Quantum-Controlled Few-Photon State Generated by Squeezed Atoms

Hiroki Saito\(^1\) and Masahito Ueda\(^1,2\)

\(^1\)Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801-3080
\(^2\)Department of Physical Electronics, Hiroshima University, Higashi-Hiroshima 739, Japan

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General principles and experimental schemes for generating a desired few-photon state from an aggregate of squeezed atoms are presented. Quantum-statistical information of the collective atomic dipole is found to be faithfully transferred to the photon state even in a few-photon regime. The controllability of few-photon states is shown to increase with increasing the number of squeezed atoms.

\[ \hat{S}_y = \sum_i \hat{\sigma}_{iy} / 2, \text{ and } \hat{S}_z = \sum_i \hat{\sigma}_{iz} / 2, \text{ where } \hat{\sigma}_{ix}, \hat{\sigma}_{iy}, \text{ and } \hat{\sigma}_{iz} \text{ denote the Pauli spin operators for the } i \text{th atom.} \]

Assuming the Jaynes-Cummings interaction [5] between the atoms and the photon field, the total Hamiltonian is given by

\[ \hat{H} = \hbar \omega_a \hat{S}_z + \hbar \omega_f \hat{a}^\dagger \hat{a} + \hbar g (\hat{a} \hat{S}_+ + \hat{a}^\dagger \hat{S}_-), \]

where \( \hat{a}^\dagger \) and \( \hat{a} \) are the creation and annihilation operators of the photon field, \( \hat{S}_+ = \hat{S}_x + i \hat{S}_y \), \( \hbar \omega_a \) is the energy difference between the two levels of the atoms, \( \hbar \omega_f \) is an energy quantum of the photon, and \( g \) is a coupling constant. When \( \omega_f = \omega_a \), we can eliminate the noninteracting part of the Hamiltonian \( H_0 = \hbar \omega_a \hat{S}_z + \hbar \omega_f \hat{a}^\dagger \hat{a} \) by working on a rotating frame \( e^{i \hat{H}_0 t / \hbar | \psi \rangle} \). Since we want to manipulate the width and the orientation of the uncertainty ellipse in phase space in any desired direction, it is convenient to introduce operators in the direction specified by the azimuth angle \( \phi \) as \( \hat{a}_\phi \equiv \frac{1}{2} (\hat{a} e^{-i \phi} + \hat{a}^\dagger e^{i \phi}) \) and \( \hat{S}_\phi \equiv \frac{1}{2} (\hat{S}_x e^{-i \phi} + \hat{S}_y e^{i \phi}) \). These operators obey the following equations of motion:

\[ \frac{d \hat{a}_\phi}{dt} = -g \hat{S}_\phi^+ \hat{S}_\phi^+ / 2, \]

\[ \frac{d \hat{S}_\phi^+}{dt} = -2g \hat{a}_\phi \hat{S}_\phi^- \]

\[ \frac{d \hat{S}_\phi^-}{dt} = 2g (\hat{a}_\phi \hat{S}_\phi^+ + \hat{a}_\phi^+ \hat{S}_\phi^-) \]  \( (4) \)

When the atoms are irradiated by coherent light with classical intensity, the mean field approximation is valid, and Eqs. (2)-(4) reduce to the familiar optical Bloch equations. Since we are interested in reducing quantum fluctuations in the few-photon regime, we have to take into account higher-order correlations. In particular, we are interested in the variance of \( \hat{a}_\phi \). Its dynamical evolution is governed by

\[ \frac{d\langle (\Delta \hat{a}_\phi)^2 \rangle}{dt} = -2g \langle (\Delta \hat{a}_\phi) (\Delta \hat{S}_\phi^+ / 2) \rangle, \]

\[ \frac{d^2\langle (\Delta \hat{a}_\phi)^2 \rangle}{dt^2} = g^2 \left( 4 \langle (\Delta \hat{a}_\phi) (\Delta \hat{S}_\phi^-) \rangle + 2 \langle (\Delta \hat{S}_\phi^- / 2) \rangle \right) \]  \( (5) \)

