Colour reconnection and Bose-Einstein effects *

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Abstract

Final-state interactions and interference phenomena that could affect the value of the W mass reconstructed from hadronic WW decays at LEP2 are reviewed, and possible areas for future investigation are identified.

1 Introduction

The accurate measurement of the W boson mass is one of the primary goals of LEP2. In terms of statistics, a precision of around $\pm 40$ MeV could eventually be obtained from reconstructing the mass in fully hadronic WW decays. However, systematic uncertainties due to interactions and interference between the W decay products could degrade this precision substantially [1, 2].

The hadronic decay properties of individual, isolated W bosons can be predicted reliably on the basis of the LEP1 data on $Z^0$ decays [3], and these predictions can be tested against the LEP2 data from semi-leptonic WW decays. The problems in fully hadronic final states come from the fact that the products of the two W decays can overlap considerably in space-time [4]. The separation of the W decay vertices at LEP2 energies is $\sim 0.1$ fm. This is small compared with the typical hadronic scale of $\sim 1$ fm, so the possibilities for overlap are great. Final-state interactions and/or identical-particle symmetrization can then lead to an apparent shift in the W mass.

In this talk, I shall review ideas on the possible effects of overlap on the W mass measurement, concentrating on phenomenological developments since the CERN Workshop [1]. The two main areas of activity are colour reconnection and Bose-Einstein correlations. I shall discuss these in turn, and then try to indicate

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promising directions for future investigation, both at the current Workshop and beyond.

2 Perturbative colour reconnection

We should start by recalling the nature of colour reconnection and its expected properties in perturbative QCD. A relevant lowest-order contribution to the cross section, $\sigma_{\text{rec}}$, is shown in fig. 1, where the colour structure is (approximately) displayed by showing each gluon as a pair of directed colour lines. The W bosons are colour singlets, but the final states from each W decay exchange colour. The fact that this graph contains only two closed colour loops, like the Born graph, means that it is colour-suppressed, by two powers of the number of colours $N$, relative to the leading $O(\alpha_s^2)$ contributions. Furthermore, the space-time separation of the decay vertices, of order $1/\Gamma_W$, is large compared with the scale for hard gluon emission, which is $O(1/M_W)$. This leads to a suppression by a further factor of $\Gamma_W/M_W$. Thus the order of magnitude of the perturbative reconnection rate is [4]

$$\frac{\sigma_{\text{rec}}}{\sigma_{WW}} \sim \frac{(C_F \alpha_s)^2}{N^2} \cdot \frac{\Gamma_W}{M_W} \sim 10^{-3}.$$  \hspace{1cm} (1)

This is small compared with the non-perturbative rate, to be estimated in the next section. Although this does not automatically mean that the effect on the W mass is negligible, it is probably within the uncertainties of the non-perturbative models, and so I shall not discuss it further here.

3 Non-perturbative reconnection

At the hadronization scale of distances ($\sim 1$ fm) the space-time separation of the W decay vertices ceases to be so important, and reconnection can take place wherever the hadronization region of the two Ws overlap. This viewpoint shows
clearly that a space-time picture of the development of the final state becomes essential for a realistic treatment of reconnection at the hadronization level.

Following hadronic decay of the Ws, there is a perturbative parton showering phase of development, in which partons radiated by the initial quark and antiquark from each decay spread out and define the regions in which hadrons will subsequently form. These regions are of limited transverse size but can extend much further in the directions of motion of the initial partons, because of time dilation. Typically, therefore, the final states from the two W decays each have a predominantly two-jet structure extending some tens of fermis along the directions of the jet axes but only one or two fermis transverse to it (fig. 2). Thus there can be a large overlap between the hadronization products of the two decays, enhancing the probability of reconnection. Notice that the overlap will depend strongly on the angle $\theta$ between the decay axes. We shall discuss this effect more quantitatively in sect. 3.2.

All the commonly-used models for non-perturbative colour reconnection are based on a space-time picture in which reconnection is a local phenomenon. Objects are formed at the hadronization stage via a local interaction which may combine products of the two W decays in regions where they overlap. We may then distinguish two classes of models, according to the types of combinations that are permitted. In singlet models only colour-singlet objects can be formed, whereas in non-singlet models there is no such constraint.

