Low Energy Grand Unification with SU(16)

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Abstract

We study the possibility of achieving low unification scale in a grand unification scheme based on the gauge group SU(16). Baryon number symmetry being an explicit local gauge symmetry here gauge boson mediated proton decay is absent. We present in detail a number of symmetry breaking patterns and the higgs field representations giving rise to the desired symmetry breakings and identify one chain giving low energy unification. These higgs field representations are constructed in such a way that higgs mediated proton decay is absent. At the end we indicate the very rich low energy physics obtainable from this model which includes quark-lepton un-unified symmetry and chiral color symmetry. In brief some phenomenological implications are also studied.
1 Introduction

Grand Unified Theories (GUTs) [1] offer the possibility of a simple, but unified description of strong and electroweak interactions. Typically in these models at some high energy all the interactions arise out of a single Lagrangian which is locally invariant under the gauge transformations of a single simple Lie group called the unification group. A large spectrum of GUTs are proposed in the literature which is broadly classified by the unification group. In the minimal $SU(5)$ model all interactions unite at a single step at an energy around $10^{14} GeV$ therefore predicting the absence of any new physics between the standard electroweak breaking scale($M_z$) and the unification scale($M_U$) while the $SO(10)$ model admits intermediate breakings of symmetry. On the other hand there are models which are inspired by superstring theories, one of them postulates the exceptional group $E_6$ as the unifying group. This specific model predicts at least 12 exotic fermions on top of the 15 standard fermions. All these theoretically very attractive models have at least one common prediction namely the decay of proton.

There has been a desperate search by the experimentalists to see the signature of proton decay for the last decade and a half. Contrary to the theoretical beliefs proton decay has not been discovered. At present the lower limit of the half life of proton is a whooping $10^{32}$ years. The nonobservation of proton decay has made all the above models a little less attractive.

At this juncture one interesting possibility is that unification is achieved at a low energy scale which means that the big desert of particle physics between the electroweak scale and
the unification scale becomes small but other experimental constraints including that on the lifetime of proton remains satisfied. A grand unification scheme based on $SU(16)$ as the unification group offers such a possibility. It is worth noting here that due to the low unification scale such models are free from the problems of grand unified monopoles [3].

Earlier works on low energy unification in GUTS considered $SU(15)$ as the unification group [4]. Here we extend the idea to the left-right symmetric version of such a theory. We show that retaining all the good features of $SU(15)$ we can also incorporate left-right symmetry in intermediate stages. Unlike the $SU(15)$ GUT here Lepton number is a local gauge symmetry which may survive to a low energy scale. Right handed neutrino can be accommodated naturally as all the fermions transform in the fundamental representation of $SU(16)$.

This paper is structured as following. In section 2 and 4 we give the symmetry breaking chains and the Higgs field representations required for the breakings of symmetry. In section 3 we present some mathematical preliminaries useful for the calculation of the mass scales. In section 5 we calculate the mass scales using the renormalization group equations. In section 6 we study some phenomenological consequences of this model and in section 7 we state the conclusions. In the appendix we give some group theoretic essentials.
To achieve low energy unification we propose a number of symmetry breaking chains. At the level of highest symmetry the theory is invariant under the gauge group $SU(16)$. At and above this level the coupling constant is that of the group $SU(16)$. With the decrease in energy, the group goes through a number of symmetry breaking phases, and the theory becomes least symmetric at the present energies with the residual symmetry of $SU(3)$ color and the symmetry of electromagnetic interactions. It is noteworthy that the baryon number symmetry remains exact up to a very low energy scale of a few $TeV$. This makes the proton stable in the sense that the gauge boson mediated proton decay is absent. Interestingly the completely un-unified symmetry group of the quarks and leptons also appears at a low energy scale together with the chiral color symmetry. The appearance of this group at a comparatively low scale makes this model worthy of phenomenological studies. In this section we illustrate one chain in some detail to draw attention to the underlying group theoretic points. In section 4 we shall consider more possible chains before finally going to calculate the symmetry breaking scales.

BREAKING CHAIN 1

Here at first we give the breaking chains that can give rise to the standard model groups $SU(3)_C \times SU(2)_L \times U(1)_Y$. We note here that there can be in general a number of chains of descent to the standard model.
Here the superscript q or l denotes that quarks or leptons have nontrivial transformation law under these groups and the subscripts L and R mean so for the left and right handed fermions. The subscript c stands for the color gauge group of QCD.

In a previous paper [5] we have shown that in SU(15) GUT the effect of Higgs bosons play a significant role in the evolution of the coupling constants with increasing energy and hence on the values of the mass scales. This is due to the presence of high dimensional Higgs fields required to obtain the desired symmetry breaking pattern. The influence of the Higgs fields on the evolution of coupling constants can be so serious that they can alter the symmetry breaking pattern altogether. In SU(16) GUT The symmetry breaking pattern is very similar to that of its SU(15) counterpart. So in SU(16) or in SU(15) GUT the Higgs
effects must be taken seriously. Here we shall consider the Higgs fields required to obtain
the breaking chain and their contribution in the renormalization group equations in detail.

