Decay of $Z$ into Three Pseudoscalar Bosons

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Abstract

We consider the decay of the $Z$ boson into three pseudoscalar bosons in a general two-Higgs-doublet model. Assuming $m_A$ to be very small, and that of the two physical neutral scalar bosons $h_1$ and $h_2$, $A$ only couples to $Z$ through $h_1$, we find the $Z \to AAA$ branching fraction to be negligible for moderate values of $\tan \beta \equiv v_2/v_1$, if there is no $\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c.$ term in the Higgs potential; otherwise there is no absolute bound but very large quartic couplings (beyond the validity of perturbation theory) are needed for it to be observable.
If the standard $SU(2) \times U(1)$ electroweak gauge model is extended to include two scalar doublets, there will be a neutral pseudoscalar boson $A$ whose mass may be small. In that case, the decay of the $Z$ boson into 3 $A$’s may not be negligible. This process was first studied\cite{1} in a specific model\cite{2}. It was then discussed\cite{3} in a more general context. More recently, it has been shown\cite{4} that there is a lower bound on $m_A$ of about 60 GeV in the Minimal Supersymmetric Standard Model (MSSM), hence the decay $Z \rightarrow AAA$ is only of interest for models with two scalar doublets of a more general structure. Even in the context of supersymmetry, this is possible\cite{5} if there exists an additional U(1) gauge factor at the TeV scale.

In this paper we consider a general two-Higgs-doublet model and identify the conditions for which the decay $Z \rightarrow AAA$ may be enhanced, despite the nonobservation of $e^+e^- \rightarrow h + A$, where $h$ is either one of the two neutral scalar bosons of the model. We will show that in principle this decay is limited only by the scalar coupling $\lambda_1 - \lambda_2$ as defined below. However, if $\lambda_5 = 0$, which is true in a large class of models\cite{6}, then it may be bounded as discussed below.

Let the Higgs potential $V$ for two $SU(2) \times U(1)$ scalar doublets $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)$ be given by

$$V = m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_5^*(\Phi_2^\dagger\Phi_1)^2,$$  \hspace{1cm} (1)

where the discrete symmetry $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$ is only broken softly by the $m_{12}^2$ term. Assume $\lambda_5$ to be real for simplicity. Define $\tan \beta \equiv v_2/v_1$ as is customary, where $v_{1,2} = \langle \phi_{1,2}^0 \rangle$ are the usual two nonzero vacuum expectation values. The pseudoscalar neutral Higgs boson is then

$$A = \sqrt{2}(\sin \beta \text{Im}\phi_{1}^0 - \cos \beta \text{Im}\phi_{2}^0),$$  \hspace{1cm} (2)
with mass given by
\[ m_A^2 = -m_{12}^2 (\tan \beta + \cot \beta) - 2\lambda_5 v^2, \]  
(3)

where \( v^2 \equiv v_1^2 + v_2^2 \), and the charged Higgs boson is
\[ h^\pm = \sin \beta \phi_1^\pm - \cos \beta \phi_2^\pm, \]  
(4)

with
\[ m_{h^\pm}^2 = m_A^2 + (\lambda_5 - \lambda_4) v^2. \]  
(5)

To get the maximum \( Z \rightarrow AAA \) rate, we let \( m_A = 0 \), i.e.
\[ m_{12}^2 = -2\lambda_5 v^2 \sin \beta \cos \beta. \]  
(6)

Then the mass-squared matrix spanning the two neutral scalar Higgs bosons \( \sqrt{2} \text{Re} \phi_{1,2}^0 \) is given by
\[ M^2 = 2v^2 \begin{pmatrix} \lambda_1 \cos^2 \beta + \lambda_5 \sin^2 \beta & (\lambda_3 + \lambda_4) \sin \beta \cos \beta \\ (\lambda_3 + \lambda_4) \sin \beta \cos \beta & \lambda_2 \sin^2 \beta + \lambda_5 \cos^2 \beta \end{pmatrix}. \]  
(7)