### 3.1 Colour singlet models

According to the above classification, all the reconnection models based on string hadronization proposed by Khoze and Sjöstrand [4] are singlet models, since each string is a colour singlet. Within this framework they investigated two classes of models, called type I and type II in analogy to the two types of superconducting vortices which could correspond to colour strings, assuming the latter are formed by a QCD dual Meissner effect. In the type I scenario the strings/vortices have a significant transverse extension, of the order of one fermi, and the probability of reconnection depends on the volume of overlap of the strings formed in the two W decays. In type II, the strings are of negligible thickness, but they reconnect with unit probability if they intersect each other.
at any time between their formation and hadronization.

The main alternative to the string hadronization model is the cluster model, in which quarks and gluons from the parton showers combine locally into clusters, which are much less extended and less massive objects than strings, typically light enough to decay more or less isotropically into a small number of hadrons each.

The most widely used cluster models have also been colour singlet models, in which only singlet combinations of partons (in practice, quarks and antiquarks) are allowed to form clusters. In the limit of a large number of colours, every quark or antiquark produced in the parton shower has a unique colour-connected partner with which it can be clustered, while every gluon has a colour and an anticolour index, each uniquely connected to another parton. In the model used in the HERWIG Monte Carlo program [5, 6], after showering the gluons are split non-perturbatively into quark-antiquark pairs, so that each may form a colour-singlet cluster with its colour-connected partner. A strong point of this scenario is that the perturbative parton shower has the property of preconfinement, which means that the resulting spectrum of cluster sizes and masses peaks at small values and is universal, i.e. independent of the nature and scale of the hard process that initiated the showers [7, 8].

As in the string model, local colour reconnection in the cluster model is natural once a space-time picture of the development of the final state is included. If, for example, a quark finds that its nearest neighbouring antiquark of the right colour is not its colour-connected partner, then it may well prefer to cluster with the former. Note that this requires us to go beyond the large-$N$ limit, since as $N \to \infty$ it becomes vanishingly improbable that any antiquark other than the colour-connected one will have the right colour to form a singlet. In the real world of $N = 3$ we would expect the probability to be $1/9$, since even if the antiquark has the right colour, two out of the three same-colour combinations are octets rather than singlets.

A scenario of this type has been implemented in HERWIG, version 5.9 [6]. After parton showering and gluon splitting, at the start of the cluster formation phase, the program looks for switches of colour connections between clusters that would reduce the space-time extension of the clusters, as illustrated in fig. 3. If one is found, reconnection occurs with probability set by a parameter PRECO, default value $1/9$.

The implications of colour-singlet string and cluster models for the W mass determination were investigated during the CERN Workshop [1]. The effects of colour reconnection were generally found to be small. Compared with the equivalent model without reconnection between the two Ws, obtained for example by enhancing the W lifetime to prevent any space-time overlap between their decays, the mass shifts obtained were of the order of 100 MeV times the WW reconnection rate per event (typically 10-50%). These results are, however, sensitive to the way in which the W mass is deduced from the data, and it would certainly be of interest to repeat the model studies using the actual
methods now being employed in the LEP experiments.

Note that in all the above models there is a possibility of reconnections amongst the hadronization products of a single parton shower, not only between those from different boson decays. This is perhaps most natural in the cluster model, but in the string model a single decay can also give rise to multiple string segments or to a single string that intersects itself. Thus a retuning of the model parameters to agree with LEP1 data on single Z boson decays is really required when reconnection is introduced. This has not been done in detail, but there is no reason to suppose that it would enhance the mass shifts due to reconnection in WW events.

3.2 Non-singlet models

Although the formation of colour-singlet strings or clusters may appear the most plausible first stage of hadron production, there are good reasons to believe that this is not the whole story. One reason is that partons can still end up far away from their colour-connected partners after parton showering. In this case they may prefer to interact non-perturbatively with nearby partons of the wrong colour and form hadrons more indirectly.

