New Probes of Supersymmetry Beyond the Minimal Framework

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If supersymmetry is discovered at future colliders, what can we learn? While our appreciation of the variety of possible supersymmetric models has grown tremendously in recent years, most attempts to answer this question have been in the context of some simple and highly restrictive framework, such as minimal supergravity. In this talk I describe new probes of phenomena that are generic in models beyond the minimal framework. These include tests of supersymmetric flavor and CP violation and probes of kinematically inaccessible superparticle sectors through “super-oblique corrections.” Such probes have wide applicability to distinguishing models, from gravity- and gauge-mediated theories to hybrid models and models with flavor symmetries. Examples of measurements at LEP II, the LHC, and the NLC are given.

1. INTRODUCTION

The discovery of supersymmetry (SUSY) in low energy experiments or in current and future high energy colliders is at present a subject of great interest. It is important to bear in mind, however, that the discovery of supersymmetry will be just the first step on the long road toward determining the particular form of supersymmetry realized in nature. Such a program will require the precise measurement of superparticle properties and will likely be the focus of experimental high energy physics for decades.

While low energy experiments are certainly promising places to look for the effects of new physics, they are unlikely to yield strong bounds on SUSY parameters. In fact, even the unambiguous identification of supersymmetry as the source of new low energy signals may be far from straightforward. It is almost certain, therefore, that input from high energy colliders will be required to measure SUSY parameters and to thereby determine which of the many possible SUSY models is viable.

Given these considerations, an obvious question is, if SUSY is discovered at colliders, what can we learn? Despite the importance of this question, the answer is, at the moment, far from thoroughly understood. The essential difficulty is that, although the low energy supersymmetric theory is weakly coupled, and so observables are in principle calculable to high precision, there are generically a large number of parameters in supersymmetric models with even the most minimal field content [1]. For this reason, in any study of possible experimental probes of SUSY, one must first choose a theoretical framework. This is illustrated in Fig. 1, where various theoretical SUSY frameworks are listed with the number of new SUSY parameters accompanying them. Minimal supergravity (mSUGRA) has its well-known 5 free parameters: \( m_0, m_{1/2}, A, \tan \beta \), and the sign of \( \mu \). However, one may consider more general theories by successively relaxing unification conditions and arriving at SU(5) grand unified theories, with 17 free parameters, and the (unified) MSSM, with 31. If one further relaxes flavor, CP, and R-parity conservation, the number of parameters increases dramatically, and, of course, ultimately one can consider models with non-minimal field content (General SUSY). On the other hand, one may consider more unified and highly constrained frameworks until one arrives at string theory, which one might hope would ultimately be a theory with no new parameters. Unfortunately, at present it is nearly equivalent to...
General SUSY in terms of its predictive power for low energy SUSY parameters suitable for experimental studies.

Fig. 1 is, of course, rather simplistic, and does not include many other possible frameworks. The essential point, however, is that there is a wide variety of theoretical frameworks, and any choice of framework is necessarily a compromise between simplicity and generality. By far the most popular choice has been minimal supergravity, the “minimal framework” referred to in the title of this talk. For example, the Snowmass ’96 studies were conducted almost exclusively in this framework [2]. While this is an excellent and sometimes necessary starting point, it has led to a lamentable, if understandable, dichotomy in the field of SUSY phenomenology. On the one hand, our understanding of the variety of possible particle spectra, flavor structures, and SUSY breaking mechanisms has grown tremendously in recent years, leading to a rapid growth in the number of viable and appealing SUSY models. At the same time, much of the effort directed toward investigating the prospects for studying SUSY at colliders has begun with the highly restrictive assumption that the underlying theory is minimal supergravity.

