ABSTRACT: The recent GUT (× SUSY) models which can predict the neutrino properties are reviewed.
The fact that talks about neutrinos dominated this workshop, shows the growing popularity of this subject. Neutrino physics will be probably the first field to teach us about the physics beyond the standard model (SM). It is interesting therefore to know what are the predictions for the neutrino-properties in possible extensions of the SM. The most natural ones are obviously the Grand Unified Theories (GUTs) and Supersymmetric-GUTs (SUSY-GUTs) and I will limit myself in this talk to those theories. “Low-energy” anti-GUT models were discussed in the talk of Valle [1].

Neutrinos and the problem of fermionic masses
The masses of the charged fermions are arbitrary in the SM and the neutrinos remain massless\(^3\). The SM must be therefore extended to be able to account for the fermionic masses. The extension into GUTs is in particular interesting as they lead to relations between the masses (but those are not enough and one needs on top of that also a family symmetry).

In contrast with the charged fermions the neutrino-masses and lepton-mixing are unknown phenomenologically. The subject of this talk is to review GUT models which can predict the neutrino sector. The idea is, to look for theories which can use known information about charged fermions to fix the neutrino Dirac mass matrix \(M_D^\nu\) as well as the RH neutrino one \(M_R^\nu\). In this way the light neutrino mass matrix is obtained using the seesaw mechanism as

\[
M_{\nu,\text{light}} = -M_D^\nu M_R^{-1} M_D^{\nu T}.
\]

This is the most natural way to obtain light neutrino masses when the model involves a large mass scale \(M_{\nu_R}\), like in the GUTs.

Why GUTs (× SUSY)?
There is a large list of indications for GUTs:

- The unification of the gauge coupling constants.
- The Yukawa unification: \(m_\tau(M_{\text{GUT}}) \simeq m_b(M_{\text{GUT}})\) and other relations between the masses which are very useful for predicting the neutrino properties.
- The dynamical electroweak symmetry breaking obtained when the soft SUSY breaking is universal at \(M_{\text{GUT}}\).

ALL AT THE SAME SCALE \(M_{\text{GUT}} \simeq 10^{16}\text{GeV}\), also

- The high scale allows for the see-saw mechanism.
- GUTs can naturally explain the baryon asymmetry directly or induced via leptogenesis.

\(^3\)as the SM has no RH neutrinos and the accidental B-L symmetry protects the masslessness.
Modern cosmology requires GUTs.

Superstring theories have GUTs as a possible intermediate effective theory.

GUTS CANOT EXPLAIN EVERYTHING BUT THEY ARE VERY GOOD EFFECTIVE THEORIES AT $M_{GUT} \simeq 10^{16}$GeV

What are the predictive GUTs for the neutrino sector?
Min. SU(5) ($\times$ SUSY) – has no RH neutrinos and the vanishing neutrino masses are protected by B-L. Small masses can be obtained via gravitations affects but no real prediction for the leptonic mixing emerges. We will discuss here therefore SO(10) and $E_6$ only.

What is the mass scale of the RH neutrinos (needed for the see-saw mechanism)?
The relevant scale for MSW solution for the Solar Neutrino Puzzle (SNP) [2] and neutrinos as dark matter is $M_{\nu_R} \simeq 10^{12}$GeV . [1]
The interesting point about this scale is that it is also the scale of other phenomenae like

- the B-L violation
- the breaking of the Peccei-Quinn symmetry
- the leptogenesis which induces the baryon asymmetry
- the SUSY breaking in the hidden space

SUSY-GUTs however do not allow for an intermediate symmetry. Hence the natural scale for SUSY-GUTs is $M_{\nu_R} \simeq M_{GUT}$ . Such a scale can be used for the vacuum oscillation solution to the SNP because $m_{\nu}^2/M_{GUT} \approx 10^{-6}$eV . But most SUSY-GUTs models for the neutrinos use still $M_{\nu_R} \simeq 10^{12}$GeV and this requires in general fine tuning$^4$.

It is interesting to note here that allowing for an intermediate scale around $M_I \simeq 10^{12}$GeV, one can break L-R symmetric GUTs, like SO(10) and $E_6$, in two steps and obtain gauge and Yukawa unification even without SUSY. Also, before going to discuss explicit models let me make one more general remark. It is relatively “simple” to fix $M_{\nu}^D$ using GUT relations, but simple GUTs cannot say a thing about $M_{\nu}^R$ . Many models use therefore ad-hoc assumptions about $M_{\nu}^R$ . E.g. taking it proportional to a unite matrix or to a diagonal one with a given hierarchy. However, TO PREDICT THE NEUTRINO PROPERTIES, THE MODEL MUST FIX THE RH NEUTRINO MASS MATRIX $M_{\nu}^R$ AND NOT ONLY THE DIRAC ONE $M_{\nu}^D$.