The Higgs structure is similar to that we proposed for $SU(15)$ GUT. We denote $1^n$ as
the totally antisymmetric nth rank tensor and $1^n1^m$ as the representation which has m and
n vertical boxes in the first and second columns of its Young’s table. For the transition from
the group $G_1$ to group $G_2$ the $G_2$ singlet component of the Higgs field should acquire vacuum
expectation value. Turning to the specific case of $SU(16)$ we note that at the scale $M_U$ the
breaking can be achieved by giving the vacuum expectation value to the $SU(12) \times SU(4)$
singlet component of $1^4$. Using the exactly same procedure we see that the breaking at the
scale $M_1$ can be done by $1^{14}1$ which leaves $U(1)_B$ unbroken. At the scale $M_2$ the breaking
of $SU(6)_L$ to its special maximal algebra requires a somewhat large dimensional Higgs field
representation. We use the 14144 dimensional Higgs field $1^{14}1^2$ to break this group. As a
passing comment we note here that this Higgs field will contribute significantly to the beta
functions of the renormalization group equations and make its presence strongly felt in the
determination of the mass scales. The group $SU(4)^4$ can be broken by a Higgs field which
transforms as a 15-plet under $SU(4)^4$ and which is contained in 255 under $SU(16)$. At
the stage $M_3$ the breaking of $SU(6)_R$ to $SU(3)_R \times U(1)_R$ is a bit complicated. 255 breaks
$SU(6)_R$ to $SU(3) \times SU(3) \times U(1)_R$ and subsequently the two $SU(2)_L$ groups of the quark and
leptonic sectors respectively are glued by $1^{14}1^2$. The breaking of the lepton number local
gauge symmetry $U(1)^{lep}$ can be achieved by either 16 or the two index symmetric Higgs field
of dimension 136. In the first case it carries a lepton number one unit and in the second
case it carries that of two units. We shall see that the choice of specific Higgs field shall give interesting difference of physics in the context of neutrino oscillations. At the scale $M_5$ the breaking is done by the $1^4$ Higgs field which is 1820 dimensional. The baryon number is broken by either $1^5$ or $1^6$. In both the cases we get interesting physics. As an example in the first case we get processes where baryon number changes by 3 units and in the second case it changes by 2 units. It is well-known that to give masses to the fermions vacuum expectation value has to be given to the component $(1, 2, -\frac{1}{2})$ which is contained in either $1^2$ or 11. These Higgs Field representations are summarized in table 1A

Let us now turn our attention to the group theoretic transformation properties of the fermions under the different symmetry groups in the symmetry breaking scheme. A minimal left-right symmetric theory should have at least one right handed neutrino ($\nu_R$) on top of the standard twelve quarks which includes three left handed doublets and six right handed singlets under the weak interaction gauge group $SU(2)_L$ and three leptons namely one left handed doublet and one right handed singlet. At grand unification energies and above this sixteen fermions should transform under some representation of the unification group. As a passing comment here we state that this requirement makes $SU(16)$ a very natural choice of the unification gauge group which has a 16 dimensional fundamental representation. In the model the fermions transform under the fundamental representation of $SU(16)$. Now as the energy becomes lower the symmetry breakings occur and the transformation properties of the fermions change with each symmetry breaking taking place. In the following we summarize these transformation properties. We use the notation that $(m, n)$ is a representation which
transforms under the semisimple group $SU(M) \times SU(N)$ as a $m$-plate under the former group and as a $n$-plate under the later group.

\[
SU(16) \rightarrow 16
\]

\[
G \rightarrow (12, 1) + (1, 4)
\]

\[
G_1 \rightarrow (1, \bar{6}, n, 1) + (6, 1, -n, 1) + (1, 1, 0, 4)
\]

\[
G_2 \rightarrow (1, 1, \bar{6}, n, 1) + (3, 2, 1, -n, 1)
\]

\[+ (1, 1, 1, 0, 4)\]

\[
G_3 \rightarrow (1, 1, \bar{3}, p, n, 1, 1, 0) + (1, 1, \bar{3}, -p, n, 1, 1, 0)
\]

\[+ (3, 2, 1, 0, -n, 1, 1, 0) + (1, 1, 1, 0, 0, 1, 2, m)\]

\[+ (1, 1, 1, 0, 0, 2, 1, -m)\]

\[
G_4 \rightarrow (1, 1, \bar{3}, p, n, 0) + (1, 1, \bar{3}, -p, n, 0)
\]

\[+ (3, 2, 1, 0, -n, 0) + (1, 1, 1, 0, 0, -2\sqrt{2} m)\]

\[+ (1, 2, 1, 0, 0, \sqrt{2} m) + (1, 1, 1, 0, 0, 0)\]

\[
G_5 \rightarrow (\bar{3}, 1, n, n) + (\bar{3}, 1, n, -n) + (3, 2, -n, 0)
\]

\[+ (1, 2, 0, n) + (1, 1, 0, -2n) + (1, 1, 0, 0)\]

\[
G_6 \rightarrow (\bar{3}, 1, -\frac{2}{3} K) + (\bar{3}, 1, \frac{1}{3} K)
\]

\[+ (3, 2, \frac{1}{6} K) + (1, 1, K) + (1, 1, -\frac{1}{2} K)\]

\[+ (1, 1, 0) \quad (1)\]
Here the $U(1)$ normalisations are defined in terms of

\begin{align*}
    n &= \frac{1}{2\sqrt{6}} \\
    m &= \frac{1}{2\sqrt{2}} \\
    p &= \frac{1}{2\sqrt{3}} \\
    K &= \sqrt{\frac{3}{20}}
\end{align*}

We know that in the electroweak breaking scale $M_Z$ the generators of electromagnetic symmetry group $U(1)_{em}$ arises out as a linear combination of the generator of the $U(1)$ part of the weak isospin group $SU(2)_L$ and that of the weak hypercharge $U(1)_Y$ by the following equation,

\begin{equation}
Q = T^3_L + Y
\end{equation}

Let us call this equation as the $U(1)$ matching condition at the scale $M_Z$. Similarly at the various symmetry breaking scales in the above breaking chain we have used different matching conditions for the groups. These matching conditions are stated below.