In this talk, I will present a number of new probes of supersymmetric models beyond the minimal framework. The motivations for relaxing the strong assumptions of mSUGRA and considering less simple frameworks are many, but a few of them may be listed here:

- Strong theoretical assumptions are inappropriate for studies hoping to determine the prospects for testing theoretical assumptions.
- Studies based on mSUGRA assumptions are fragile. If a single prediction of mSUGRA is disproven, studies based on mSUGRA must be revised, or, worse, may become inapplicable.
- SUSY alone is already a highly constrained framework, and so strong assumptions are not (all) necessary to obtain meaningful results. For example, studies have shown that superparticles with masses up to $\sim 1$ TeV may be discovered at the LHC, even without the assumption of R-parity conservation [3]. Other studies find that gaugino mass unification need not be assumed, but in fact may be tested both at LEP II [4] and future linear $e^+e^-$ colliders [5,6].
- Strong assumptions artificially suppress phenomena generic in SUSY. For example, the scalar mass matrices are generically new sources of flavor and CP violation in SUSY, but such violations are absent in the minimal framework. The presence (or the absence) of such phenomena may in fact be powerful tools for excluding some models and favoring others, as we will see below.
- Strong assumptions preclude important lessons for collider plans. In the present climate for building new colliders, it is essential that general scenarios be considered in evaluating collider proposals. As we will see below, studies of SUSY beyond the minimal framework have important lessons for collider parameters and options that are not evident in the minimal framework.

At present, one of the leading motivations for models beyond minimal supergravity is the desire to find new solutions to the supersymmetric flavor problem, i.e., the problem that low energy constraints are violated for generic sfermion masses and mixings. The problem may be solved by sfermion degeneracy, fermion-sfermion alignment, very massive sfermions, or some combination of these three. In this talk, I will describe a number
of collider probes that may help determine which solution is realized in nature, and may therefore be used to eliminate some models in favor of others. In Sec. 2, probes of scalar flavor structures at both $e^+e^-$ and hadron machines will be described. In Sec. 3, I will discuss probes of models with very heavy particles. Such particles will not be directly discovered at colliders, but their properties may be probed through “super-oblique parameters,” supersymmetric analogues of the oblique parameters of the standard model which will be introduced below. This talk is an overview of work presented in Refs. [7–10].

2. SFERMION FLAVOR MIXING

If supersymmetric particles are discovered, measurements of their masses will be of primary importance. Such measurements have been well-studied. Their intergenerational mixings will also be of great interest, however, especially given the importance of the SUSY flavor problem. In the standard model, all flavor mixing is confined to the CKM matrix. However, in any supersymmetric extension of the standard model, both lepton and quark flavor are typically violated by supersymmetric interactions. These new violations arise because the scalar partners of the fermions must be given mass, and the scalar mass matrices are generally not diagonal in the same basis as the fermion masses. There are then 7 new independent flavor matrices, $W_{\alpha a}$, $a = u_L, d_L, e_L, \nu_L$, which are analogues of the CKM matrix and provide a rich variety of new phenomena that may be studied at colliders.

Here we will focus on the leptonic sector. In the standard model, lepton flavor is conserved, and so any lepton flavor violation observed in the processes discussed below is necessarily supersymmetric in origin. The $W$ matrices appear in gaugino/Higgsino vertices. For neutralinos $\chi^0$, these vertices are given by the interactions

$$
\bar{e}_L W_{Lia} e_L a \chi^0 + \bar{\nu}_L W_{Lia} \bar{\nu}_L a \chi^0 + \bar{e}_R a W_{Ria} e_R \chi^0 + \bar{\nu}_R a W_{Ria} \bar{\nu}_R \chi^0 ,
$$

where the Latin and Greek subscripts are generational indices for scalars and fermions, respectively. For full three generation mixing, the two matrices $W_L$ and $W_R$ may be parametrized by 6 mixing angles and 4 phases.