Experimental evidence for an important non-singlet component of hadronization comes from the failure of singlet models to account for the data on production of heavy quarkonia. The scale for the formation of the heavy quarks themselves is set by the quark mass, whereas that for formation of the observed quarkonium states is set by the binding energy of the latter. There is thus a particularly large gap in this case between the scale at which parton showering effectively stops and that at which the final hadrons are formed. The hypothesis that the heavy quarks must be in a singlet state at the end of showering in order to bind greatly underestimates the amount of quarkonium production in hadron collisions [9, 10]. A large, indeed dominant, colour-octet component is also required [11]. There are also indications of an octet contribution.
to $J/\psi$ production in $Z^0$ decays [12, 13]. The octet states must then evolve by non-perturbative gluon emission into the observed singlets. In the case of quarkonium there is plenty of time for this to happen, owing to the scale discrepancy mentioned above. In the case of light-quark hadrons there is not a mismatch of scales, but the hadronization timescale is just as long, and so it would be surprising if a similar mechanism did not contribute at some level.

The only general hadronization model available at present which includes a non-singlet component is that of Ellis and Geiger [14], which is based on a transport-theoretical treatment of parton showering and cluster formation. A novel feature of the model is that it uses an effective Lagrangian containing both partonic and hadronic degrees of freedom to generate the parton shower. The two components have scale-dependent couplings that imply dominance of the partonic degrees of freedom at short distances and of the hadronic ones at long distances. As a consequence, whenever partons start to move too far away from their neighbours, cluster formation begins and prevents them from becoming widely separated, as required for colour confinement.

In [14], three scenarios for the mechanism of cluster formation are considered. The first two are, roughly speaking, colour-singlet models similar to that used in HERWIG, discussed above. Like the other singlet models, they imply relatively small shifts in the $W$ mass due to colour reconnection. However, the third scenario, which Ellis and Geiger call “colour-full”, includes non-singlet clustering: nearest-neighbour parton pairs in any colour combination are clustered if their separation exceeds a critical value. The net colour of the cluster, if any, is carried off by a secondary parton, and the process continues until all partons have been clustered. This scenario leads to much larger mass shifts, possibly as large as 400 MeV (in the positive direction).

In view of the big difference between the $W$ mass shifts in the colour-singlet models and the only available non-singlet model, and the experimental evidence for a non-singlet mechanism in quarkonium production, it is clearly important to investigate the predictions of the Ellis-Geiger colour-full scenario as fully as possible. First of all, as with the singlet models in which colour reconnection has been introduced as a new feature, one should tune the parameters of the model to achieve optimal agreement with LEP1 data. A point of particular interest would be to compare the predictions of this model\(^1\) and the singlet models with data on quarkonium production in both hadron-hadron and electron-positron collisions.

In addition, the Ellis-Geiger model predicts some striking reconnection effects in $WW$ events, besides the large $W$ mass shift [14]. In particular, a substantial reduction in the hadron multiplicity coming from the overlap of the hadronization regions of the two $W$s is expected. Since the overlap increases as the angle $\theta$ between the jet axes of the two $W$ decays decreases (see fig. 2), this

\(^1\)Predictions for quarkonium hadroproduction have become available since the Workshop [15].
implies a reduced multiplicity at small values of $\theta$, as shown fig. 4. The decrease is predicted to be mainly in the region of central rapidities relative to the thrust axis (fig. 5).

It is important to establish how well such an effect could be seen in the data, taking into account the fact that events with small values of $\theta$ tend to be removed by cuts designed to eliminate QCD background. Presumably the multiplicity in the overlap region falls because the enhanced parton density there leads to the production of lower-mass clusters, so that the hadron density does not rise proportionately. In non-singlet models, secondary parton emission from non-singlet clusters contributes to increasing the parton density, enhancing the effect. In the Ellis-Geiger model, it is noticeable that in single $Z^0$ decay the colour-full scenario has the highest parton and hadron densities in the central rapidity region, where most overlap will occur in WW events. Here again, however, it will be important to achieve good agreement with the LEP1 data before quantitative predictions for WW can be made.

A reduction of multiplicity at low rapidities in overlapping W decays would be expected to increase the measured value of the W mass in WW events, as observed in [14]. This is because the misassignment of wide-angle particles amongst jets, which tends to decrease the mass, will be less important if there are fewer such particles.