Consider now the following two linear combinations:
\[ h_1 = \sqrt{2}(\sin \beta \text{Re} \phi_1^0 - \cos \beta \text{Re} \phi_2^0), \]  
(8)
\[ h_2 = \sqrt{2}(\cos \beta \text{Re} \phi_1^0 + \sin \beta \text{Re} \phi_2^0). \]  
(9)

It is well-known that \( h_1 \) couples to \( AZ \) but not \( ZZ \), whereas \( h_2 \) couples to \( ZZ \) but not \( AZ \). However, the process \( e^+e^- \rightarrow h + A \) is in general possible because \( h \) will normally have a \( h_1 \) component, thereby putting a constraint on \( m_A \) if kinematically allowed. For our purpose, we will require \( h_1 \) and \( h_2 \) to be mass eigenstates, in which case \( m_A \) is unconstrained by the nonobservation of \( e^+e^- \rightarrow h + A \) even if \( m_2 \) is small, as long as \( m_1 \) is larger than the \( e^+e^- \) center-of-mass energy. This allows us to have the maximum effective coupling of \( Z \) to \( AAA \) as shown below.

The requirement that \( h_1 \) and \( h_2 \) be mass eigenstates leads to the condition
\[ \lambda_2 \sin^2 \beta - \lambda_1 \cos^2 \beta + (\lambda_3 + \lambda_4 + \lambda_5)(\cos^2 \beta - \sin^2 \beta) = 0. \]  
(10)
As a result, the masses of $h_{1,2}$ are given by

$$m_1^2 = [\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta + \lambda_5 - \lambda_3 - \lambda_4]v^2, \quad (11)$$

$$m_2^2 = [\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta + \lambda_5 + \lambda_3 + \lambda_4]v^2. \quad (12)$$

Note that in the MSSM, Eq. (10) cannot be satisfied in the presence of radiative corrections.

We now extract the $h_1 AA$ coupling from Eq. (1), using Eqs. (2) and (8). We find it to be given by

$$\sin 2\beta \sqrt{2} (\lambda_1 - \lambda_2)v, \quad (13)$$

where Eq. (10) has been used. As a function of $\beta$, this expression is obviously maximized at $\sin 2\beta = \pm 1$. On the other hand, our conditions so far do not limit the combination $\lambda_1 - \lambda_2$, hence there is no absolute bound on $Z \rightarrow AAA$ in this general case.

Let us consider the case $\lambda_5 = 0$. This is natural in a large class of models where the two Higgs doublets are remnants[6] of a gauge model larger than the standard model such that they are distinguishable under the larger symmetry. In that case, we have

$$m_1^2 = 2(\lambda_1 - \lambda_3 - \lambda_4)v^2 \cos^2 \beta = 2(\lambda_2 - \lambda_3 - \lambda_4)v^2 \sin^2 \beta, \quad (14)$$

and we can rewrite (13) as

$$-\frac{m_1^2}{v \sqrt{2}} \cot 2\beta. \quad (15)$$

The above expression appears to be unbounded as $\sin 2\beta \rightarrow 0$. However, that would require very large quartic scalar couplings. This can be seen two ways. First, since (15) is equal to (13), we need an extremely large value of $\lambda_1 - \lambda_2$. Second, from Eq. (14), we see also that if $\sin \beta$ is small, then $\lambda_2 - \lambda_3 - \lambda_4$ has to be big, and if $\cos \beta$ is small, then $\lambda_1 - \lambda_3 - \lambda_4$ has to be big. Thus we will choose moderate values of $\tan \beta$ in (15) for the following discussion.