2.1. Flavor Violation

In this section we investigate probes of flavor mixing, and for simplicity, we will set the CP-violating phases to zero and consider two generation mixing. It is important to remember that low energy bounds already provide significant constraints on the $W$ matrices. The most stringent constraints are from $B(\mu \to e\gamma) < 4.9 \times 10^{-11}$ [11], $B(\tau \to e\gamma) < 2.7 \times 10^{-6}$ [12], and $B(\tau \to \mu\gamma) < 3.0 \times 10^{-6}$ [12]. We will be ambitious by considering $\bar{\nu}_R - \bar{\nu}_R$ mixing, which competes directly with the strongest of these, the bound on $\mu \to e\gamma$. (13 and 23 mixing, as well as mixings among the left-handed sleptons, may also be considered in analyses similar to the one described below.) The mixing matrix $W_R$ may then be parametrized by a single mixing angle $\theta_{12}$. With this mixing, the leading contributions to $\mu \to e\gamma$ [13] in the gaugino region are from the two diagrams of Fig. 2. (In the mixed or Higgsino regions, additional contributions reduce $B(\mu \to e\gamma)$ [14].) Note that the rate depends on many SUSY parameters, and so $\mu \to e\gamma$ alone will not provide model-independent constraints on the SUSY parameters. Both amplitudes are proportional to $\sin 2\theta_{12}$ and are superGIM suppressed in the limit of degenerate sleptons. The second amplitude is also proportional to the left-right mixing parameter $\hat{\ell} \equiv (-A + \mu \tan \beta)/\hat{m}_{12}$. The total rate then has the form

$$
B(\mu \to e\gamma) \sim f(\hat{\ell}) \left( \frac{\Delta m_{12}^2}{\hat{m}_{12}^2} \sin 2\theta_{12} \right)^2 ,
$$

where $\Delta m_{ij}^2 = \langle m_i^2 - m_j^2 \rangle/2$ is the average squared slepton mass, and $\Delta m_{ij}^2 = \langle m_i^2 - m_j^2 \rangle/2 \approx 2m\Delta m_{ij}$, with $\Delta m_{ij} = m_i - m_j$. Note that the superGIM mechanism suppresses the rate for $\Delta m < m$.

We now consider the rate for flavor-violating signals at colliders. These signals arise from processes involving on-shell slepton production with unlike flavor leptons in the final state. For sim-
contributions from this reaction as $M$ denote sets of particles. This process is more complicated and is presented in Ref. [8]; however, the essential conclusions given below remain unchanged.

As with the low energy signal, the flavor-violating collider signal vanishes in the limit of degenerate sleptons, as it must. However, in marked contrast with the low energy signal, the collider signal is suppressed only for $\Delta m < \Gamma$. There is thus a large region of parameter space with $\Gamma \sim \Delta m \sim m$ in which the low energy signal is highly suppressed, but the collider signal may be large.

We now determine the reach of various colliders in the flavor mixing parameter space. We begin with LEP II, and assume an integrated luminosity of 500 pb$^{-1}$ at $\sqrt{s} = 190$ GeV. To present quantitative results, we choose slepton masses just beyond the current bounds $\tilde{m}_{12} \approx 80$ GeV and assume the LSP is Bino-like with mass $M_1 = 50$ GeV. The effects of variations from this scenario are discussed in Ref. [7].

With these choices, the flavor-violating signal we are looking for is $e^+e^- \rightarrow e^\pm\mu^\mp\tilde{\chi}_i^0\tilde{\chi}_j^0$. The largest background is $W$ pair production, but this may be reduced significantly with appropriate cuts. In Fig. 4, flavor-violating cross sections are given by the solid contours, with the 5$\sigma$ discovery signal given by the thick contour. Bounds from $\mu \rightarrow e\gamma$ are also plotted — note that the two diagrams of Fig. 2 interfere destructively, and so $B(\mu \rightarrow e\gamma)$ is not monotonic in $t$ and even vanishes for certain $t$. We see that for low values of $\hat{t}$, LEP II extends the reach in parameter space, and for mixing angles $\theta_{12} \gtrsim \theta_C$, where $\theta_C$ is the Cabibbo angle, lepton flavor violation may be discovered at LEP II. It is remarkable...
that for a significant region in parameter space, LEP II, which will produce at most a few hundred sleptons, is more sensitive to lepton flavor violation than $\mu \rightarrow e\gamma$, with its astounding statistics.\(^5\) This example indicates the extremely promising prospects for precision SUSY measurements once superpartners are discovered.

At future linear colliders, the prospects for lepton flavor violation discovery are even brighter. Highly polarized $e^-$ beams may also be used to reduce backgrounds. In Fig. 5, the discovery reach at the NLC is shown for $\int L = 50 \text{ fb}^{-1}$, $\sqrt{s} = 500 \text{ GeV}$, a 90% right-polarized $e^-$ beam, and suitably scaled up SUSY parameters. The NLC is sensitive to mixing angles $\theta_{12} \gtrsim 0.05$, and extends the reach in parameter space beyond current $\mu \rightarrow e\gamma$ bounds even for $\hat{t} = 50$. In fact, these bounds can be improved still further by using the $e^- e^-$ mode of the NLC [7].