At the scale $M_A$ the lepton number symmetry breaks as the generator of $U(1)_{lep}$ and the diagonal generator of $SU(2)_L$ mixes with each other in the following way to generate the group $U(1)^l$,

\begin{equation}
Y^l = \sqrt{\frac{1}{3}} T^3_L + \sqrt{\frac{2}{3}} Y_{lep}
\end{equation}

At the scale $M_S$ $U(1)_R$ and $U(1)^l$ breaks to make $U(1)_h$.

\begin{equation}
Y_h = \sqrt{\frac{1}{2}} Y_R + \sqrt{\frac{1}{2}} Y^l
\end{equation}
At the scale $M_6$, baryon number cease to be a local gauge symmetry and conventional hypercharge appears from the linear combination of $U(1)_B$ and $U(1)_h$.

\[ Y = -\sqrt{\frac{1}{10}} Y_B - \sqrt{\frac{9}{10}} Y_h \]  

(5)

### 3 Mathematical Preliminaries

In this section we briefly touch two more mathematically involved topics. To begin with we note that the generators of $SU(16)$ and that of the standard model groups cannot be normalized in the same way. We proceed further in the section by giving a short discussion of the process of calculating the contribution of the Higgs fields to the beta functions. Let us fix that all the generators of $SU(16)$ are normalized to 2. In that case at the standard model energies the generators of $SU(3)_C$ and $SU(2)_L$ automatically becomes the generators of $SU(16)$. In contrast the generators of $U(1)_Y$ are normalized to $\frac{1}{2}$. So in the renormalization group equations we have to multiply the beta function corresponding to $U(1)_Y$ group by the appropriate factor of 4. Similarly it is easy to see that all other $U(1)$ groups in the symmetry breaking chain has to be multiplied by 4. Turning to the non-Abelian groups it can be checked that the group $SU(2)_L^q$ in all stages is normalized to $\frac{3}{2}$ hence to treat it at par with all other groups one has to multiply the beta function corresponding to this by a factor of $\frac{4}{3}$. $SU(3)_L$ and $SU(3)_R$ in all the stages are normalized to 1 hence one finds the aforesaid factor to be 2. To complete the discussion on the normalization factors we note that all other groups are normalized to $\frac{1}{2}$ hence the relevant factor is 4.
At this point let us turn our attention to the expression of the beta function \( b_i(N) \), for the group \( SU(N) \)

\[
b(N) = -\frac{1}{(4\pi)^2} \left[ \frac{11}{3} N - \frac{1}{6} T - \frac{4}{3} n_f \right]
\]

(6)

For \( U(1) \) groups \( N \) vanishes. Here \( n_f \) denotes the number of families of fermions and \( T(R) \) denotes the contribution of the Higgs fields which transform nontrivially under the group under consideration. To calculate \( T \) we have followed the following sum rule\([9]\):

Suppose \( R_i \) and \( r_i \) \((i = 1, 2, \ldots)\) are different representations of a group \( SU(N) \), which when vectorically multiplied satisfies the following relation.

\[
R_1 \times R_2 = \sum_{i=1} r_i
\]

(7)

Also let for the representation of dimension \( r \), the contribution to the renormalization group equation is \( T(R) \). Then,

\[
T(R_1 \times R_2) = R_2 T(R_1) + R_1 T(R_2) = \sum_{i=1} T(r_i)
\]

(8)

To use these equations one uses the following information to start with

\[
T(N) = \frac{1}{2}
\]

\[
T(N^2 - 1) = N
\]

\[
T\left[ \frac{N(N - 1)}{2} \right] = \frac{N - 2}{2}
\]

\[
T\left[ \frac{N(N + 1)}{2} \right] = \frac{N + 2}{2}
\]

\[
T(1) = 0
\]
As an example consider \(3\) and \(\bar{3}\) representations of \(SU(3)\). When vectorically multiplied they give

\[
3 \times \bar{3} = 1 + 8
\]  

so using the sumrule

\[
T(8) = 3T(3) + 3T(\bar{3}) - T(1) = 3
\]

4 Other paths to the Standard Model

We have already noted that there can be a number of paths to the standard model groups starting from the unification group \(SU(16)\). Let us consider here two typical chains of descent. In the first case (chain 2) here we shall break the \(U(1)\) groups as low as possible. It is in a sense one extreme case as the beta function coefficients for the \(U(1)\) groups are very small in magnitude compared to that of the other groups (the eigenvalue of the casimir operator vanish).

\[
\text{BREAKING CHAIN 2}
\]

\[
SU(16) \xrightarrow{\mathcal{M}_1} G_1 [SU(12) \times SU(4)]^L \\
\xrightarrow{\mathcal{M}_1} G_1 [SU(6)_L \times SU(6)_R \times U(1)_B \times SU(4)]^L \\
\xrightarrow{\mathcal{M}_2} G_2 [SU(3)_L \times SU(2)_L \times SU(6)_R \times U(1)_B \times SU(4)]^L
\]
\[ M_3 \rightarrow G_3[SU(3)_L \times SU(2)_L^L \times SU(3)_R \times U(1)_{B-L}^R \times SU(2)_R^L \times SU(2)_{B-L}^L \times U(1)^{lep}] \]

\[ M_4 \rightarrow G_4[SU(3)_c \times SU(2)_R^q \times U(1)_B^q \times SU(2)_{L}^L \times SU(2)_{R}^L \times U(1)^{lep}] \]

\[ M_5 \rightarrow G_5[SU(3)_c \times SU(2)_L^q \times SU(2)_R^L \times U(1)_B^q \times U(1)_R^q \times U(1)^{lep}] \]

\[ M_6 \rightarrow G_6[SU(3)_c \times SU(2)_L^q \times U(1)_Y] \]

\[ M_7 \rightarrow G_7[SU(3)_c \times U(1)_{em}] \]

It is to note that all the \( U(1) \) groups remain ununified till the scale \( M_6 \). At the scale they merge together to give the familiar hypercharge of the standard model. At the scale \( M_6 \) the matching condition is

\[
Y = -\sqrt{\frac{2}{20}} Y_B - \sqrt{\frac{9}{20}} Y_R^q - \sqrt{\frac{3}{20}} Y_L^L - \sqrt{\frac{6}{20}} Y_{lep} \quad (11)
\]