In colour-singlet models one would also expect a $\theta$ dependence of the multiplicity, although presumably much smaller and not necessarily of the same sign. Thus a thorough investigation of these effects would be worthwhile as a direct test of the models in WW events. In the HERWIG model discussed above, the amount of colour reconnection can be varied by changing the input parameter PRECO; setting this to unity would correspond to ignoring colour altogether dur-
Figure 5: Charged multiplicity and rapidity distributions.
ing cluster formation. This goes some way towards a non-singlet mechanism, although without the secondary parton emission of the Ellis-Geiger colour-full scenario. If an effect strongly correlated with a W mass shift were seen in a variety of models, its measured value could serve to calibrate the magnitude of the mass shift.

4 Modelling Bose-Einstein effects

Bose-Einstein correlations between identical bosons (in practice, pions) are also a potential source of a W mass shift in WW events. There are almost four times as many identical-pion pairs in the WW final state as in a single W decay, and so some kind of non-linear effect, possibly leading to a mass shift, would be expected. The problem with estimating such effects is that they are purely quantum mechanical in nature, whereas the Monte Carlo programs used to generate simulated events are based on classical models.

In fact it is not quite true that classical Monte Carlo programs cannot simulate purely quantum phenomena. Spin correlations that embody the Einstein-Podolsky-Rosen “paradox” can be simulated in full because the program does not have to respect causality: information can be propagated backwards in time as long as the program is not performing a real-time simulation. In this way acausal correlations can be incorporated [16]. However, the problem with simulating Bose-Einstein effects is not so much with causality as with the computational complexity of symmetrizing with respect to a very large number of variables. In response to this problem, several methods for grafting Bose-Einstein correlations onto existing event generators have been proposed. We deal briefly with each of them in turn below.

4.1 Redistribution method

The most developed technique is that proposed and incorporated into the JETSET generator by Sjöstrand [17, 18]. In this approach the momenta of identical final-state particles are redistributed (shifted) to reproduce the expected two-boson momentum correlations. The advantage of this method is that, since it involves no reweighting of events, it remains a true (unit-weight) event generator and does not suffer any loss of efficiency. The main disadvantage is that the momentum shifts spoil overall energy-momentum conservation and so one has to modify also some momenta of non-identical particles in order to compensate for this. In addition, Bose-Einstein correlations involving more than two particles are omitted. Nevertheless the approach is a convenient way of incorporating the two-particle correlations observed at LEP1 and investigating their possible consequences in WW events. It was found [18] that the implications for the W mass measurement could be quite severe.

A worrying feature of the current approaches to Bose-Einstein effects – the
redistribution method and the reweighting methods to be discussed below – is that they take little account of the space-time development of the final state. Identical bosons are supposed to originate from a common production region characterized by a few parameters, which cannot do justice to the complexity of the actual hadronization region, especially in WW final states. In reality, the effects of overlapping regions discussed above for colour reconnection will also be relevant for Bose-Einstein effects. Identical pions from different W decays would only be expected to interfere if there is significant overlap. The components of the two-pion correlation function due to same-W and different-W interference can be separated by comparing fully hadronic and semi-leptonic final states. At present there is no evidence of a different-W Bose-Einstein effect in the data [19]. Here again it would be most interesting to explore the correlation between such an effect and the W mass shift in a variety of models.

4.2 Reweighting methods

Another method for imposing Bose-Einstein correlations on classically simulated events is by weighting each event with a symmetrized weight function. In principle, all the multiparticle correlations could be fully included in this way. The problem is that, even if we knew how to compute the correct weight function, its calculation would be too laborious, involving a sum over all permutations of particles. In addition, the distribution of event weights would probably be so broad that the simulation would become hopelessly inefficient. This has led to the investigation of ‘partial symmetrization’ procedures that aim to include the most important permutations for each event. Two such procedures are discussed in more detail below.

4.2.1 Clustering.

One possible way of limiting the amount of computation is to organize the identical particles into clusters such that the significant weight factors are likely to be limited to permutations within clusters. The invariant measure of closeness in phase space, for two identical bosons \( i \) and \( j \) of mass \( m \), is

\[
Q_{ij}^2 = -(p_i - p_j)^2 = M_{ij}^2 - 4m^2. \tag{2}
\]

The Bose-Einstein weight will approach unity for large \( Q_{ij} \), and so one defines a cluster as a set of identical pions such that each member \( i \) has at least one neighbour \( j \) with \( Q_{ij} < Q_0 \). Here \( Q_0 \) is a parameter, ideally such that \( RQ_0 \gg 1 \) where \( R \) is the size of the source region. Then only permutations within clusters are considered in computing the Bose-Einstein weights.