### 2.2. CP Violation

In the presence of CP violation in the slepton mass matrix, the cross sections $\sigma_{\alpha\beta} = \sigma(f_1, f_2 \rightarrow e_\alpha^+ X e_\beta^- Y)$ and $\sigma_{\beta\alpha} = \sigma(f_1, f_2 \rightarrow e_\beta^+ X e_\alpha^- Y)$ are no longer equal [8,15].\(^6\) It is easy to show from Eq. (4) that

$$\Delta_{\alpha\beta} \equiv \sigma_{\alpha\beta} - \sigma_{\beta\alpha} = -4\sigma_0 \times$$

$$\sum_{i<j} \text{Im} \left[ W_{i\alpha} W_{i\beta}^* W_{j\beta}^* W_{j\alpha} \right] \text{Im} \left[ \frac{1}{1 + i x_{ij}} \right]. \ (6)$$

Again, the sum is over sleptons that may be produced on-shell. We therefore reproduce the familiar result that CP violation requires the presence of at least two amplitudes that differ both in their CP-odd (“weak”) phases and their CP-even (“strong”) phases. For what parameters can

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\(^5\)Of course, in the best case scenario in which $\mu \rightarrow e\gamma$ is also discovered, both low and high energy measurements will be useful for determining the flavor-violating parameters.

\(^6\)Phases in SUSY parameters such as $\mu$ and the gaugino masses may change these cross sections, but do not distinguish between flavors, and so do not contribute to the CP-violating difference.
this asymmetry be large? The slepton mass-dependent (CP-even) part is \(-x_{ij}/(1 + x_{ij}^2)\). The
\(W\)-dependent (CP-odd) part may be written as
\[
\text{Im} \left[ W_{\alpha L} W_{\beta L}^* W_{\alpha R}^* W_{\beta R} \right] = \bar{J} \sum_{k\gamma} \varepsilon_{ijk} \varepsilon_{\alpha\beta\gamma},
\]
where \(\bar{J}\) is the supersymmetric analogue to the Jarlskog invariant, and is the unique re-phase invariant that may be formed from a single \(W\) matrix. In a standard parametrization \([16]\), \(\bar{J} = \sin \theta_{12} \cos \theta_{13} \sin \theta_{13} \sin \theta_{23} \cos^2 \theta_{23} \sin \delta\), where \(\theta_{ij}\) are the 3 mixing angles, and \(\delta\) is the CP-violating phase. We see then that the CP-violating collider signal is maximal for \(\Delta m \sim \Gamma\) and requires full 3 generation mixing and, of course, a significant CP-violating phase \(\delta\).

The two rephase invariants \(\bar{J}_{L,R}\), each formed from a single \(W\) matrix, govern the sizes of CP-violating collider signals. As in the case of flavor violation, it is important to understand to what extent these signals are already bounded by low energy constraints, such as, in this case, the electric dipole moment (EDM) of the electron. Recall, however, that there are 4 irreducible phases in the two \(W\) matrices. The EDM of the electron constrains the rephase invariants \(\bar{K}_{12} = \text{Im} \left[ W_{L21} W_{L23}^* W_{R21}^* W_{R22} \right]\) and \(\bar{K}_{13} = \text{Im} \left[ W_{L31} W_{L33}^* W_{R31}^* W_{R33} \right]\). The four independent invariants \(\bar{J}_{L,R}\) and \(\bar{K}_{12,13}\) therefore form a convenient basis - colliders probe \(\bar{J}_{L,R}\) and the electron EDM probes \(\bar{K}_{12,13}\). Formally, we see that electron EDM constraints provide no bounds on collider signals. Of course, it would be unnatural to expect a large hierarchy between the \(\bar{J}\)’s and \(\bar{K}\)’s. However, one may check that for \(\Delta m \sim \Gamma\), whose collider probe may be relevant, the bounds on the \(\bar{K}\)’s from \(\delta_e < 4 \times 10^{-2}\) cm \(\sim \bar{K}_{12,13}\) are extremely weak.