Another interesting possibility is to break \( SU(16) \) via the left-right symmetric group of Pati and Salam\(^2\). The low energy Phenomenology of the Pati-Salam group is widely studied. So it will be interesting to see how low the intermediate scales can come down to so we can make some concrete predictions of the model in view of the on coming experiments. Hence in the second chain that is discussed here (chain 3) the left-right symmetric group \( SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) will be kept as low as possible

**BREAKING CHAIN 3**

\[ SU(16) \rightarrow G[SU(12) \times SU(4)] \]
We notice that $U(1)_{B-L}$ group is formed at the scale $M_5$ when baryon number symmetry and the lepton number symmetry is broken together at the same scale. The matching condition is

$$Y_{B-L} = \sqrt{\frac{1}{4} Y_B} - \sqrt{\frac{3}{4} Y_{lep}}$$

(12)

At the scale $M_6$ the generator of the group $U(1)_{B-L}$ and the diagonal generator of the right handed $SU(2)$ group form a linear combination to generate the conventional hypercharge.

$$Y = \sqrt{\frac{9}{10} T^3_R} - \sqrt{\frac{1}{10} Y_{B-L}}$$

(13)

Applying exactly the similar principles that we used for calculating the Higgs structure for the first breaking chain, We can calculate the Higgs fields required to break $SU(16)$ in the fashion of chain 1. The essential difference in chain 2 is that the breaking of $SU(2)_L^t$ to $U(1)_R^t$ which can be conveniently done by 255 which has a component (1,15) under G and at the scale $M_6$ the four $U(1)$ groups are glued by a combination of Higgs fields $1^4$ and $1^3$.
The breaking chain 3 is much more symmetric and simpler too. The Higgs fields that we require to break the chain is also less complicated. To break the left right symmetry group we need the Higgs fields \((1, 1, 3, -\sqrt{3}/8), (1, 2, 2, 0), (1, 2, 2, 0)\) and \((1, 1, 3, \sqrt{3}/8)\) which can be easily embedded in the group \(G\) in the representations \((143, 6), (1, 15)\) and \((78, 1)\) and hence in \(SU(16)\). The details of the Higgs fields required for the chains 2 and 3 are given in table 2A and table 3A.

5 Mass scales

To evaluate the mass scales we use the standard procedure of evolving the couplings with energy. The energy dependence of the couplings with energy [7]. The energy dependence of the couplings are completely determined by the particle content of the theory and their couplings inside the loop diagrams of the gauge bosons. This is expressed by the renormalization group equation. The one-loop RG equation is given by the following equation.

\[
\mu \frac{d}{d\mu} \alpha(\mu) = 2b \alpha^2(\mu)
\]

where

\[
\alpha = \frac{g^2}{4\pi}
\]

the beta function coefficients are already defined. Now, using the above informations and the matching conditions given with each symmetry breaking chain one can relate the \(SU(16)\) coupling constant \(\alpha_{SU(16)}\) with the standard model couplings \(\alpha_{3c}, \alpha_{2L}\) and \(\alpha_{1Y}\) at the scale of the mass of the Z particle \(M_Z\). At this point let us remember that there are three quark doublets and one leptonic doublet under the group \(SU(2)_L\) in the standard model hence
in the evolution of coupling $\alpha_{2L}$ the quark and leptonic groups $SU(2)^q_L$ and $SU(2)^l_L$ do not contribute equally to the standard model group $SU(2)_L$ instead they contribute with a relative factor 3

CHAIN 1

$$g^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) +$$

$$2b_{12} M_{U1} + (b_{6L} + b_{6R}) M_{12} + (b_{3L} + b_{6R}) M_{23} +$$

$$(b_{3L} + b_{3R}) M_{34} + (b_{3L} + b_{3R}) M_{45} + 2b_{3c} M_{56} + 2b_{3c} M_{6z}$$

$$g^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) +$$

$$\left(\frac{3}{2} b_{12} + \frac{3}{2} b_{4q} \right) M_{U1} + \left(\frac{3}{2} b_{6L} + \frac{3}{2} b_{4q} \right) M_{12} +$$

$$\left(\frac{3}{2} b_{12} + \frac{3}{2} b_{4q} \right) M_{34} + \left(\frac{3}{2} b_{6L} + \frac{3}{2} b_{4q} \right) M_{45} +$$

$$2b_{12} M_{56} + 2b_{12} M_{6z}$$

$$g^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) +$$

$$\left(\frac{9}{10} b_{12} + \frac{1}{2} b_{4q} \right) M_{U1} + \left(\frac{9}{10} b_{6L} + \frac{1}{2} b_{4q} \right) M_{12} +$$

$$\left(\frac{9}{10} b_{6R} + \frac{1}{2} b_{4q} \right) M_{23} + \left(\frac{9}{10} b_{1R} + \frac{1}{2} b_{4q} \right) M_{34} +$$

$$\left(\frac{9}{10} b_{1R} + \frac{1}{2} b_{4q} \right) M_{45} +$$

$$\left(\frac{9}{10} b_{6h} + \frac{1}{2} b_{4q} \right) M_{56} + 2b_{1Y} M_{6z}$$

(16)