$^4$Note, however, that $(M_{GUT})^2 / M_{Pl}^2 \approx 10^{13}$GeV what suggests that non-renormalizable contributions due to an underlying theory above the GUT (superstrings?) may generate such a scale.
Examples for masses models for the charged fermions and their extension into the neutrino sector:

**Texture Zeros**

Zero entries in the mass matrice lead to relations between mixing angles and mass ratios. In the framework of SUSY-GUTs, Roberts, Ramond and Ross [3] found five possible sets of symmetric mass matrices with more than three texture zeros which are consistent with the charge fermions phenomenology. Those can be extended into the leptonic sector using the good SO(10) (E$_6$) GUT relations:

a) $Y_\tau(M_{GUT}) = Y_d(M_{GUT})$ (induced via one $H_{10}$) using the renormalization group equations to run the relation down, one obtains the “observed”

$$3Y_\tau(1\text{GeV}) \simeq Y_\theta(1\text{GeV})$$

b) Georgi-Jarlskog [4] relations (induced via $H_{126}$):

$$Y_\alpha(M_{GUT}) = \frac{1}{3} Y_\mu(M_{GUT}); \quad Y_d(M_{GUT}) = 3Y_\tau(M_{GUT})$$

i.e.

$$\text{det}M_\ell(M_{GUT}) = \text{det}M_d(M_{GUT})$$

c) similar relations between $M_u$ and $M^D_\nu$ are possible but GUT relations do not say what $M^R_\nu$ is.

*How can we predict $M^R_\nu$?*

**Several examples and ideas:**

(i) *The simplest possibility:*

To use a symmetry which forces all Yukawa matrices (including $M^R_\nu$) to have the same texture [5].

This horizontal symmetry, on top of the SUSY-GUT gives additional relations between the entries. In particular $M^D_\nu$ and $M^R_\nu$ are fixed in terms of the matrices of the charged fermions. E.g. The symmetric Fritzsch texture [6] known to give a large lepton-mixing [7].

One can use therefore

$$|M^R_\nu| \simeq M_{GUT}$$

to solve the SNP via vacuum oscillation of $\nu_1 - \nu_2$.

Problem: the model requires $m_t < 150\text{GeV}$ (can be avoided with small changes?)

(ii) *Minimality + Predictability* [8] [9]

All matrix elements of the Yukawa matrices are due to one VEV only. The horizontal symmetry and the minimal Higgs structure fix $M^R_\nu$ in terms of the parameters of the quark and charged leptons. Results: “The standard scenario”: I.e. small leptonic mixing angles which can solve the solar neutrino puzzle via MSW. Also $\nu_3$ can be the hot dark matter.
Asymmetric textures
Most recent models for the quark mass (and mixing) use asymmetric mass matrices but these were not yet applied for the neutrino-sector. An interesting exception is, however, models of Babu and Barr [10] with large leptonic mixing. In these models new vector like fields (which get explicit large masses) are added. The latter have mixed mass terms with the light fermions in such a way that the LH mixing angles of the quarks are small while the RH ones are large, but the opposite is true for the lepton.

Broken gauged Abelian symmetry $U(1)_X$
Based on the idea of Froggat and Nielsen [11] as applied to SUSY-GUTs [12]. $U(1)_X$ dictates zero entries in the mass matrices by forbidding certain Yukawa couplings. The general idea is that the heavy family acquires masses by direct coupling to the light Higgs. The light families obtain their masses when $U(1)_X$ is spontaneously broken. This is done by giving a VEV to a chiral SM singlet field $\theta$ with the charge $X_\theta = -1$. The GUT obtains then non-renormalizable contribution like

$$n_{ij} \bar{\psi}_i \psi_j H(\frac{\theta}{M})^{(Q_i+Q_j+Q_H)}$$

where $Q_i$ are the $U(1)_X$ charges of the field and $M$ the scale of the $U(1)_X$ invariant underlying theory (e.g. $M_{\text{Planck}}$).

Noting $\epsilon = \frac{<\theta>}{M} << 1$ and $Q_{ij} = Q_i + Q_j + Q_H > 0$, the Yukawa matrices have then the general form

$$Y = \begin{pmatrix}
n_{11} e^{Q_{11}} & n_{12} e^{Q_{12}} & n_{13} e^{Q_{13}} \\
n_{21} e^{Q_{21}} & n_{22} e^{Q_{22}} & n_{23} e^{Q_{23}} \\
n_{31} e^{Q_{31}} & n_{32} e^{Q_{32}} & n_{33} e^{Q_{33}}
\end{pmatrix}$$

Assuming $n_{ij} \sim O(1)$ one can choose the $Q_{ij}$ such that a fit to the quark masses and mixing is obtained. At this point GUT relations to obtain $M_\ell$ and $M^{D}_\nu$ can be used. The models [13] differ in details especially for $M^R_\nu$. The resulting neutrino properties give in general the “Standard Scenario”. This is by far the most popular type of models for the leptonic sector. Its nice property is that it “explains” the observed hierarchy in masses and mixing angles of the charged fermions. It is however, NOT SO PREDICTIVE FOR THE NEUTRINO-SECTOR. This is because of the $n_{ij} \sim O(1)$ unknown factors. $M^{light}_\nu$ in the see-saw expression eq.(1) is a product of three matrices, hence THERE IS AN UNKNOWN CORRECTION OF $[O(1)]^3$! Also, the charges $Q_{ij}$ are fixed by hand.