We now consider possible probes of the CP-violating cross section asymmetries at future colliders. First, note that we are looking for a statistically significant asymmetry \(\Delta_{\alpha\beta}\) in a sample of dilepton events. Contributions to this sample from non-slepton events dilute this asymmetry, and so we must isolate a large pure sample of slepton events. Second, it is interesting that Eqs. (6) and (7) imply that all three possible asymmetries are equal, up to a sign: \(|\Delta_{12}| = |\Delta_{13}| = |\Delta_{23}|\). Of course, backgrounds and other experimental issues entering the measurements of these asymmetries differ from channel to channel. We will only consider the 12 asymmetry here, but note that ultimately all three asymmetries may be combined to provide the most powerful probe of \(\bar{J}\). Finally, as we must now consider general three generation mixing, the parameter space is fairly complicated. To present our results, we characterize the mixing by a single angle \(\theta = \theta_{12} = \theta_{13} = \theta_{23}\), and similarly, we fix \(\Delta m = \Delta m_{12} = \Delta m_{23}\). Variations from these assumptions may be found in Ref. [8].

Let us begin by considering the NLC. Again, we must choose parameters to present quantitative results, and we consider the NLC parameters chosen above with \(m_{\tilde{g}} \approx 200\) GeV. Right-handed beam polarization and appropriate cuts effectively reduce the number of dilepton events from the main backgrounds \(WW, e\nu W, \) and \(ee WW\). The resulting \(3\sigma\) CP violation discovery contours are presented in Fig. 6 for various integrated luminosities. We see that, for \(\Delta m \approx \Gamma, \bar{J}\) as low as \(10^{-3}\) to \(10^{-2}\) may be probed. The NLC provides a robust probe of slepton CP violation, for the most part requiring only that slepton pairs be kinematically accessible. In addition, left- and right-handed slepton flavor and CP violation may be disentangled by gradually raising the beam energy or using beam polarization.

Probes of slepton CP violation are also possible at hadron machines if a large sample of slepton events may be isolated. At the LHC with \(\sqrt{s} = 14\) TeV, the most promising source of sleptons is in cascade decays of squarks and gluinos. To study this possibility, we consider a scenario studied in Ref. [18], in which the following cascade decays occur (sparticle masses in GeV and branching ratios in each step are indicated):
\[
\tilde{g} (767) \xrightarrow{31\%} \tilde{t}_L (688) \xrightarrow{32\%} \tilde{t}_R (231) \xrightarrow{36\%} \tilde{t}_R (157).
\]
Left-handed sleptons are heavier than \(\tilde{\chi}_2^0\) and so are almost never produced. The \(\tilde{t}_R\) events may be isolated with the cuts of Ref. [18], and the resulting reach is given by the thick contour of Fig. 7. The reach is, of course, highly sensitive to the choice of SUSY parameters. For example, we also present results for different gluino/squark
Figure 6. CP Violation at the NLC. $3\sigma$ slepton CP violation discovery contours for the integrated luminosity given (in fb$^{-1}$). The CP-violating phase is fixed to $\sin \delta = 1$. The SUSY parameters are as given in the text, with $\Gamma = 0.58$ GeV $= 2.9 \times 10^{-3} m$.

Figure 7. CP Violation at the LHC. $3\sigma$ slepton CP violation discovery contours for an integrated luminosity of 100 fb$^{-1}$. The CP-violating phase is fixed to $\sin \delta = 1$. The wide contour is for the parameters discussed in the text, where $m_\tilde{\chi} = 767$ GeV, and $\Gamma = 0.13$ GeV $= 8.1 \times 10^{-4} m$. The other contours give an indication of possible discovery reaches for scenarios with $m_\tilde{\chi} = m_\tilde{q}$ shown (in GeV). See full discussion in the text.

masses as indicated, where we have made the naive assumption that the dilepton signal and background cross sections simply scale with the total squark/gluino pair production cross section. We see that for lighter squarks and gluinos, dramatic improvements result from the increased statistics. Of course, the results are also dependent on the various branching ratios, and may be improved by considering both LHC detectors and multi-year runs. We see, however, that if sleptons are produced in gluino and squark cascades, probes of $J$ to the level of $10^{-3}$ may be possible.