CHAIN 2

15
\[ g_{3c}^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) + \]
\[ 2b_{12} M_{U1} + (b_{6L} + b_{6R}) M_{12} + (b_{3L} + b_{6R}) M_{23} + \]
\[ (b_{3L} + b_{3R}) M_{34} + 2b_{3c} M_{45} + 2b_{3c} M_{56} + 2b_{3c} M_{6z} \]
\[ g_{2L}^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) + \]
\[ \left( \frac{3}{2} b_{12} + \frac{1}{2} b_{1L} \right) M_{U1} + \left( \frac{3}{2} b_{3L} + \frac{1}{2} b_{3L} \right) M_{12} + \]
\[ \left( \frac{3}{2} b_{2L} + \frac{1}{2} b_{2L} \right) M_{23} + \left( \frac{3}{2} b_{2L} + \frac{1}{2} b_{2L} \right) M_{34} + \]
\[ \left( \frac{3}{2} b_{2L} + \frac{1}{2} b_{2L} \right) M_{45} + \left( \frac{3}{2} b_{2L} + \frac{1}{2} b_{2L} \right) M_{56} + 2b_{2L} M_{6z} \]
\[ g_{1Y}^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) + \]
\[ \left( \frac{11}{10} b_{12} + \frac{9}{10} b_{1L} \right) M_{U1} + \left( \frac{9}{10} b_{6R} + \frac{1}{5} b_{1B} + \frac{9}{10} b_{1L} \right) M_{12} + \]
\[ \left( \frac{9}{10} b_{6R} + \frac{1}{5} b_{1B} + \frac{9}{10} b_{1L} \right) M_{23} + \left( \frac{9}{10} b_{6R} + \frac{1}{5} b_{2L} + \frac{3}{10} b_{1B} + \frac{6}{10} b_{1L} \right) M_{34} + \]
\[ \left( \frac{9}{10} b_{1R} + \frac{1}{5} b_{2L} + \frac{3}{10} b_{2L} + \frac{6}{10} b_{1L} \right) M_{45} + \]
\[ \left( \frac{9}{10} b_{1R} + \frac{1}{5} b_{1B} + \frac{3}{10} b_{1R} + \frac{6}{10} b_{1L} \right) M_{56} + 2b_{1Y} M_{6z} \] (17)

CHAIN 3

\[ g_{3c}^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) + \]
\[ 2b_{12} M_{U1} + (b_{6L} + b_{6R}) M_{12} + (b_{3L} + b_{6R}) M_{23} + \]
\[ (b_{3L} + b_{3R}) M_{34} + 2b_{3c} M_{45} + 2b_{3c} M_{56} + 2b_{3c} M_{6z} \]
\[ g_{2L}^{-2}(M_z) = g_{SU(16)}^{-2}(M_U) + \]

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Here $M_{ij}$ is defined as $\ln(\frac{M_{ij}}{M})$. As a comment we note that generally one would expect that the coefficients of $b_{qL}^{q+t}$ and $b_{1(B-L)}$ to be $\frac{6}{5}$ and $\frac{4}{5}$ respectively as it appears in the $SO(10)$ model. It is worth noting that these factors are dependent on the normalization of the $U(1)$ part of $SU(2)_{R}^{q+t}$ and that of the group $U(1)_{B-L}$. This point will be elaborated further in the appendix.

To calculate the mass scales we also have to know the numerical values of the beta function coefficients. To know them one has to know the contribution of the Higgs scalars to the beta functions ($T$). In the following three tables we give these values.

Now with the quantities $g_{W}^{-2}(M_{z})$, $g_{2L}^{-2}(M_{z})$ and $g_{3c}^{-2}(M_{z})$ at hand one can construct two different linear combinations with them to form the experimentally measured quantities at
Table 1: Contributions of the Higgs scalar to the R-G equation at various energy scales in chain1.

<table>
<thead>
<tr>
<th>$G$</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$G_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left[1_B\right] = 45$</td>
<td>$[1_B] = 45$</td>
<td>$[3_R] = 15$</td>
<td>$[3_R] = 9$</td>
<td>$[1^q_R] = 3.16$</td>
<td>$[1^q_R] = 0.575$</td>
<td></td>
</tr>
<tr>
<td>$[4'] = 63$</td>
<td>$[1^q_R] = 7.58$</td>
<td>$[1^q_R] = 7.58$</td>
<td>$[1^q_R] = 1.25$</td>
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<td>$[1^q_R] = 0.575$</td>
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</tr>
<tr>
<td>$[1^q_L] = 27$</td>
<td>$[1^q_L] = 20$</td>
<td>$[1^q_L] = 2$</td>
<td>$[1^q_L] = 1$</td>
<td>$[1^q_L] = 0.5$</td>
<td>$[1^q_L] = 0.5$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Contributions of the Higgs scalar to the R-G equation at various energy scales in chain2.

the energy scale $M_z$. It easy to check that the following relations hold between them.

$$\sin^2(\theta_w) = \frac{3}{8} - \frac{5}{8} e^2 (g_{1Y}^{-2} - g_{2L}^{-2})$$

$$1 - \frac{8}{3} \frac{\alpha}{\alpha_s} = e^2 (g_{2L}^{-2} + \frac{5}{3} g_{1Y}^{-2} - \frac{8}{3} g_{2c}^{-2})$$

From the present experimental measurements at LEP the value of $\sin^2(\theta_w)$ and $\alpha_s$ has been very accurately measured. We use for our purpose the following values[11] of them and the $U(1)$ coupling $\alpha$ at the scale $M_z$

$$\sin^2(\theta_w) = 0.2336 \pm 0.0018$$
### Table 3: Contributions of the Higgs scalar to the R-G equation at various energy scales in chain3.

<table>
<thead>
<tr>
<th>$G$</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$G_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[4'] = 89$</td>
<td>$[1_R] = 3$</td>
<td>$[2_R] = 75$</td>
<td>$[1_R] = 1.5$</td>
<td>$[2_R] = 6$</td>
<td>$[1_{B-\Lambda}] = 2.25$</td>
<td></td>
</tr>
<tr>
<td>$[4'] = 51$</td>
<td></td>
<td>$[1_R] = 3$</td>
<td></td>
<td>$[2_R] = 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$[4'] = 51$</td>
<td></td>
<td>$[1_{\text{eff}}] = 3.5$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Having these informations at hand one can straightaway go to calculate the mass scales of symmetry breaking.