This approach has been applied in [20]. A rather small value of the cluster cutoff, \( Q_0 = 0.2 \) GeV, was used, thereby limiting the typical cluster size and simplifying the weight calculation. It would be good to investigate the stability of the results with respect to variation of \( Q_0 \). The cluster weights were computed
using the model of [21]. To keep the total cross section and the mean value of the pion multiplicity $n$ unchanged, a further overall weight factor of $c\lambda^n$ was applied, with $c$ and $\lambda$ determined retrospectively. The conclusion of [20] is that the shift in the reconstructed W mass due to Bose-Einstein correlations is not more than 30 MeV.

4.2.2 Limited permutation.

Another possibility is to organize permutations into those of exactly 2, 3, 4, . . . identical particles, and then take into account only those up to some maximum number $K$ [22]. The validity of the truncation can be checked to some extent, by comparing the results for $K - 1$ and $K$. In [22], this method has been applied to minimum-bias proton-antiproton collisions; results are presented for $K = 4$ and 5, showing good stability. Once again the weights were rescaled by $c\lambda^n$ to keep the total cross section and mean pion multiplicity unchanged. Clearly it would be interesting to apply this method to WW final states.

4.3 Other methods

An important point to bear in mind is that much work on Bose-Einstein correlations has also been undertaken in the context of nuclear physics, and we may be able to borrow some ideas from that field. For example, in [23] it is proposed to treat the classical space-time points of origin of pions in Monte Carlo simulations of heavy ion collisions as sources of Gaussian wave packets. The two-particle correlation function can then be computed, with the width of the packets as a free parameter. Such an approach looks feasible for WW events, and it might meet the objection that existing models do not take adequate account of the space-time structure of the process.

Another interesting recent development is a treatment of Bose-Einstein correlations which emerges naturally from the Lund string model of hadronization [24]. Here again the space-time structure is included and therefore a more satisfactory treatment of WW final states may be possible.

5 Summary

Unfortunately, the possible effects of colour reconnection and Bose-Einstein correlations on the reconstructed W mass are still uncertain. The necessity for a large colour-octet contribution to quarkonium production suggests that one should take seriously the possibility of a non-singlet hadronization mechanism for light quarks as well, which could lead to a larger mass shift than is currently estimated using colour-singlet models. As for Bose-Einstein correlations, studies done so far using the reweighting method suggest smaller mass shifts than
those found with the redistribution method, although further work is needed to confirm this\textsuperscript{2}.

Some topics on which further work would appear worthwhile are:

- To update the CERN Workshop studies of colour reconnection effects in various models, using the methods now being employed to extract the value of the W mass.
- To tune the parameters of models which allow reconnection in single $Z^0$ decays (especially the Ellis-Geiger model), to optimize agreement with LEP1 data.
- To add a non-singlet hadronization component to singlet models, and to investigate how it affects the W mass determination.
- To study quarkonium production in singlet hadronization models, to see whether a non-singlet contribution is essential.
- To establish whether a reduction in multiplicity at small values of the WW relative decay angle $\theta$ could be seen in the data, after cuts to remove QCD background.
- To study the same effect in a variety of models and see whether any drop in multiplicity at small $\theta$ is correlated with a W mass shift.
- To look for a correlation between a different-W Bose-Einstein effect and a W mass shift in various models.
- To apply the limited-permutation approximation for Bose-Einstein weights to WW final states.
- To compute Bose-Einstein correlations in WW final states by using the space-time configurations generated by existing Monte Carlo programs as sources of Gaussian wave packets.

Perhaps the most useful outcome would be to establish a convincing correlation between some measurable effect and the W mass shift, which would help us to estimate the latter in a slightly less model-dependent way. For this purpose the multiplicity at small $\theta$ and the different-W Bose-Einstein effect look promising.

\textbf{References}


\textsuperscript{2}A comparative study of reweighting schemes has been completed recently [25]; some results are presented later in these Proceedings [26].


[26] Report of W mass working group