omega_\phi h^2 \equiv \frac{\rho_\phi h^2}{\rho_c} \geq 1.5 \times 10^{-2} \left( \frac{m_\phi}{\text{MeV}} \right)^{-3/4},
\]

where \( h \) is the present Hubble parameter in units of 100km/sec/Mpc. We show in Fig. 1 this lower bound. We see that the predicted lower bound of the dilaton density may be taken below the critical density for \( m_\phi > 70 \text{keV} \).

Now we consider the case of \( m_\phi < 130 \text{GeV} \). In this case we use the reheating temperature \( T_R \) given in Eq. (7) to write \( M \) in terms of \( T_R \) and \( m_\chi \) as \( M \approx 10^{-4} m_\chi^{3/2} M_G^{1/2} T_R^{-1} \). Then \( V_0 \) is written from Eqs. (3) and (8) as \( V_0 \approx 10^{-8} (2n + 4)^{-1} m_\chi^5 M_G T_R^{-2} \). The abundance of the dilaton are given by

\[
\frac{n_\phi}{s}_{BB} \approx 4 \left( \frac{T_c}{m_\chi} \right)^3 \left( \frac{\phi_0}{M_G} \right)^2 \left( \frac{M_G}{m_\phi} \right)^{1/2} \frac{(2n + 4) 10^8 T_R^3}{m_\chi^2 M_G}.
\]

and

\[
\frac{n_\phi}{s}_{TI} \approx \frac{3}{8} \left( \frac{\phi_0}{M_G} \right)^2 \frac{10^{-8} m_\chi^5}{(2n + 4) m_\phi^3 M_G T_R}.
\]

The lower bound of the total abundance is achieved when \( (n_\phi/s)_{BB} = (n_\phi/s)_{TI} \), namely

\[
m_\chi,\text{min} \approx 190 (2n + 4)^{2/7} M_G^{1/14} T_R^{4/7} m_\phi^{5/14}.
\]

This yields (from Eq.(14))

\[
\Omega_\phi h^2 \geq \frac{1.5 \times 10^{-2} \left( \frac{m_\chi,\text{min}}{130 \text{GeV}} \right)^{3/2} \left( \frac{m_\phi}{\text{MeV}} \right)^{-3/4}}{2.3 \times 10^{-3} \left( \frac{m_\phi}{\text{MeV}} \right)^{-3/14}}.
\]

Here we have taken \( (2n + 4)^{3/7} \approx 2, \phi_0 \approx M_G, T_c \approx m_\chi \) and \( T_R \approx 10 \text{MeV} \). We also show this lower bound in Fig. 1. Notice that a kink appears at \( m_\phi \approx 20 \text{MeV} \). This is because \( m_\chi,\text{min} \) in Eq. (19) exceeds 130GeV for \( m_\phi \approx 20 \text{MeV} \) and the dilaton
abundance takes its minimum at $m_\chi = 130\text{GeV}$. We see from Fig. 1 that for all region of $m_\phi \simeq 10^{-2}\text{keV} - 1\text{GeV}$ relevant to gauge-mediated SUSY breaking models the lower bound of $\Omega_\phi h^2$ is below the critical density $\Omega h^2 \simeq 0.25$ in the present universe.

We are now at the point to derive a constraint from the observed $X(\gamma)$-ray backgrounds.

First, we should estimate the lifetime of the dilaton. The main decay mode of dilaton is a two-photon process, $\phi \rightarrow 2\gamma$, since the decay mode to two neutrinos has a chirality suppression and vanishes for massless neutrinos \cite{11}. The dilaton $\phi$ has a couplings to two photons as

$$L_\phi = \frac{b}{16\pi\alpha_{em}M_{G}^2} F_{\mu\nu} F^{\mu\nu}. \tag{21}$$

Here, $b$ is a parameter of the order 1 which depends on the type of superstring theories and compactifications.\textsuperscript{11} We simply take $b = 1$ in the present analysis.\textsuperscript{12}