Let us discuss the calculation of the first chain in some detail. Let us now assume that $M_4 = M_3 = M_A$. This means that the groups $SU(6)_L, SU(6)_R$ and $SU(4)^t$ happens to break at the same scale. Similarly let us also assume that $M_4 = M_5 = M_B$. Now using the values of the $T(R)$s table2 and solving for $M_{U1}$ and $M_{B6}$ in terms of the other variables one gets,

\[ M_{U1} = -.28 - .10 M_{1A} - .10 M_{6x} + .04 M_{AB} \]

\[ M_{B6} = 19.80 - 4.81 M_{1A} - 2.93 M_{6x} - .21 M_{AB} \]  \hspace{1cm} (21)

As the symmetry breaking at $M_U$ occurs before it happens at $M_1$ $M_{U1}$ is at least positive. So from the first equation one infers that for a specific set of values of the other parameters in the right-hand side there is a minimum value to $M_{AB}$. Varying the parameters of the
equations one gets the following subset of the solution set allowed by the equations. Taking $M_z$ to be around 91 GeV one can also calculate the unification scale and the scale $M_6$ where the completely un-unified symmetry of the quarks and leptons and the chiral color symmetry is broken. We note that as the parameter $M_{AB}$ increases i.e. as the separation between the scale $M_A$ and the scale $M_B$ increases the scale $M_B$ comes down. Exactly in a similar way let

<table>
<thead>
<tr>
<th>$M_{AB}$</th>
<th>$M_{1A}$</th>
<th>$M_{6z}$</th>
<th>$M_{B6}$</th>
<th>$M_{U1}$</th>
<th>$M_B$</th>
<th>$M_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>18.4</td>
<td>0</td>
<td>$10^9$</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>9.5</td>
<td>1</td>
<td>0</td>
<td>12.9</td>
<td>0</td>
<td>$10^8$</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>10.75</td>
<td>1.5</td>
<td>0</td>
<td>10.3</td>
<td>0</td>
<td>$10^7$</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>0</td>
<td>8.7</td>
<td>0</td>
<td>$10^6$</td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>14.5</td>
<td>3</td>
<td>0</td>
<td>2.3</td>
<td>0</td>
<td>$10^4$</td>
<td>$10^{11}$</td>
</tr>
</tbody>
</table>

Table 4: Mass scales from chain1.

us see the breaking scales that we may get from the solution of the chain2. Here we keep the $U(1)$ groups as low as possible in the hope that it will give rise to distinct phenomenology at the low energy. To begin with let us keep $M_2=M_3=M_A$ and $M_4=M_5=M_B$. The solutions are

$$M_{U1} = -.70 - .01 M_{1A} - .02 M_{6z} + .01 M_{AB}$$

$$M_{B6} = 5.35 - .44 M_{1A} - .78 M_{6z} - .81 M_{AB}$$

(22)

To keep $M_{U1}$ positive we have to have $M_{AB}$ larger than 70. This pushes the unification scale beyond the Planck scale and hence makes the breaking chain uninteresting.
The third option that we have considered here is to come to the low energy groups via the left-right symmetric Pati-Salam group in the chain3. The solutions in this case are

\[
M_{U1} = -0.99 + 0.13 M_{1A} + 0.01 M_{56} + 0.02 M_{A5}
\]

\[
M_{62} = 6.84 + 2.64 M_{1A} - 0.22 M_{56} - 0.1 M_{A5}
\]

The solution set of these equations are interesting though low energy unification is not possible here. Let us at first state a sample solution set. This is obvious from the equations

<table>
<thead>
<tr>
<th>$M_{1A}$</th>
<th>$M_{A5}$</th>
<th>$M_{56}$</th>
<th>$M_{U1}$</th>
<th>$M_{62}$</th>
<th>$M_6$</th>
<th>$M_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.61</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>26.92</td>
<td>$10^{13}$</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>6.84</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>24.89</td>
<td>$10^{12}$</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>6.07</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>21.86</td>
<td>$10^{11}$</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>5.30</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>19.35</td>
<td>$10^{10}$</td>
<td>$10^{39}$</td>
</tr>
<tr>
<td>4.53</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>16.79</td>
<td>$10^{9}$</td>
<td>$10^{39}$</td>
</tr>
</tbody>
</table>

Table 5: Mass scales from chain3.

that to keep $M_{U1}$ positive one need a rather large value of $M_{1A}$ which on the other hand pushes $M_{62}$ up. The minimum value of $M_{1A}$ is around 7.6 which gives the minimum value of the unification scale which is around $10^{17}$ GeV. In a previous paper[6] we have shown that with the precisely measured value of $\sin^2(\theta_W)$ that is available now left-right symmetry at the low energy coming from a grand unified scenario is ruled out. This analysis comes as a confirmation of that result and it shows that even having a number of parameters that we have in the form of a number of breaking stages, Left-Right symmetry cannot come down to a low energy for any choice of the parameter space.
6 Phenomenology

6.1 Proton Decay

Having the mass scales and Higgs structure in hand we proceed in this paper to discuss the issue of proton decay now. In all the breaking chains that we have considered here, the quark lepton unification is broken at the scale $M_U$ while the quark antiquark unification is broken at the scale $M_1$. As a result the leptoquark gauge bosons ($X_\mu$) will acquire mass at the scale $M_U$ while the di-quark gauge bosons ($Y_\mu$) acquire mass at the scale $M_1$. Under the group $G_1$ their transformation properties are

$$X_\mu \implies (6, 1, -B, \bar{4}) + (1, \bar{6}, B, \bar{4}) + (\bar{6}, 1, B, 4) + (1, 6, -B, 4)$$

$$Y_\mu \implies (6, 6, -2B, 1) + (\bar{6}, \bar{6}, 2B, 1)$$

where

$$B = \frac{1}{2\sqrt{6}}$$

(24)

Now $U(1)_B$ being an explicit local gauge symmetry of the model, $X_\mu$ and $Y_\mu$ contains different ” Baryon Numbers ” and hence cannot mix directly to form an $SU(16)$ invariant operator.