where \( \Delta \hat{O} \equiv \hat{O} - \langle \hat{O} \rangle \) for an arbitrary operator \( \hat{O} \). When the field is initially in the vacuum state, there is no initial correlation between the atoms and the field, so the right-hand side of Eq. (5) vanishes at \( t = 0 \) and
Im distribution defined by ures show the initial squeezed atom states, prepared by squeezed state from squeezed fifty atoms. The left figure is obtained only by replacing and below the standard quantum limit at times much shorter than ~ g\(^{-1}\). The field displacement \(\langle \hat{a}_\phi \rangle\) and its variance \((\langle \Delta \hat{a}_\phi \rangle^2)\) can be controlled independently because their time evolutions are governed respectively by \(\langle \dot{\hat{S}}_{-\phi + \pi/2} \rangle\) and \(\langle \Delta \hat{S}_{-\phi + \pi/2} \rangle\). From Eqs. (2)-(6) we find that the amplitude squeezed state is obtained from the atomic state that satisfies, e.g., \(\langle \hat{S}_x \rangle = 0, \langle \hat{S}_y \rangle \neq 0, \langle \hat{S}_z \rangle < 0\), and \((\langle \Delta \hat{S}_y \rangle^2) < \langle \langle \Delta \hat{S}_x \rangle^2 \rangle/2\). The phase squeezed state is obtained only by replacing \((\langle \Delta \hat{S}_y \rangle^2)\) for \((\langle \Delta \hat{S}_x \rangle^2)\). The right figures in Fig. 1 (a) and (b) illustrate generation of the amplitude squeezed state and the phase squeezed state from squeezed fifty atoms. The left figures show the initial squeezed atom states, prepared by the scheme discussed below, in the spin quasi-probability distribution defined by \(\langle \theta, \phi | \hat{\rho}_{\text{atom}} | \theta, \phi \rangle\), where \(\langle \theta, \phi \rangle = e^{-i\theta \hat{S}_z}e^{-i\phi \hat{S}_y} | S, S_z = S \rangle\) is the coherent state of spin or angular momentum and will be referred to as the Bloch state [6]. The quasi-probability distribution of the photon field \(Q(\alpha) = \text{Tr}_{\text{atom}}[\langle \alpha | \hat{\rho} \langle \alpha \rangle] / \pi\) (right figures in Fig. 1 (a) and (b)) is obtained by numerical diagonalization of the Jaynes-Cummings Hamiltonian, where \(\langle \alpha \rangle\) is the coherent state of the radiation field with amplitude \(\alpha\), and \(\hat{\rho}\) is the density operator of the entire system when the maximal squeezing is obtained. From Fig. 1 we find that the profile of \(Q(\alpha)\) follows that of \(\langle \theta, \phi | \hat{\rho}_{\text{atom}} | \theta, \phi \rangle\) projected on the \(S_x\)-\(S_y\) plane, where \(S_x\) and \(S_y\) correspond to \(\text{Im} \alpha\) and \(\text{Re} \alpha\). This rather faithful transfer of quantum information from the atomic system to the photon system holds in general, and tells us how to prepare the collective atomic state in order to produce a desired few-photon state. Figure 1(c) shows the time evolutions of the atomic and field observables for the case of Fig. 1(b). The radiation-field amplitude \(\langle \hat{a} \rangle\) grows as the mean spin vector tilts towards the negative z-axis (i.e., \(\theta \rightarrow 0\)). We also note that quantum fluctuations in the radiation field, \((\langle \Delta \hat{a}^2 \rangle^2)\), decreases in time at the expense of increasing atomic fluctuations \((\langle \Delta \hat{S}_z \rangle^2)\).

For a collection of atoms to be able to radiate a photon state that is squeezed in any desired direction in phase space, we will refer to as tailor-made radiation, condition (7) has to be met for arbitrary \(\phi\). Thus the necessary and sufficient condition for the tailor-made radiation is given by

\[
\langle (\Delta \hat{S}_{\phi + \pi/2})^2 \rangle < \frac{|\langle \hat{S}_z \rangle|}{2},
\]

where \((\langle \Delta \hat{S}_{\phi}^\text{min} \rangle^2)\) denotes the minimum value of the variance perpendicular to the mean spin vector \(\langle \hat{S} \rangle\). In a special case where the total spin is conserved, Eq. (8) reduces to the criterion obtained in Refs. [4,7] which discussed the interferometric phase sensitivity. A crucial observation is that phase squeezing (as in Fig. 1 b) can only be obtained by states satisfying the condition (8). This is because fluctuations projected on the \(S_x\)-\(S_y\) plane cannot be reduced to below \((\langle \Delta \hat{S}_{\phi}^\text{min} \rangle^2)\) in the direction of \(\phi\) by any rotation of the spin vector. This is why the Bloch state which has an isotropic uncertainty distribution with respect to the plane perpendicular to the mean spin vector cannot radiate the phase squeezed state. The
The photon fluctuation (10) attains a minimum value given the number of atoms. We seek the maximally of the spin obtained by the interaction with the coherent state, since these models are mathematically equivalent [14], and interaction of the spins through nonlinear Hamiltonians [11].