2.3. Implications for Models

The discovery of slepton flavor or CP violation will have many strong implications for models. If slepton flavor violation is discovered, it will be clear that the stringent low energy constraints on lepton flavor violation are satisfied not by very massive sfermions or by sfermion-fermion alignment, but by sfermion degeneracy. Such a discovery therefore excludes pure mSUGRA and pure gauge-mediated models, in which the flavor-blind mediation of SUSY breaking guarantees the degeneracy of sleptons at a certain mass scale, and hence renders the slepton mixing matrices trivial. Such measurements also have strong implications for models with flavor symmetries, as they make definite predictions for the $W$ matrices.

At the same time, we will know that the sleptons cannot be completely degenerate and have mass splittings $\lesssim \Gamma$. It is worthwhile to note that lepton flavor violation is sensitive to mass splittings at the scale of $\Gamma \approx 0.1 - 1$ GeV, a level that may be extremely difficult to achieve by conventional kinematic techniques.

The discovery of CP violation will have even greater consequences. As noted above, CP violation requires not only a significant CP-violating phase, but also full three generation mixing. To-
gether with the low energy constraints, CP violation implies that all three sleptons are degenerate to $\sim \Gamma$. These requirements strongly suggest that the slepton mass matrix has the form

$$m^2_{\tilde{l} \tilde{l}} = m^2_0 \left[ 1 + \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \right],$$

(8)

where the $\epsilon_{ij}$ are of order $\Gamma/m$ and are of comparable size for all $i, j$. That any one measurement leads to such a specific texture is rather remarkable. Such a texture could arise in hybrid models, in which the large flavor-blind piece arises from one source (e.g., gauge-mediated SUSY breaking), and the small $\epsilon_{ij}$ contributions arise from another (e.g., supergravity with a large fundamental SUSY-breaking scale) [19].

3. VERY MASSIVE SUPERPARTNERS

As seen in Sec. 2, in models beyond the minimal framework, a wide variety of flavor mixing phenomena may be present, and, if sfermions are produced at future colliders, the study of such phenomena may lead to important insights. It is possible, however, that some part of the particle spectrum will be beyond the reach of future colliders. In fact, this possibility is realized in a wide variety of models, and is often found in theories designed to solve the supersymmetric flavor problem. These models may be roughly divided into two categories. In the first class of models, which we will refer to as “heavy QCD models,” the gluino and all the squarks are heavy. Such may be the case in models with gauge-mediated SUSY breaking, where strongly-interacting particles get large contributions to their masses, in no-scale supergravity, and generically in models with a heavy gluino, which drives the squark masses up through renormalization group evolution. In a second class of models, “2–1 models,” the first and second generation sfermions have masses $\mathcal{O}(10 \text{ TeV})$, while the third generation sfermions are at the weak scale [20]. Such models are motivated by the desire to satisfy low energy constraints from, for example, $K^0 - \bar{K}^0$ mixing and $\mu \rightarrow e\gamma$, without the need for sfermion universality, sfermion alignment, or small CP-violating phases. At the same time, the extreme fine-tuning problem arising from very massive third generation sfermions is alleviated.

3.1. Super-oblique Corrections

In all of the models described above, the heavy superpartners may well be beyond the discovery reach of planned future colliders and decouple from most experimental observables. However, the mass scale, and possibly even other properties, of such a sector may be probed by a class of precision measurements we now discuss [9,10,21–23]. These probes rely on the fact that SUSY alone, without further assumptions, already provides stringent constraints on the form of the low energy theory. In particular, let us denote the standard model gauge couplings by $g_i$ and let $h_i$ be their supersymmetric analogues, the gaugino-sfermion-sfermion couplings, where the subscript $i = 1, 2, 3$ refers to the U(1), SU(2), and SU(3) gauge groups. SUSY implies that the relations

$$g_i = h_i$$

(9)

hold to all orders in the limit of unbroken SUSY. However, SUSY-breaking mass differences within supermultiplets with standard model quantum numbers lead to corrections to Eq. (9) that grow logarithmically with the superpartner masses. Such deviations from Eq. (9) are thus unambiguous, non-decoupling, model-independent signals of SUSY-breaking mass splittings, and by precisely measuring such deviations in processes involving accessible superparticles, bounds on the mass scale of the kinematically inaccessible particles may be determined.