The mixing can be induced indirectly through the term $D_\mu \phi_\alpha D^\mu \phi_\beta$, where $D_\mu$ is the
covariant derivative of the $SU(16)$ invariant theory, $D_\mu \phi_a D^\mu \phi_b$ will contain a term $X_\mu \phi_a X^\mu \phi_b$. When $\phi_a$ and $\phi_b$ acquires vacuum expectation value the mixing between $X_\mu$ and $Y^\mu$ occurs. But this can occur only at the scale $M_0$ hence the amplitude is suppressed by a factor of $O\left(\frac{M_0}{M_T^2}\right)\frac{M_0}{M_T^2}$.

To see how the gauge bosons couple to the Higgs fields we note that all the gauge bosons at the $SU(16)$ level transform under the 224 dimensional adjoint representation. We also note the following tensor product at the $SU(16)$ level

$$224 \times 224 = 1 + 224 + 224 + 14175 + 10800 + 12376 + 12376$$

(25)

Being the product of two selfconjugate representations all the terms in the right hand side are selfconjugate which couples to only self conjugate representations. From the table 1A that the the Higgs field that carries Baryon Number is $1^5$. So the only Higgs field which can induce a Baryon Number violating effect is $1^5$ which is 4368 dimensional.

The only self conjugate combination made up with $1^5$’s is $< 4368 > < 4368 >$ which again carries no baryon number hence not giving rise to any baryon number violating process[8].

To see the higgs field mediated proton decay at first we note that the fermions are in the 16 dimensional fundamental representation. To give mass to the fermions the coupling of the form $\bar{\psi}_L^c \psi_L \phi$ must exist. The minimum dimensional Higgs field which can do the job is 120. This field can give rise to Higgs mediated proton decay if $1^6$ breaks the Baryon Number due to the presence of the term $< 1^6 > < 1^6 > < 1^2 > < 1^2 >$ in the Lagrangian. In that case
we can choose $136$ to give mass to the fermions. In our choice $1^5$ breaks the baryon number hence it does not couple to $120$. Hence there is no Higgs mediated proton decay.

6.2 $N - \bar{N}$ oscillations

Let us consider the $SU(16)$ level operator $<1^5><1^5><1^5><16>$. This forms a singlet under $SU(16)$ and hence allowed in the Lagrangian. This term give rise to $\Delta B = 3$ processes. If instead we choose $136$ to break the Lepton Number symmetry, then this process vanishes.

In the last subsection we noted that if $1^6$ breaks the Baryon Number symmetry then one has to choose $136$ to give mass to the fermions here we note that then the term $<1^{14}1^2><136><136><1^6><1^6>$ will be be allowed in the Lagrangian which may give rise to $\Delta B = 3$ processes. As the term is of dimension five it will be suppressed by $M_U$. With $1^2$ we can construct the $SU(16)$ level operator $<1^5><1^5><1^4><1^2>$ which can break the Baryon Number by two units and hence giving rise to gauge boson mediated $N - \bar{N}$ oscillations. To see the Higgs field mediated processes we note that if $120$ dimensional Higgs field couples to the fermions and $1^6$ breaks the Baryon number then the operator $<1\bar{2}0><1\bar{2}0><1\bar{2}0><1^6>$ can give rise to Higgs field mediated $N - \bar{N}$ oscillations.

7 Conclusion

In this paper we have seen that there exists one possible breaking chain in a Grand Unified Theory based on the group $SU(16)$ where a unification scale of the order of $10^{11}$ GeV is
possible. There exists a very low energy scale \( (M_B) \) which may be almost anywhere between the unification scale and the electroweak scale where completely ununified symmetry of quarks and leptons may exist together with chiral color symmetry. The scale \( M_B \) comes lower when the separation between the scale \( M_A \) and the scale \( M_B \) is increased. Qualitatively we understand it in the following way. The beta function coefficients can be looked into as the slope of the lines if one plots the inverse coupling constants with respect to energy. It can be easily checked that as at the \( SU(16) \) level all the fermions transform under the fundamental representation of the group and in the other levels they transform in a more complicated way under the various groups in the intermediate stages all the groups cannot be normalized in the same way. To compensate for the mismatch in the normalisations the beta function coefficients has to be multiplied by appropriate factors. Due to that the slope of the curves representing the inverse couplings also gets multiplied by the appropriate factors and the couplings get united earlier giving rise to low energy unification.

We have also seen that this model satisfies the experimental constraints coming from proton decay experiments in the sense that proton decay is suppressed. We have shown that there exists at least one choice of the Higgs sector where there is no Higgs mediated proton decay either.

For some specific choice of the Higgs fields there may exist interesting physical consequences like the \( N - \bar{N} \) oscillation. There is also the possibility of having the sea-saw mechanism to give Majorana mass to the neutrinos and this also may have observable consequences.
Last but not the least we emphasize again that there exists very rich low energy physics coming from this model hence keeping in mind the forthcoming high-energy experiments at SSC, LHC and other places this model is worthy of further investigation.

Acknowledgement

It is a pleasure to thank Utpal Sarkar for introducing the problem and also for a number of valuable discussions. The author would also like to thank A. Joshipura and S. Rindani for discussing some relevant points when this work was being presented in a group seminar at P.R.L.