Correlated zeros in $M^{D}_\nu$ and $M^R_\nu$.
This is a way suggested by Binétruy et al [14] to solve the last problem in case (iv). The correlation induces a natural mass degeneracy (independent of $n_{ij}$) and large mixing angles. Its breaking leads to only small $O(1)$ deviation from the full degeneracy (and not $[O(1)]^3$ as in the previous case). They give several explicit examples and compare them with the neutrino-anomalies. The idea is very nice but the models use in the present form many ad hoc requirements.

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5Note, that the $n_{ij}$ can be quite large like the Georgi- Jarlskog $n_{22} = -3$ factor.
(vi) Non-Abelian family symmetries
are the natural way to avoid $O(1)$ factors. One can fix at least part of the coefficients
of the non-renormalizable contributions in terms of Clebsch-Gordans of the non-Abelian
group [8].

Another advantage here is that one can account for the asymmetry between the heavy
family and the light ones. This is done by putting the families in a $1 + 2$ representation
under U(2) [15] or $S(3)^3$ [16].

An interesting new model based on the family group $\Delta(48) \times U(1)$ is due to Chou and
Wu [17]. They added a sterile neutrino and get:

$$m_2 \approx m_3 >> m_1$$

(vii) Models with sterile neutrinos.
The only case which can account for all the neutrino-anomalies. Needs a SM singlet
fermion which is light $\sim M_\nu$ and mixed with the light neutrinos. Questions: Why is it
so light? How is it mixed? why only one sterile neutrino? A possible solution [18] is to
add new matter and use again a kind of see-saw. A U(1) symmetry is then needed to
protect the sterile neutrino mass. It was pointed out by Ma [19] that $E_6$ GUT (in contrast
with SO(10)) has all the right ingredients for models with a sterile neutrino: I.e. a SM
singlet fermions and heavy matter in the 27 representation, as well as additional U(1)
symmetries.

(viii) GUT models (without low energy SUSY) 6.
One uses in those models an intermediate Pati-Salam like symmetry at $M_I \approx 10^{12} GeV$
to get Gauge and Yukawa unification with $M_{GUT} > 5 \times 10^{15} GeV$ so that problems with too
fast proton decay are avoided. $M_I$ is then a natural scale for $M_{\nu_R}$. In the corresponding
models:

a) Achiman and Lukas [20] generate the mixing by GUT radiative corrections induced via
$M_\nu^R$. This gives large leptonic mixing. The consistency of the Renormalization Group
Equations for this model require $m_t = 175 GeV$ [21].

b) Babu and Mohapatra[22] show that mixing can be generated via non-renormalizable
contributions. They get however small neutrino-mixing.

c) Lee and Mohapatra [23] - use see-saw $II^{ed}$ art i.e. $(M_\nu^{light})_{11} \neq 0$. This gives approxi-
mately degenerate neutrinos.

d) Buccella et al [24] suggested recently a new version based on SO(10).

Concluding remarks
Recent models incline toward using asymmetric mass matrices (more freedom) as well

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6SUSY is probably required to include Gravity but is broken at $M_{Planck}$ in this case.
as giving the neutrinos degenerate masses, which go usually with large leptonic mixing angles.

GUTs are a natural framework for the study of the fermionic masses and mixing. And this is true in particular for the neutrinos as they have the large scale needed for the see-saw mechanism.

Low energy theories for the neutrino-masses [1] cannot use the advantages of GUTs. They need a lot of ad-hoc assumptions and new “light” particles. These lead obviously to many new phenomena one can look experimentally for.

We have seen that different models can give similar results. To distinguish between models one needs additional information from outside the neutrino sector. E.g. effects due to RH mixing can play this role. Recall that mass matrices are diagonalized via a bi-unitary transformation

\[ M_{\text{diag.}} = U_R M U_L^\dagger. \]

If \( M \) is not symmetric (or hermitian) \( U_R \neq U_L \). \( U_R \) is not relevant for the SM as the RH components of the fermions are singlets. The RH rotation play however an important role in GUTs. E.g. in the proton decay[25] or in \( \nu_R \) decays which lead to leptogenesis.

References