The corrections to Eq. (9) are highly analogous to the oblique corrections of the standard model [9,21]. In the standard model, non-degenerate SU(2) multiplets lead to inequivalent renormalizations of the propagators of the $(W, Z)$ vector multiplet, inducing non-decoupling effects that grow with the mass splitting. Similarly, in supersymmetric models, non-degenerate supermultiplets lead to inequivalent renormalizations of the propagators within each (gauge boson, gaugino) vector supermultiplet, inducing non-decoupling effects that grow with the mass splitting. We will therefore refer to the latter effects
as “super-oblique corrections” and parametrize them by “super-oblique parameters” [9]. These corrections are particularly important because they receive additive contributions from every split supermultiplet and so may be significantly enhanced. Furthermore, the simple nature of the corrections allows one to bound them with many processes in a model-independent fashion.

The coupling constant splittings between $g_i$ and $h_i$ result from differences in wavefunction renormalizations, and so are most analogous to the oblique parameter $U$. We therefore define

$$\tilde{U}_i \equiv h_i/g_i - 1,$$

(10)

where the subscript $i$ denotes the gauge group. These parameters receive corrections from superpartners of the standard model particles, and also from possible exotic supermultiplets. For example, we find [9]

$$\tilde{U}_1 \approx 0.35\% (0.29\%) \times \ln R$$

(11)

$$\tilde{U}_2 \approx 0.71\% (0.80\%) \times \ln R$$

(12)

$$\tilde{U}_3 \approx 2.5\% \times \ln R$$

(13)

for 2–1 models (heavy QCD models), where $R = M/m$ is the ratio of heavy to light mass scales.\(^7\) (In heavy QCD models, the gluino is decoupled, and so no measurement of $\tilde{U}_3$ is possible.) We see that measurements of these parameters at the percent level may be able to detect variations from exact SUSY, and may even be able to measure the heavy scale $M$. Contributions from vector-like (messenger) sectors have also been calculated [9], and were found to be significant only for highly split supermultiplets with masses $\lesssim 100$ TeV.

3.2. Measurements of Super-oblique Parameters

Super-oblique corrections are present in all processes involving gauginos, and so may be measured at colliders in a variety of ways, depending on what sparticles are available for study. The possible observables at both future lepton and hadron colliders were systematically classified in Ref. [10], where detailed and representative studies of each of the three super-oblique parameters were also presented.

Here we will focus on a test of the U(1) couplings, using selectron production at a future linear collider. We will consider the $e^-e^-$ option at such machines, where a number of beautiful properties make an extremely precise measurement possible. Let us consider first right-handed selectrons. At an $e^-e^-$ collider, $\tilde{e}_R$ production takes place only through $t$-channel neutralino exchange (see Fig. 8). Note that while the SUSY process we are interested in is allowed because the neutralino is a Majorana fermion, many other would-be backgrounds are absent; for example, $W^-W^-$ pair production is forbidden by total lepton number conservation. The backgrounds that do remain, such as $e^-\nu W^-$, are small and may also be further suppressed by polarizing both beams.

The cross section possesses a number of important properties. First, it tends to be large relative to the $e^+e^-$ cross section. For $m_{\tilde{e}_R} \approx 150$ GeV, $M_1 = 100$ GeV, and a total integrated luminosity of 50 fb\(^{-1}\), the statistical error is only 0.3%. Second, the cross section is proportional to $h_1^2$ and is therefore highly sensitive to super-oblique corrections. Third, and most importantly, a Majorana mass insertion in the neutralino propagator is needed. This greatly reduces theoretical systematic errors, which are typically dominated by uncertainties in the $\tilde{e}_R$ and $\tilde{\chi}_1^0$ masses. These masses are constrained by electron energy distribution endpoints. The resulting allowed masses are positively correlated; for example, typical al-