8 Appendix

8.1 SU(16) Tensor Products

\[ 16 \times 16 = 120_a + 136_s \]
\[ \bar{16} \times 16 = 1 + 255 \]
\[ 16 \times 120 = 560_a + 1360 \]
\[ 120 \times 120 = 1 + 255 + 14144 \]
\[ 136 \times 136 = 1 + 255 + 18240 \]
\[ 560_a \times 16 = 1820_a + 7140 \]
\[ 1820_a \times 16 = 4368_a + 24752 \]

(26)

8.2 SU(16) Branching Rules

\[ SU(16) \Longrightarrow SU(12) \times SU(4) \]

\[
\begin{align*}
16 & = (12, 1) + (1, 4) \\
136 & = (78, 1) + (12, 4) + (1, 10) \\
120 & = (66, 1) + (12, 4) + (1, 6) \\
255 & = (143, 1) + (12, \bar{4}) + (\bar{1}2, 4) + (1, 15) + (1, 1) \\
560 & = (220, 1) + (66, 4) + (12, 6) + (1, 4) \\
1820 & = (495, 1) + (220, 4) + (66, 6) + (12, \bar{4}) + (1, 1) \\
14144 & = (1, 1) + (1, 35) + (12, \bar{4}) + (12, \bar{2}0) + (\bar{1}2, 4) + (\bar{1}2, 20) + (66, 6) + (66, \bar{6}) + (143, 1) + (143, 15) + (70, \bar{4}) + (780, 4) + \ldots
\end{align*}
\]
8.3 SU(12) Tensor Products

\[ 12 \times 12 = 66_0 + 78_s \]

\[ \tilde{12} \times 12 = 1 + 143 \]

\[ 12 \times 66 = 220_a + 572 \]

\[ 78 \times 78 = 1 + 143 + 5940 \]

\[ 66 \times 66 = 1 + 143 + 2412 \]

\[ 220_a \times 12 = 495 + 2145 \]

\[ 495_a \times 12 = 792 + 5148 \]

(28)

8.4 SU(12) Branching Rules

\[ SU(12) \implies SU(6) \times SU(6) \times U(1) \]

\[ 12 = (6, 1, -B) + (1, \bar{6}, B) \]

\[ 66 = (15, 1, -2B) + (1, \bar{15}, 2B) + (6, \bar{6}, 0) \]

\[ 78 = (21, 1, -2B) + (1, 2\bar{1}, 2B) + (6, \bar{6}, 0) \]
\[143 = (35, 1, 0) + (6, 6, 2B) + (6, 6, -2B) + (1, 1, 0) + (1, 35, 0)\]

\[220 = (20, 1, -3B) + (1, 20, 3B) + (6, \bar{5}, B) + (15, \bar{6}, -B)\]

\[495 = (15, 1, -4B) + (20, 6, -2B) + (15, \bar{15}, 0) + (6, 20, 2B) + (1, \bar{15}, 4B)\]

\[792 = (\bar{6}, 1, -5B) + (15, 6, -3B) + (20, \bar{15}, -B) + (15, 20, B) + (6, \bar{15}, 3B) + (1, 6, 5B)\]

\[572 = (70, 1, -3B) + (15, \bar{6}, -B) + (6, \bar{15}, B) + (21, \bar{6}, -B) + (6, 21, B) + (1, \bar{70}, 3B)\]

\[4212 = (189, 1, 0) + (15, 15, -4B) + (6, 6, -2B) + (84, 6, -2B) + (\bar{15}, \bar{15}, 4B) + (1, 35, 0) + (1, 189, 0) + (6, \bar{84}, 2B) + (\bar{84}, \bar{6}, 2B) + (6, 84, -2B) + (1, 1, 0) + (35, 1, 0) + (35, 35, 0) + (6, 6, 2B)\]
8.5 $SU(6)$ Branching Rules

\[SU(6) \implies SU(3) \times SU(2)\]

\[
\begin{align*}
6 &= (3, 2) \\
15 &= (6, 1) + (\bar{3}, 3) \\
20 &= (1, 4) + (8, 2) \\
21 &= (\bar{3}, 1) + (6, 3) \\
35 &= (1, 3) + (8, 1) + (8, 3) \\
70 &= (1, 2) + (8, 4) + (8, 2) \\
    &\quad + (10, 2)
\end{align*}
\]

\[(30)\]

8.6 Normalization of $U(1)_{(B-L)}$ and $SU(2)^{q+l}_R$

Consider chain3. Under the group $G_6$ the sixteen fermions transform as

\[(\bar{3}, 1, 2, \frac{1}{\sqrt{2}t}) + (3, 2, 1, -\frac{1}{\sqrt{2}t}) + (1, 1, 2, -\frac{3}{\sqrt{2}t}) + (1, 2, 1, \frac{3}{\sqrt{2}t})\]

The $T_{2R}^3$ parts of the right handed $SU(2)$ group are to be taken as $\pm \frac{1}{\sqrt{2}t}$ so as to get the correct $U(1)$ charges at the Standard Model level. $U(1)_{B-L}$ is normalized to 2 while the $U(1)$ generator of the right handed $SU(2)$ is normalized to $\frac{8}{2t}$ i.e $\frac{1}{3}$. Taking this factor of 6 in
the relative normalization one can easily get the familiar matching conditions of the $SO(10)$ model.

### 8.7 Anomaly Cancellation and Mass Scales

16 dimensional fundamental representation of $SU(16)$ is not anomaly free. To get the cancellation of anomaly one has to introduce mirror fermions. But these fermions will not alter the values of the mass scales obtained here. This is because in the two equations used for $\sin^2(\theta_w)$ and $1 - \frac{8}{3} \frac{\alpha}{\alpha_s}$ respectively the fermion contributions to the beta function coefficients cancel.

### References