Then, the lifetime $\tau_\phi$ of the dilaton is estimated as

$$\tau_\phi \simeq \frac{1}{20} \frac{M_{pl}^2}{m_\phi^3} \simeq 6 \times 10^{21} \text{sec} \left(\frac{m_\phi}{\text{MeV}}\right)^{-3}. \tag{22}$$

The $X(\gamma)$-ray flux from the dilaton decay is given by \cite{11}

$$F_\gamma(E) = \frac{n_{\phi,0}}{2\pi \tau_\phi H_0 \Omega_0} \left(\frac{E}{m_\phi}\right)^{3/2} \exp \left[ - \frac{2}{3\tau_\phi H_0 \Omega_0^{1/2}} \left(\frac{E}{m_\phi}\right)^{3/2} \right], \tag{23}$$

where $n_{\phi,0}$ is the present dilaton number density, $H_0$ the present Hubble parameter, $\Omega_0$ the total density of the present universe in units of the critical density, and $E$ the energy of $X(\gamma)$-ray. The flux $F_\gamma$ takes the maximum value $F_{\gamma,max}$ at

$$E_{max} = \frac{m_\phi}{2} \quad \text{for} \quad \tau_\phi > \frac{2}{3} H_0^{-1} \Omega_0^{-1/2}$$

$$= \frac{m_\phi}{2} \left(\frac{3\tau_\phi H_0 \Omega_0^{1/2}}{2}\right)^{2/3} \quad \text{for} \quad \tau_\phi < \frac{2}{3} H_0^{-1} \Omega_0^{-1/2}. \tag{24}$$

By requiring that $F_{\gamma,max}$ should be less than the observed $X(\gamma)$-ray backgrounds \cite{16, 17, 18}, we obtain a constraint on $\Omega_\phi h^2$ which is also shown in Fig. 1. We see

\textsuperscript{11}For example we get $b = \sqrt{2}$ \cite{14} for a compactification of the M-theory \cite{15}.

\textsuperscript{12}We note that our conclusion does not depend heavily on the parameter $b$. If one takes $b = 0.3$, for instance, the excluded region ($100\text{keV} \lesssim m_\phi \lesssim 1\text{GeV}$) derived in the present analysis moves slightly to $200\text{keV} \lesssim m_\phi \lesssim 1\text{GeV}$. 

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that the mass region $100\text{keV} \lesssim m_\phi \lesssim 1\text{GeV}$ is excluded by the observed X($\gamma$)-ray backgrounds.

In summary, we have shown that generic models for the thermal inflation are successful to dilute the energy density of the coherent dilaton oscillation below the critical density of the present universe. However, we have found that the constraint from the experimental upperbounds on the cosmic X($\gamma$)-ray backgrounds is much more stringent and it excludes the dilaton mass region, $100\text{keV} \lesssim m_\phi \lesssim 1\text{GeV}$.

This raises a new problem in recently observed interesting models [5, 6] for gauge-mediated SUSY breaking as long as $m_\phi \approx m_3/2$ as expected in a large class of superstring theories. On the other hand, the region, $10^{-2}\text{keV} \lesssim m_\phi \lesssim 100\text{keV}$, survives the constraint. In this region the lower bounds of the dilaton density $\Omega_\phi h^2$ are achieved when $m_\chi \approx 10\text{GeV}$ and $M \approx 10^9\text{GeV}$ which implies the cut-off scale $M_\ast \approx 10^{17}\text{GeV}$ for $n = 1$. It may be encouraging that the cut-off scale $M_\ast$ is not far below the gravitational scale $M_G \approx 2.4 \times 10^{18}\text{GeV}$.

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References


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If one assumes $\phi_0 \approx 0.1M_G$, a small window around $m_\phi \approx 1\text{GeV}$ appears.


Figure 1: The lower bounds of the dilaton density $\Omega_\phi h^2$ in the presence of the thermal inflation for $m_\phi > 130\text{GeV}$ (long-dashed line) and $m_\phi < 130\text{GeV}$ (short-dashed line). The dotted line represents $\Omega h^2 = 0.25$. The experimental upperbound on $\Omega_\phi h^2$ from the X-ray backgrounds is shown by solid curve.