\(^7\)Finite corrections may be absorbed through a small shift in this definition of $R$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Figure8}
\caption{Selectron pair production at an $e^-e^-$ collider.}
\end{figure}
Figure 9. The allowed regions, “uncertainty ellipses,” of the $(m_{\tilde{e}_R}, m_{\tilde{\chi}_0^1})$ plane, determined by measurements of the end points of final state electron energy distributions with uncertainties $\Delta E = 0.3$ GeV and 0.5 GeV. The underlying central values are $(m_{\tilde{e}_R}, m_{\tilde{\chi}_0^1}) = (150$ GeV, 100 GeV), and $\sqrt{s} = 500$ GeV. We also superimpose contours (in percent) of the fractional variation of $\sigma_R$ with respect to its value at the underlying parameters.

allowed regions for a year’s worth of data and the SUSY parameters above are given by the ellipses in Fig. 9. Contours of constant cross section are also given. We see that, since the cross section decreases for increasing $m_{\tilde{e}_R}$, but increases for increasing $M_1$ because of the Majorana mass insertion, the contours of constant cross section run nearly parallel to the major axes of the ellipses. Thus, the systematic error on the cross section from the mass measurements is less than 0.5%.

Combining these errors, we find that the cross section can be measured to $\lesssim 0.6\%$, and so the super-oblique parameter $U_1$ can be measured to $\lesssim 0.15\%$. Such a measurement constrains the heavy superpartner scale $M$ to within a factor of 1.5. For left-handed selectrons, the dominant diagram will be $\tilde{W}$ exchange, and so the cross section will be proportional to $h^2_\gamma$. The heavy superpartner scale may then be constrained even more accurately.

Of course, at the percent level, experimental systematic errors may become important; such uncertainties depend on collider designs and are subjects of current investigation. It is clear, however, that if selectrons are produced at future colliders, the prospects for precision measurements of super-oblique parameters are indeed promising. Measurements of $U_2$ from chargino production [6,10] and selectron production [22] in the more conventional $e^+ e^-$ mode of linear colliders, and of $U_3$ from squark branching ratios [10] have also been studied, with generally weaker but still promising results.

3.3. Implications for Models

The implications of measurements of the super-oblique parameters depend strongly on what scenario is realized in nature. As we have seen, if some number of superpartners are not yet discovered, bounds on the super-oblique parameters may lead to bounds on their mass scale. In addition, if measurements of more than one super-oblique parameter may be made, some understanding of the relative splittings in the heavy sector may be gained. Inconsistencies among the measured values of the different super-oblique parameters could also point to additional inaccessible exotic particles with highly split multiplets that are not in complete representations of a grand-unified group. All such insights will provide important input for model building.

If, on the other hand, all superpartners of the standard model particles are found, the consistency of all super-oblique parameters with zero will be an important check of the supersymmetric model with minimal field content. If instead deviations of the super-oblique parameters from zero are found, such measurements will provide exciting evidence for exotic (messenger) sectors with highly split multiplets not far from the weak scale [9]. These insights could also provide a target for future sparticle searches, and could play an important role in evaluating future proposals for colliders with even higher energies, such as the muon collider or higher energy hadron machines.
4. CONCLUSIONS

The discovery of supersymmetry will be just the beginning of a long and exciting road toward determining which supersymmetric theory is realized in nature. Our understanding of the variety of possible supersymmetric theories has grown dramatically in recent years, and it is important to determine what colliders and what techniques may provide stringent tests to distinguish such models. In this talk, I have discussed examples of phenomena that are generic in supersymmetry beyond the minimal supergravity framework and may in fact be very useful for differentiating the vast array of possible models. Future colliders may be able to provide stringent probes of sfermion flavor and CP violation, well beyond current low energy bounds. In addition, as many well-motivated models predict that some superpartners are beyond the discovery reach of future colliders, we have described methods of probing these sectors through non-decoupling “super-oblique corrections.” Just as the oblique corrections of the standard model provide strong constraints on technicolor models and other extensions of the standard model, the super-oblique parameters may provide powerful constraints otherwise inaccessible physics, and may also have wide implications for theories beyond the minimal supersymmetric standard model.

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REFERENCES

2. For a review, see J. Bagger, these proceedings.
20. See A. Nelson, these proceedings, and references therein; N. Polonsky, these proceedings.