In Ref. [13] the Jaynes-Cummings Hamiltonian is used, where the state of atoms is indicated in the spin quasi-probability distribution at each stage. It consists of three stages: (1) The excited two-level atoms are injected into the first cavity in which the atoms become squeezed by interacting with the coherent state of the radiation field [\(\alpha\)] prepared by laser or maser [13]. (2) The output squeezed atoms pass through the coherent field with classical intensity. This field rotates the mean spin vector in the spin space to the desired direction. To control the rotation axis the coherent field and the classical field must be driven synchronously with an appropriate phase difference provided by the phase shifter. (3) The atoms go into the third vacuum cavity, radiate photons and come out of the cavity before reabsorbing the emitted photons. Left in the third cavity is the desired photon state which we can take out by switching the Q-factor of the cavity mechanically or by applying a magnetic field.

We propose two possible experimental schemes to implement our theory. The first one is a scheme using the micromaser technique [15] as illustrated in Fig. 3, where the state of atoms is indicated in the spin quasi-probability distribution at each stage. It consists of three stages: (1) The excited two-level atoms are injected into the first cavity in which the atoms become squeezed by interacting with the coherent state of the radiation field [\(\alpha\)] prepared by laser or maser [13]. (2) The output squeezed atoms pass through the coherent field with classical intensity. This field rotates the mean spin vector in the spin space to the desired direction. To control the rotation axis the coherent field and the classical field must be driven synchronously with an appropriate phase difference provided by the phase shifter. (3) The atoms go into the third vacuum cavity, radiate photons and come out of the cavity before reabsorbing the emitted photons. Left in the third cavity is the desired photon state which we can take out by switching the Q-factor of the cavity mechanically or by applying a magnetic field.

![FIG. 2. Possible range of amplitude \(|\langle \hat{a}\rangle|\) and its variance \(|\langle (\Delta \hat{a})^2 \rangle|\) of the radiation field that can be obtained by 2, 5, 10, 50, and 100 atoms prepared by interaction with the coherent state.](image-url)
In conclusion, we have shown that the quantum-statistical information of collective atomic dipoles is faithfully transferred to the radiation field even in a few-photon regime. This implies that we can produce a desired few-photon state by preparing atoms in an appropriate squeezed state. This idea can be tested using a high-Q cavity that sustains more than one atom undergoing the Jaynes-Cummings interaction with the radiation field.

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FIG. 3. Schematic illustration of an experimental setup to generate a few-photon state that features any desired quantum statistics. The state of the atoms at each stage is shown by the spin quasi-probability distribution. The two-level excited atoms go into the first cavity and interact with a coherent state of the radiation field $|\alpha\rangle$. The output atoms are in a squeezed spin state (SSS). By interaction with a classical field in the second cavity, the mean spin vector is rotated to a desired direction, where the rotation axis can be specified by the phase shifter. The atoms then go into the third cavity and emit photons there. Left in the third cavity is the desired few-photon state which can be extracted by our changing the cavity quality factor.

The second scheme is to employ the atom trapping and the laser-cooling, in which the above three stages are implemented at the same place. For this purpose, the optical cavity should be off-resonant during the preparation of the atomic state, and be resonant only at the time of radiation. Interaction with the coherent state corresponding to the first stage above can be realized by interaction with the center-of-mass oscillation of atoms through the stimulated Raman transition [16]. This second scheme has the advantage of producing a large number of squeezed atoms.

In an actual experiment, we must finish the whole sequence of processes before the two-level atoms decay into other levels. If we use, for example, the $63p_{3/2} \rightarrow 61d_{3/2}$ transition of rubidium atoms, the lifetime is of order millisecond and the coupling constant is $g \sim 10^4$ Hz. Since the required interaction time is $gt \sim 1$, i.e., $t \sim 10^{-4}$ sec, the whole procedure can be accomplished within the atomic lifetime. The finite Q-factor of the cavity will not be an obstacle, since the cavity lifetime now reaches $t_c \sim 10^{-1}$ sec in the microwave regime [2]. Thermal photons, however, must be carefully suppressed. If we use the above-mentioned transition (21.5 GHz), the temperature should be below, say, 0.2 K in order to suppress the average number of thermal photons in the cavity to below 0.01.

[12] S. M. Barnett and M.-A. Dupuis, J. Opt. Soc. Am. B 4, 505 (1987). Although this reference used condition (9) as a criterion for dipole squeezing, we wish to point out that the state constructed there also satisfies condition (8), and hence may be used for the tailor-made radiation.
[13] Heidmann, et al. considered the situation in which atoms are initially in a coherent atomic state and the field...


[14] From the relation $e^{i\pi S_x}S_\pm e^{-i\pi S_x} = S_\mp$, we find that time evolution with $\hat{H}_2$ can be rewritten by

$$e^{-i\hat{H}_2t}\left|\overline{S}\right\rangle = e^{-i\hat{H}_2t}e^{-i\pi S_x}\left|S\right\rangle = e^{-i\pi S_x}e^{-i\hat{H}_2t}\left|S\right\rangle,$$

which indicates that both models are equivalent rotating the initial and final spin state.