In Figure 1 we show the diagram for the decay $Z \rightarrow AAA$ with an intermediate virtual $h_1$. To maximize this rate, we minimize $m_1$ to be just above the maximum experimental
$e^+e^-$ center-of-mass energy, which is 172 GeV up to now but will soon be 183 GeV. As for $h_2$, it interacts exactly as the one Higgs boson of the standard-model, from which we have the experimental limit\[7\] of $m_2 > 65$ GeV. However, $m_2$ is not directly involved in the $h_1AA$ coupling here. Note also that $\lambda_4$ by itself must be large and negative so that $m_{h^\pm}$ of Eq. (5) can be greater than $m_t - m_b$ for $m_A = 0$, so as to prevent the decay $t \rightarrow b + h^\pm$. This condition is not satisfied in the MSSM where $\lambda_4 = -g_2^2/2$, hence $m_A = 0$ is not allowed there\[4\].

Assuming $\lambda_5 = 0$ and using Eq. (15) with $m_1 = 180$ GeV and $|\cot 2\beta| = 1$ (i.e. $\tan \beta = 0.4$ or 2.4), we now calculate the $Z \rightarrow AAA$ decay rate, following Ref. [1]. The amplitude is given by

$$\mathcal{M} = g_Z m_1^2 \sqrt{2} v \left[ \frac{\epsilon \cdot k_1}{(p-k_1)^2 - m_1^2} + \frac{\epsilon \cdot k_2}{(p-k_2)^2 - m_1^2} + \frac{\epsilon \cdot k_3}{(p-k_3)^2 - m_1^2} \right], \quad (16)$$

where $g_Z = e / \sin \theta_W \cos \theta_W$, $p$ is the four-momentum of the $Z$ boson, and $k_{1,2,3}$ are those of the $A$'s. The effective coupling used in Ref. [1] is now determined to be

$$\lambda_{\text{eff}} = \frac{m_1^2 \sqrt{2}}{v^2} \approx 1.5. \quad (17)$$

Using the estimate of Ref. [1], this $Z \rightarrow AAA$ rate is then about $1.0 \times 10^{-7}$ GeV. Hence its branching fraction is about $4 \times 10^{-8}$ which is clearly negligible. To obtain a branching fraction of $10^{-6}$, we need $\cot 2\beta = 5$ (i.e. $\tan \beta = 0.1$ or 10). In this case, either $\lambda_1 - \lambda_3 - \lambda_4$ or $\lambda_2 - \lambda_3 - \lambda_4$ in Eq. (14) has to be about 53.5. If $\lambda_5 \neq 0$, then we cannot use Eqs. (14) and (15), but Eq. (13) is still valid. To obtain a branching fraction of $10^{-6}$, we will then need $|\lambda_1 - \lambda_2|$ to be about 53.5. Thus in both scenarios, one or more quartic scalar couplings have to be very large and beyond the validity of perturbation theory.

If $h_1$ and $h_2$ are not exact mass eigenstates, then there is an additional contribution from $h_1 - h_2$ mixing which is necessarily very small from the constraint of experimental data if
$m_2$ is below 172 GeV. The $h_2AA$ coupling is given by

$$
\frac{v}{\sqrt{2}} \left( \frac{m_2^2}{2v^2} - 2\lambda_5 [1 - \sin^2 \beta \cos^2 \beta] \right). \quad (18)
$$

If $\lambda_5 = 0$, this expression is bounded independent of $\tan \beta$ and the overall contribution (including the small $h_1 - h_2$ mixing) is negligible. If $\lambda_5 \neq 0$, then its value has to be huge for the process to be observable.

The reason that $\Gamma(Z \to AAA)$ is so small is twofold. One is that with the higher energy reached by LEP2, the nonobservation of $Z \to h + A$ forces $m_1$ to be much greater than $M_Z$. The other is that for $m_1 >> M_Z$, the leading term in $M$ vanishes because $\epsilon \cdot (k_1 + k_2 + k_3) = 0$, resulting in a very severe suppression factor[1]. Our conclusion is that the decay $Z \to AAA$ is not likely to be observable in a general two-Higgs-doublet model with parameters in the perturbative regime.

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References


Figure Caption

Fig. 1. One of 3 diagrams for the decay $Z \rightarrow AAA$. The other 2 are obvious permutations.